

Name:.....

Student Number:

Teacher's Name:.....



Pymble Ladies' College

Mathematics Advanced HSC Trial Examination Term 3 2025

**General
Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning or calculations

**Total marks
100****Section 1 – 10 marks** (pages 1-4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7-31)

- Attempt Questions 11-31
- Allow about 2 hours and 45 minutes for this section

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SECTION I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the Multiple-Choice Answer sheet for Questions 1-10

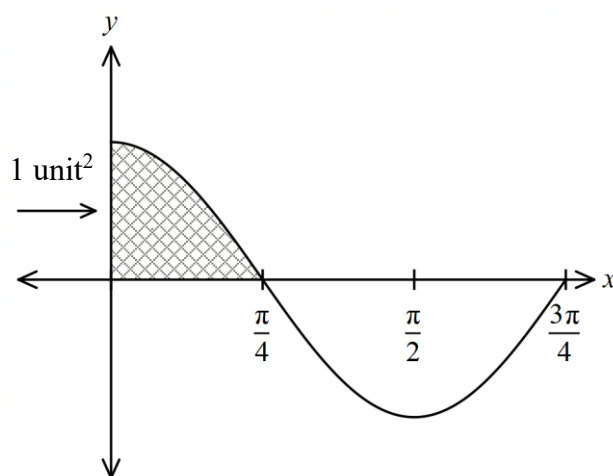
- 1 Which expression is equivalent to $\left(\frac{2x}{5}\right)^{-3}$?
- (A) $\frac{8x^3}{125}$
- (B) $\frac{125x^3}{8}$
- (C) $\frac{125}{8x^3}$
- (D) $\frac{8}{125x^3}$
- 2 A curve with equation $y = f(x)$ passes through the point $(3, 7)$. Given $f'(3) = 0$, what is the equation of the normal to the curve at $(3, 7)$?
- (A) $y = \frac{7}{3}x$
- (B) $y = 0$
- (C) $x = 3$
- (D) $x = 7$
- 3 A and B are mutually exclusive events. Which of the following statements must be correct?
- (A) $P(A \cup B) = P(A) \times P(B)$
- (B) $P(A \cup B) = P(A) - P(B)$
- (C) $P(A \cap B) = 0$
- (D) $P(A \cap B) = 1$

- 4 What is the domain of the function $f(x) = \frac{1}{\sqrt{x^3 - 1}}$?
- (A) $x \in [-1, 1]$
- (B) $x \in (-\infty, -1] \cup [1, \infty)$
- (C) $x \in (-1, 1)$
- (D) $x \in (1, \infty)$
- 5 In a group of 100 students, 35 play basketball, 38 play hockey and 30 do not play either basketball or hockey.
How many students play both basketball and hockey?
- (A) 70
- (B) 3
- (C) 27
- (D) 33
- 6 If $y = \sqrt{x^2 + 8}$ what is an expression for $\frac{dy}{dx}$?
- (A) $\frac{dy}{dx} = \sqrt{2x}$
- (B) $\frac{dy}{dx} = \frac{x}{2\sqrt{x^2 + 8}}$
- (C) $\frac{dy}{dx} = \frac{2x}{\sqrt{x^2 + 8}}$
- (D) $\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 8}}$

- 7 A and B are two independent events such that $P(\bar{A} \cap B) = 0.4$ and $P(B) = 0.5$.
What is the value of $P(A)$?

- (A) 0.1
- (B) 0.2
- (C) 0.3
- (D) 0.4

- 8 The diagram shows part of the graph $f(x) = a \cos bx$.



The area of the shaded region is equal to 1 unit². What is the value of $\int_0^{\frac{3\pi}{4}} f(x) dx$?

- (A) -2
- (B) -1
- (C) 1
- (D) 3

- 9 The graph of $y = \frac{5}{x+2}$ is translated 4 units to the right then dilated vertically by a factor of $\frac{1}{2}$.

Which of the following gives the equation of the new function?

