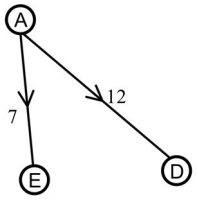
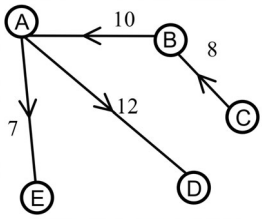
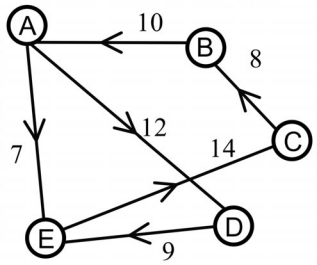
Western Mathematics Exams

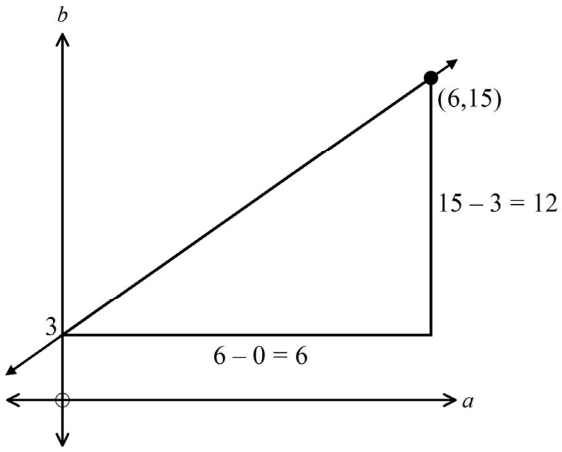
Mathematics
Standard 2

SOLUTIONS

Multiple Choice Worked Solutions

No	Working	Answer
1	The shape is a sector and we want its area, so from the formula sheet the formula is $A = \frac{\theta}{360} \times \pi r^2$. The radius, $r = 24$ with an angle of 60^o, so the calculation is $A = \frac{60}{360} \times \pi \times 24^2$	C
2	Unordered selections of 3 from 10 $= \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$ $= 120$	A
3	Taxable income = 69000 – 1255 = 67745 Income Tax = 3572 + (67745 – 37000) × 0.325 = 3572 + 30745 × 0.325 = 3572 + 9992.125 = \$13 564.13 (nearest cent)	C
4	A goes to D(12) and E(7) B goes to A(10) and C to B(8)   D goes to E (9) and E goes to C(14)  	D
5	$3ab - 3a(2a - 4b) - a^2 = 3ab - 6a^2 + 12ab - a^2$ $= 15ab - 7a^2$	D

6	$\tan \angle WTK = \frac{230}{320}$ $\angle WTK = \tan^{-1} \frac{230}{320}$ $= 36^\circ$ Bearing $= 270 - 36$ $= 234^\circ$	**C**
7	Looking down the interest column, the interest amount increased in June. Int Rate May = \$1,242.95 ÷ \$248,589.46 = 0.0050 (4 dec places) Int Rate June = \$1,342.43 ÷ \$247,832.41 = 0.0054 (4 dec places) Increase in annual interest rate = $0.0004 \times 12 \times 100 = 0.5\%$ (1 dec place)	**B**
8	One variable decreases as the other increases, so the correlation will be negative. The data is grouped close to the line, so the value will be close to -1. So -0.95 is the most likely.	**A**
9	Quarterly interest rate =0.12 ÷ 4 =0.03 After first year the investment= $1450 \times (1.03)^4$ = \$1 631.99 After she added \$1 280 value = \$1 631.99 + \$1 280 = \$2 911.99 After second year the investment = \$2 911.99 $\times (1.03)^4$ = \$3 277.47	**C**
10	Area of the cloth = $30^2 = 900$ cm^2. An area of 1 m^2 = $100 \times 100 = 10\,000$ cm^2 Area of cloth = $\frac{900}{10000} = 0.09\ m^2$ Amount absorbed = $0.09 \times 5 = 0.45$ L = 450 mL	**B**
11	Under the declining balance method, each succeeding years price is a percentage of the previous year's price, which means the value goes down by gradually lesser amounts each year. Graph B indicates this. Graph C is a straight line depreciation and A and D are erratic amounts.	**B**
12	80% as a z score = $\frac{80 - 90}{5} = -2$.	**A**
13	Both C and D have the correct shape for an exponential with a negative in the index, but it should have a y intercept of 1, so only D is correct.	**D**