(A) $\frac{y}{2} = \frac{5}{x-2}$

(B) $2y = \frac{5}{x-4}$

(C) $2y = \frac{5}{x-2}$

(D) $\frac{y}{2} = \frac{5}{x-4}$

- 10 Let $h(x) = f(g(x))$, with $f(4) = 1$, $f'(4) = 6$, $g(2) = 4$ and $g'(2) = 4$.

What is the equation of the tangent to the graph of $y = h(x)$ at $x = 2$?

(A) $y = x - 6$

(B) $y = 4x - 2$

(C) $y = 6x - 11$

(D) $y = 24x - 47$

End of Section I

Name.....

Student
Number

Teacher's Name.....

Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11-31

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer in the spaces provided. These spaces provide guidance for the expected length of the answer.
- Extra writing space is provided at the end of the paper. If you use the additional space, clearly indicate which question you are answering.
- Your responses should include relevant mathematical reasoning and/or calculations.

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Question 11 (5 marks)

Phoebe and Sophie's bakery sells bread rolls for \$0.80 each. The cost, C dollars, of making n bread rolls is given by $C = 300 + 0.2n$.

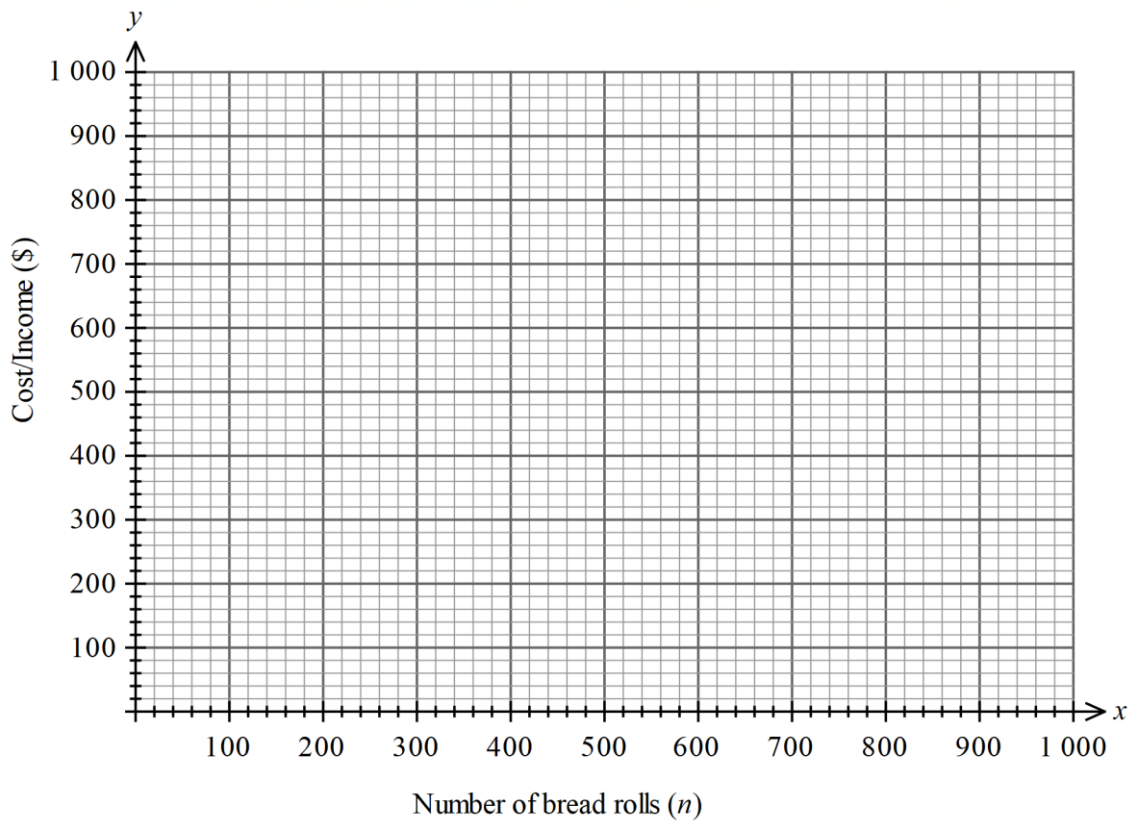
- (a) How much does every 100 bread rolls made add to the cost of production?

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- (b) On the grid below, draw the graphs of the cost, C , and the income, I .

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Hence, find the number of bread rolls that must be sold to break even.

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- (c) How many bread rolls must be sold to make a profit of \$240?

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Question 12 (3 marks)

Solve the equation

$$2\log_{10} x = \log_{10} 4 + \log_{10} (x + 8).$$

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Question 13 (3 marks)

The curve $y = x^2 - \frac{a}{x}$ has a stationary point at $(-3, b)$.

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Find the values of the constants a and b .

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Question 14 (3 marks)

Solve $\tan x + 2 \sin x = 0$, for $x \in [0, 2\pi]$.

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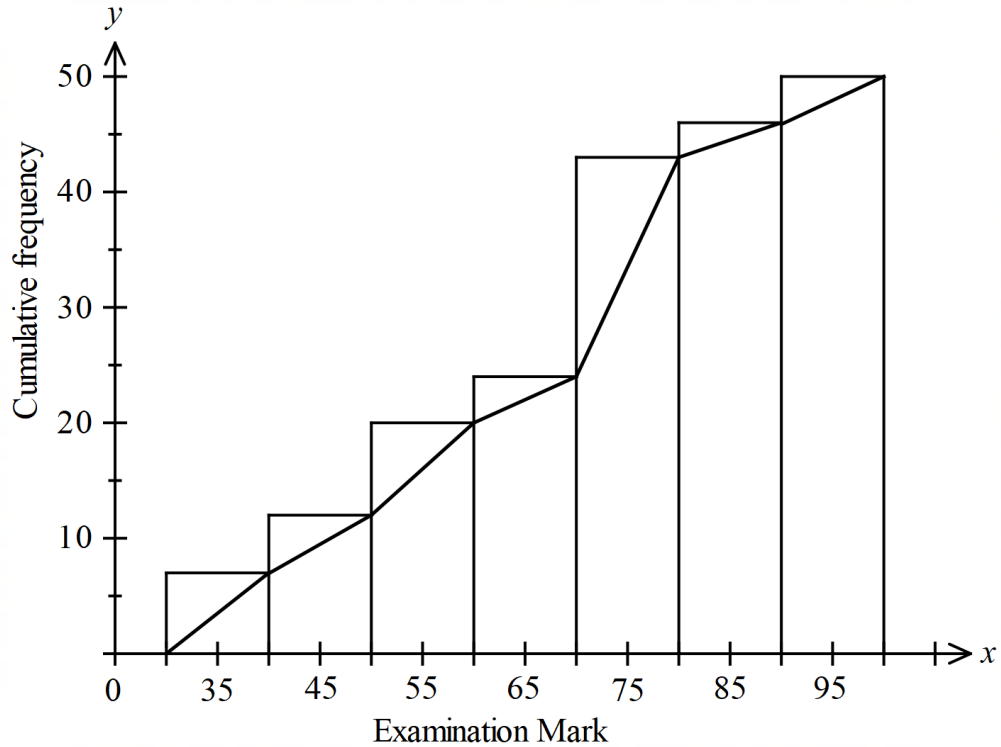
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Question 15 (3 marks)

A set of examination results is displayed in a cumulative frequency histogram and polygon (ogive). The marks have been sorted into groups and the marks given on the diagram are class centres.



- (a) Find the median examination mark.

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- (b) Estimate the percentage of students that achieved a mark below 45.

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- (c) A student knows that their mark is between the 6th and 7th decile.
 Give a possible examination mark for the student.

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Question 16 (4 marks)

- (a) Given that $y = x\sqrt{x}$, find $\frac{dy}{dx}$.1

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- (b) The line, L, has equation $6x - 2y + 5 = 0$. L is a tangent to the curve with equation $y = x\sqrt{x} + k$, where k is a constant.3

Find the value of k .

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Question 17 (4 marks)

Jessica has two types of coins in her money box. 20% of the coins are gold and 80% are silver. Jessica takes a coin from the money box, notes it's type and replaces it. She then takes a second coin and repeats the process.