14	 $m = \frac{12}{6} = 2$ y int = 3 Equation: $b = 2a + 3$	**A**
15	Network B is not spanning as it does not include vertex L Networks A and C include all of the vertices, but also include a cycle or a loop. Only network D, is a spanning tree.	**D**

2019 Trial Higher School Certificate Examination
Mathematics Standard 2

Name _____ Teacher _____

Section I – Multiple Choice Answer Sheet

Allow about 25 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1.	A ○	B ○	C ●	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ○	C ○	D ●
5.	A ○	B ○	C ○	D ●
6.	A ○	B ○	C ●	D ○
7.	A ○	B ●	C ○	D ○
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ○	B ●	C ○	D ○
11.	A ○	B ●	C ○	D ○
12.	A ●	B ○	C ○	D ○
13.	A ○	B ○	C ○	D ●
14.	A ●	B ○	C ○	D ○
15.	A ○	B ○	C ○	D ●

	Solution	Marks
Question 16		
(a)	The method will not provide a random sample and is likely to be biased based on the common background and interests of Barbara's friends.	1
(b)	The data is categorical and ordinal.	1
Question 17		
	(i) Stamp duty = 3% of 45 000 + 5% of 15 000 = .03 × 45 000 + .05 × 15 000 = 1 350 + 750 = \$2 100 Cost of car = 60 000 + 2 100 = \$62 100	2
Question 18		
(a)	One has been played from the three by 5*SOS* leaving 2 One track has been played from the 12 leaving 11 Missing probability is $\frac{2}{11}$	1
(b)	$P(Same) = = \frac{5}{12} \times \frac{4}{11} + \frac{1}{3} \times \frac{3}{11} + \frac{1}{4} \times \frac{2}{11}$ $= \frac{19}{66}$	1
(c)	$P(Different) = 1 - P(\ same)$ $= 1 - \frac{19}{66}$ $= \frac{47}{66}$	1
Question 19		
	50 km at 15 litres/100 km = 15 ÷ 2 = 7.5 litres 250 km at 10 litres/100 km = 10 × 2.5 = 25 litres Total = 25 + 7.5 = 32.5 litres	2
Question 20		
	(ii) Convert to z-scores. For a diameter of 6.4 $z = \frac{6.4 - 6.3}{0.05}$ $= 2$ For a diameter of 6.25 $z = \frac{6.25 - 6.3}{0.05}$ $= -1$ Approximately 68% of scores have z-scores between –1 and 1 Percentage of z-scores less than -1 $= \frac{100 - 68}{2} = 16\%$ Approximately 95% of scores have z-scores between –2 and 2 Percentage of z scores more than 2 $= \frac{100 - 95}{2} = 2.5\%$ Total percentage rejected = 16 + 2.5 = 18.5%	2