- (a) Find the probability that both coins are the same type. 2

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- (b) Find the probability that both coins are gold, given that at least one of the coins is gold. 2

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Question 18 (6 marks)

A random sample of eight cars was selected from a large data set. The masses of these cars, in kilograms, were as follows:

950, 989, 1247, 1415, 1506, 1680, 1833, 2040

It is given that for the population of cars in the large data set, the lower quartile = 1167 kg, the median = 1393 kg and the upper quartile = 1570 kg.

- (a) Are any of the cars from the sample of eight outliers?
Justify your answer mathematically.
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The table shows the probability distribution of the number of previous owners, N , for a sample of cars taken from the large data set.

n	0	1	2	3	4	5	6 or more
$P(N = n)$	0.14	0.37	$0.9k$	0.25	$0.4k$	$1.7k$	0

- (b) Find the value of k .
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- (c) Hence, find the value of $P(1 \leq N < 5)$.
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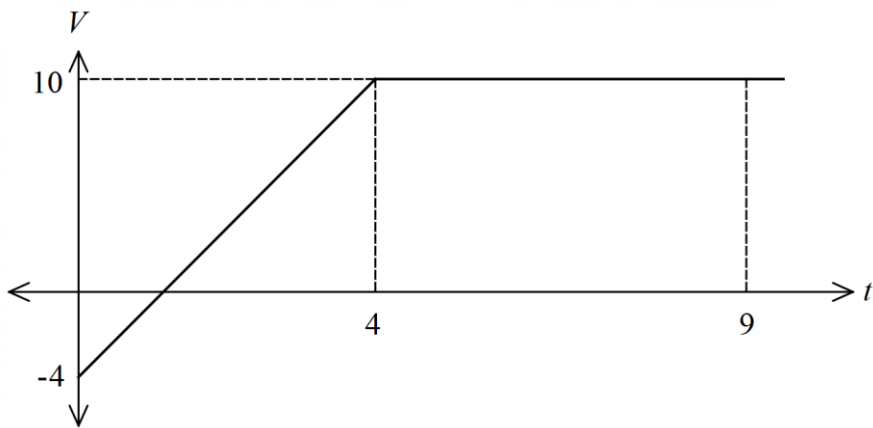
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Question 19 (3 marks)

A particle is moving in a straight line such that it's velocity, V m/s, changes with respect to time, t seconds, as shown in the graph below.



- (a) What is the acceleration of the particle over the first 4 seconds? 1

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- (b) The particle is initially at a fixed point P . What is the displacement of the particle from P when $t = 9$? 2

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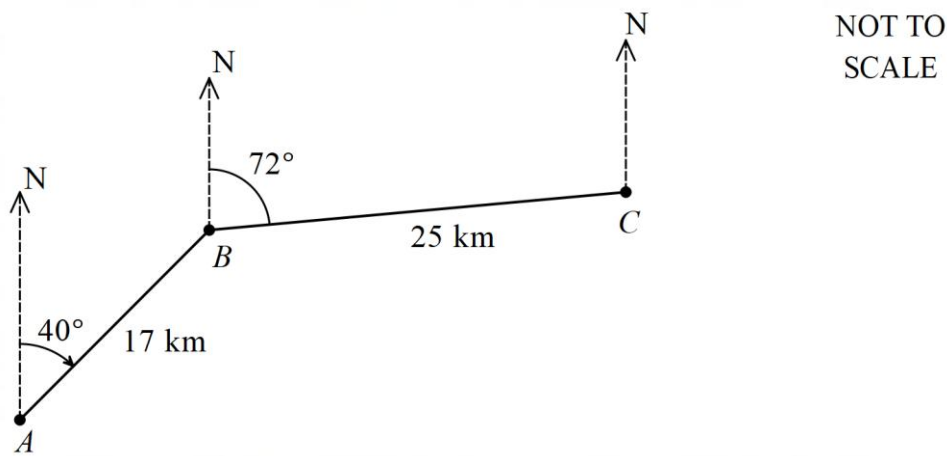
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Question 20 (6 marks)

The positions of three towns A , B and C are shown below.



Town A and Town B are 17 km apart. Town B is on a bearing of 040° from Town A .
Town B and Town C are 25 km apart. Town C is on a bearing of 072° from Town B .