Question 21

Absolute Error = half of the smallest unit (0.5 cm) = 0.25 cm

$$\% \text{ error } = \pm \frac{0.25}{12.5} \times 100$$

$$= \pm 2\%$$

2

Question 22

Hourly earnings = $35 \times 19.5 = \$682.50$

Commission = $0.04 \times 16000 = \$640$

Total Earnings = $\$ 682.50 + \$640 = \$ 1322.50$

2

Question 23

(a) Mean = 42.5 — 1

(b) Standard deviation = 15.4 — 1

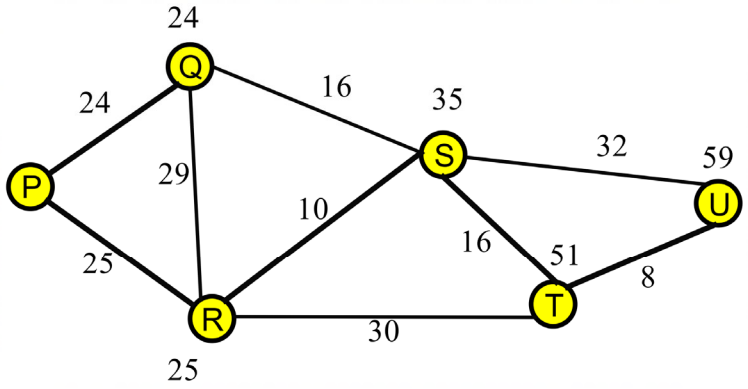
Question 24

(a) Time difference = 10 + 5 = 15 hours
So Sydney is 15 hours ahead. — 1

(b) Sydney is 15 hours ahead of 6 am Wednesday 7th.
6 hours ahead of 6 am is 12 pm Wed.
9 hours more ahead is **9 pm on Wednesday 7th.** — 1

(c) The time she leaves New York (Sydney time) is 9 pm Wednesday.
Time for flight is 20 hours, so 3 hours to midnight and 17 hours after midnight so arrives at 5 pm Thursday 8th. — 1

Question 25



Starting at P, shortest to R is 25 and to Q is 24, shortest to S is through R
25+10 = 35 then to T is through S 35+16 = 51 and hence to U is through T
51 + 8 = 59
So shortest time is 59 minutes going from P to R, to S, to T to U
Or abbreviated to PRSTU. — 3

Question 26

The heater is 2 000 watts which is 2 kw.
It is run for $120 \times 8 = 960$ hours per year
Power used = $960 \times 2 = 1920$ kwh
Cost at \$0.30 per kwh = $1920 \times 0.3 = \$576$ — 2

Question 27

(a) 1

$$B = 2\pi\left(R + \frac{T}{2}\right) \times \frac{A}{360}$$

$$B = 2\pi\left(60 + \frac{90}{2}\right) \times \frac{120}{360}$$

$$= 2\pi(60 + 45) \times \frac{120}{360}$$

$$= 2\pi(105) \times \frac{1}{3}$$

$$= 70\pi$$

$$= 219.9114$$

$$= 220 \text{ (nearest whole number)}$$

(b) 2

$$B = 2\pi\left(R + \frac{T}{2}\right) \times \frac{A}{360}$$

$$55 = 2\pi\left(20 + \frac{30}{2}\right) \times \frac{A}{360}$$

$$55 = 2\pi(35) \times \frac{A}{360}$$

$$55 = 220 \times \frac{A}{360}$$

$$A = 55 \times \frac{360}{220}$$

$$A = 90$$

Question 28

(a) 1

(i) 6.6% pa % $= \frac{6.6}{12} = 0.55\%$ per month.

From table interest factor for 0.55% for 12 years (120 months) is 169.3267.

Future value $= 450 \times 169.3267 = \$76\ 197$ (nearest dollar)

(b) 1

(ii) 9.0% pa % $= \frac{9.0}{12} = 0.75\%$ per month.

Let I be the required interest factor

Then $500 \times I = \$100\ 000$

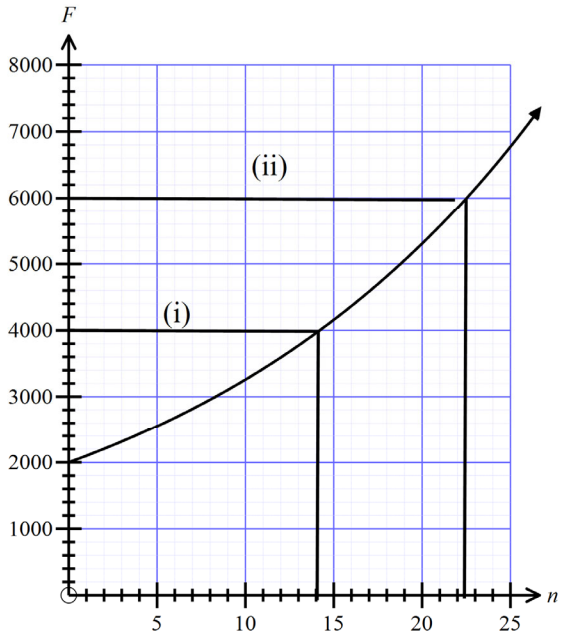
$$I = \frac{100000}{500} = 200$$

From table, in the column for 0.75% the first interest factor to exceed 200 is for 123 months (200.9236),