- (a) Find the size of the obtuse angle ABC . 1

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- (b) What is the distance between Town A and Town C , correct to the nearest kilometre? 2

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- (c) What is the bearing of Town A from Town C ? 3

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Question 21 (6 marks)

Zarina buys a car for \$18 000 on 1st January 2023. The value of the car decreases exponentially. The car has a value of \$12 000 on 1st January 2025.

Zarina says “because the car has lost \$6000 after 2 years, after another 2 years, it will be worth \$6000”.

Zarina’s friend Sarah says “because the car has lost one-third of its value after 2 years, after another two years it will be worth \$8000”.

- (a) Explain whose statement is correct, using mathematical reasoning.

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- (b) The value of Zarina’s car \$ V , t years after 1st January 2023 may be modelled by the equation

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$$V = Ae^{-kt} \text{ where } A \text{ and } k \text{ are positive constants.}$$

Find the value of t when the car has a value of \$10 000, giving your answer correct to 2 significant figures.

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Question 21 continues on page 17

Question 21 continued.

- (c) Give a reason why the model will not be suitable to calculate the value of her car when $t = 30$.

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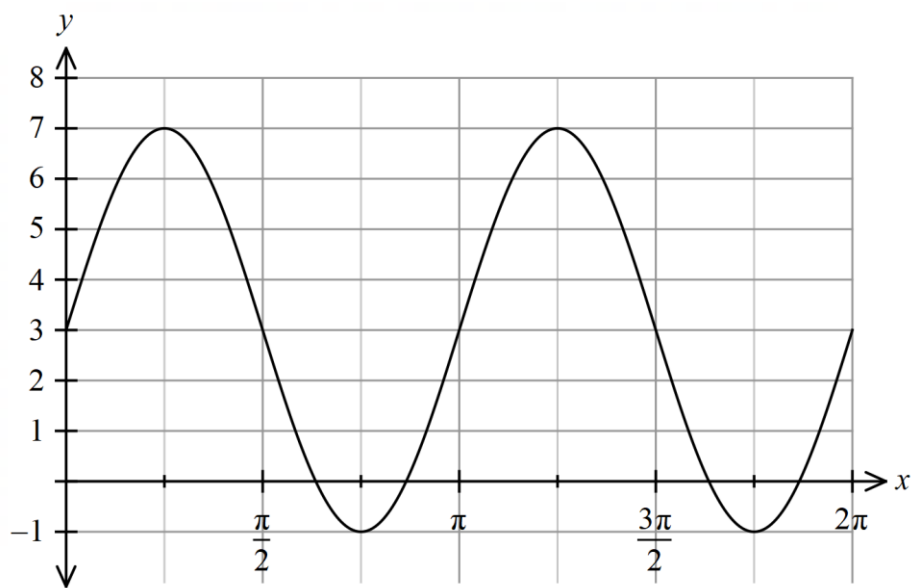
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Question 22 (4 marks)



The diagram shows the curve with equation $y = a \sin(bx) + c$ for $0 \leq x \leq 2\pi$, where a , b and c are positive constants.

- (a) State the values of a , b and c . 3

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- (b) For these values of a , b and c , determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for $a \sin(bx) + c = 7 - x$. 1

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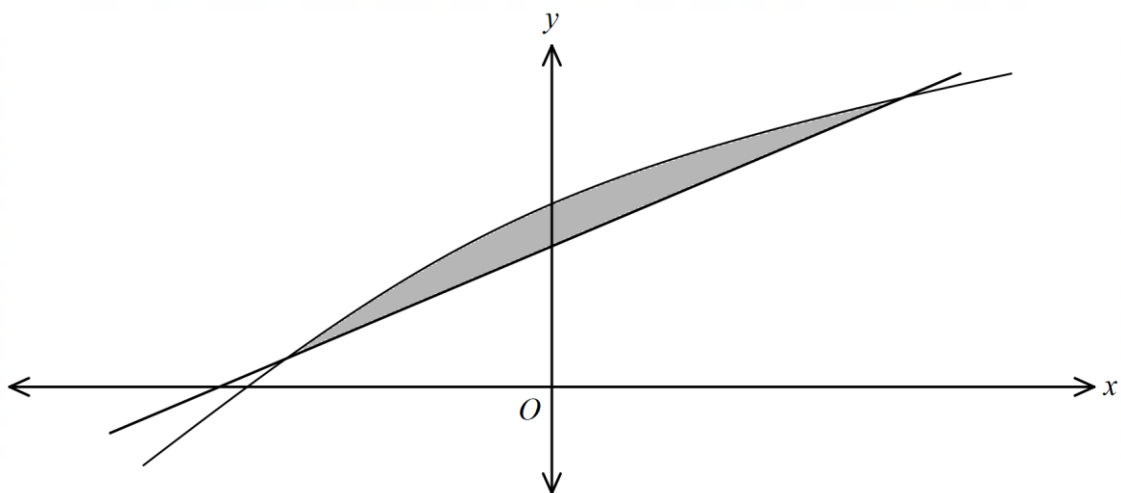
Question 23 (4 marks)

The first term of an arithmetic series is -20 and the common difference is 5 .

- (a) Find the sum of the first 20 terms of the progression.1

- (b) It is given that the sum of the first $2k$ terms is ten times the sum of the first k terms. Find the value of k .3

Question 24 (4 marks)



The diagram shows part of the curve with equation $y = -2x^2 + 8x + 11$ and the line with equation $y = 8x + 9$.

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Find the area of the shaded region.

[illegible]

Question 25 (3 marks)

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The shoe size of women in a town is normally distributed with a mean size of 8 and a standard deviation of 1.
A shoe shop in the town sells women’s shoes ranging from size 5 to size 10.
Given that there are 4000 women in the town, how many would be expected to find shoes that fit them in this shop?

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Question 26 (5 marks)

A continuous random variable, x , has the following probability density function

$$f(x) = \begin{cases} \frac{A}{\sqrt{3x+1}} & 1 \leq x \leq 5 \text{ (where } A > 0) \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the value of A is $\frac{3}{4}$.3

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Question 26 continues on page 23

Question 26 continued

- (b) Given that $P(X < C) = 3P(X > C)$, find the value of C .

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[illegible]

Question 27 (7 marks)

Consider the curve $y = (x + 2)^2(x - 4)$.

- (a) Find the stationary points and determine their nature.

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- (b) Show that $(0, -16)$ is a point of inflexion.

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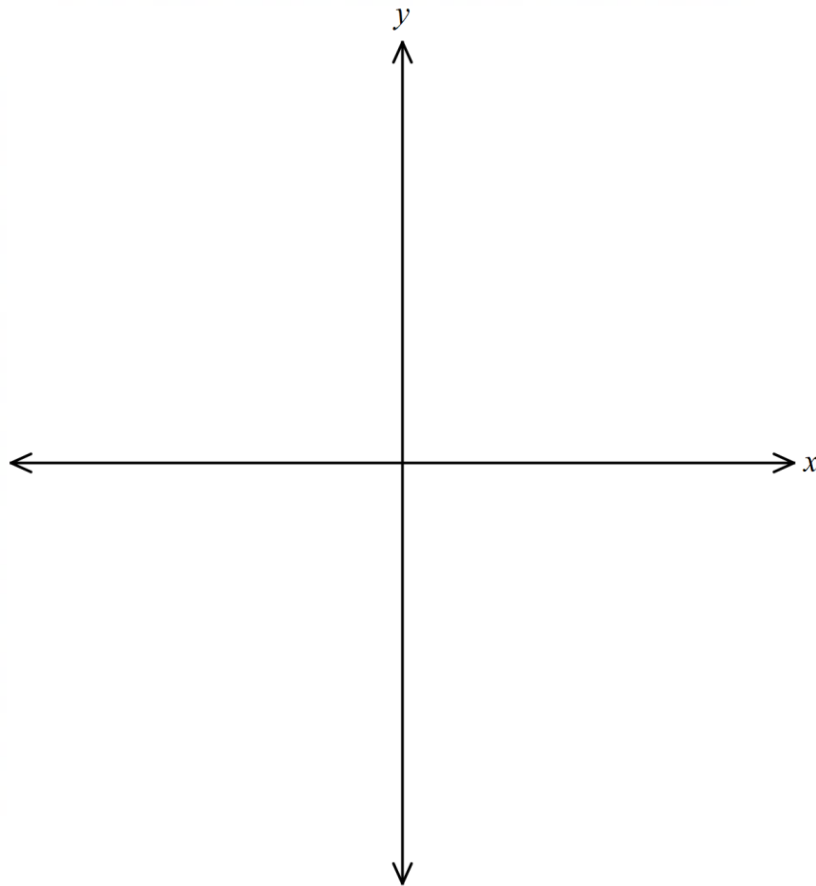
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Question 27 continued.

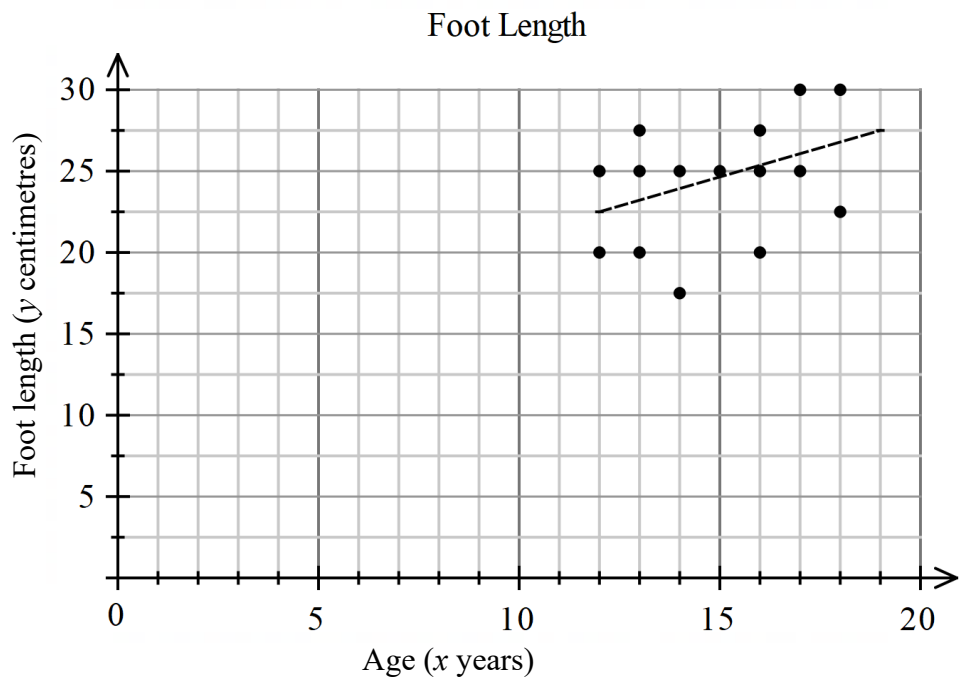
(c) Sketch the curve showing all relevant features.

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Question 28 (4 marks)

For a sample of 15 high school students, their age in years, x , and the length of their right foot in centimetres, y , were recorded.



The graph shows the data as well as a regression line which passes through (12, 22.5) and (19, 27.5).

- (a) Find the equation of the regression line in the form $y = mx + c$.

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Question 28 continues on page 27

Question 28 continued

- (b) Lisa has a five-year old brother whose right foot is 13 centimetres in length. 2

She extends the line of best fit and notes that it predicts that a five-year old should have a foot length of approximately 17.5 centimetres.

Give two reasons why Lisa is incorrect to assume from the bivariate data that her brother has smaller than normal feet for a five-year old.

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Question 29 (2 marks)

Use two applications of the trapezoidal rule to find an approximate value of 2

$$\int_1^2 \ln x \, dx .$$

Give your answer correct to 2 decimal places.

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Question 30 (6 marks)

(a) Show that $\int_1^a \left(6 - \frac{12}{\sqrt{x}} \right) dx = 6a - 24\sqrt{a} + 18$.

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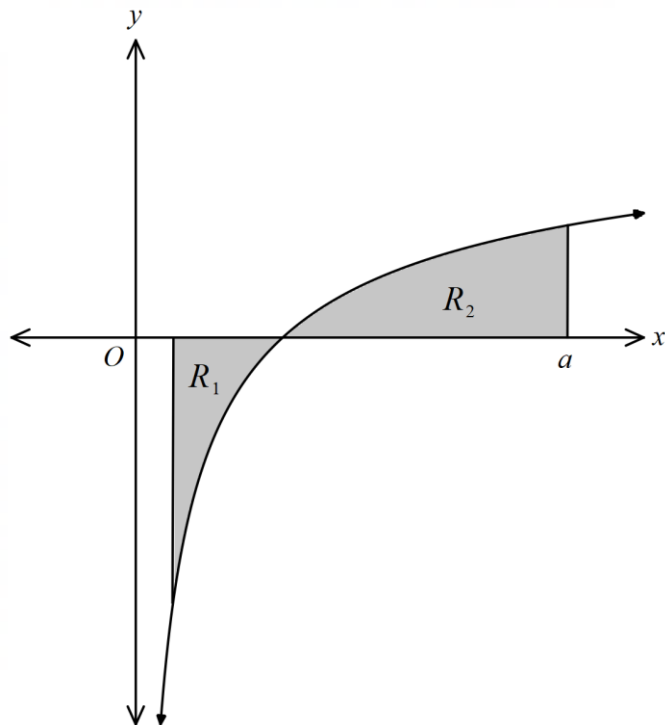
(b) The curve, $y = 6 - \frac{12}{\sqrt{x}}$, the line $x = 1$ and the line $x = a$ are shown on the diagram

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below.

The shaded region R_1 is bounded by the curve, the line $x = 1$ and the x -axis.

The shaded region R_2 is bounded by the curve, the line $x = a$ and the x -axis.



Question 30 continues on page 29

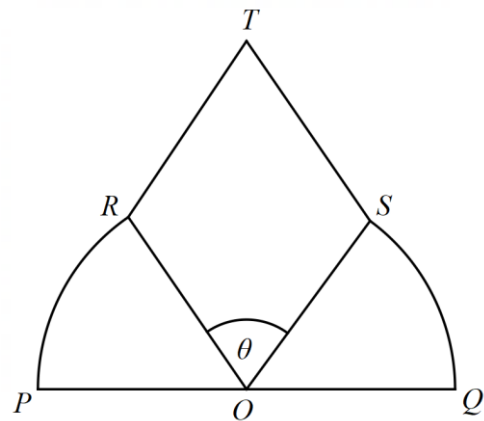
Part (b)

Find the value of a . Justify your answer mathematically.

[illegible]

Question 31 (5 marks)

A new design for a company logo is to be made from two equal sectors of a circle, ORP and OQS , and a rhombus $OSTR$, as shown in the diagram below.



The points P , O and Q lie on a straight line and the angle ROS is θ radians.
A large copy of the logo, with $PQ = 5$ metres, is to be put on a wall.

- (a) Show that the area of the logo, A square metres, is given by

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$$A = \frac{25}{8}(\pi - \theta + 2 \sin \theta)$$

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Question 31 continued on page 31

Question 31 continued

- (b) What value of θ will produce a maximum area for the logo.
Fully justify your answer.

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NSW Education Standards Authority

HIGHER SCHOOL CERTIFICATE

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

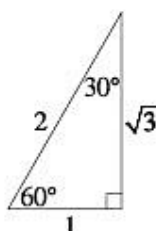
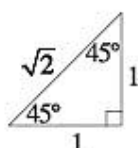
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

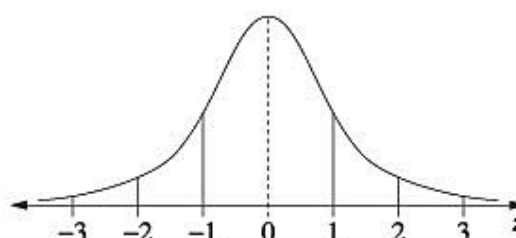
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x_1\underline{i} + y_1\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$