So the minimum term is 10 years and 3 months

Question 29

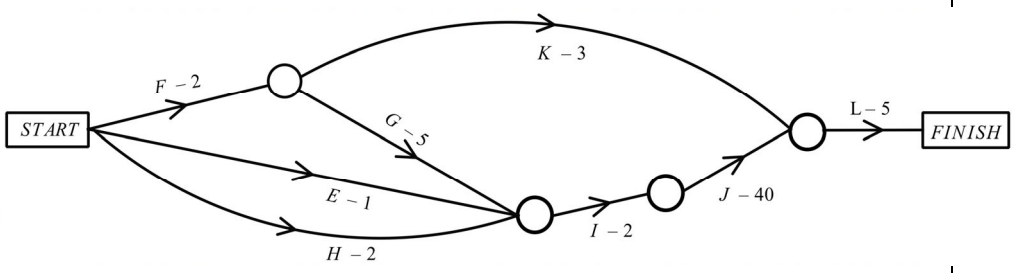
(a) (i)



It takes 14 years (from the graph). 1

(b) (ii) The time to triple is about 22.5 years, compared to 14 years to double, so it is increasing at an increasing rate, or it is increasing exponentially. 2

Question 30



2

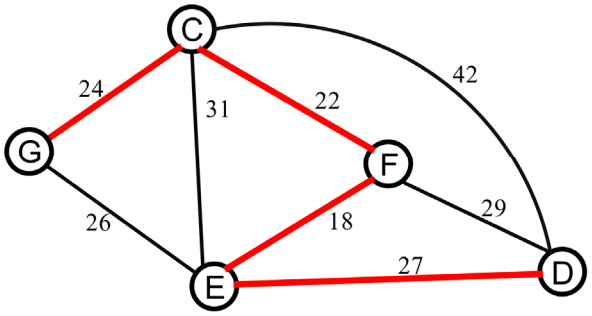
Question 31

Monthly repayment =\$193.33 from table.
There are $5 \times 12 = 60$ repayments.
Amount paid = $\$193.33 \times 60 = \$11\,599.80$ 2

Question 32

(a) A minimum spanning tree — 1

(b) Using Prims Algorithm — 2



Select C as the first vertex (arbitrary as any other could be chosen)
Highlight CF as lowest weight leaving C
Highlight FE as lowest weight leaving the tree.
Highlight CG as lowest weight leaving the tree.
Highlight ED as lowest weight leaving the tree not creating a cycle.
Every vertex is now included, so we have our minimum spanning tree
Length of road to re-seal = 24 + 22 + 18 + 27 = **91 km**

Question 33

2

$$\text{Dosage for child 1-12} = \frac{\text{age of child (in years)} \times \text{adult dose}}{\text{age of child (in years)} + 12}$$

$$= \frac{8 \times 15}{8 + 12}$$

$$= \frac{120}{20}$$

$$= 6 \text{ mL}$$

Question 34

2

$$\text{area of cross section} = \pi \times 3^2 = 28.3\ m^2$$

$$\text{volume} = 28.3 \times 16 = 452.4\ m^3$$

$$\textit{capacity} = 452.4 \text{ kL}$$

$$\text{Number of 150 kL tanks} = \frac{452.4}{150} = 3.015$$

3 tanks could be filled.

Question 35

(a) 1

From 19 calls the median is the 10th which is a 3
From lower 9 calls Q_L is 5th which is a 2.
From upper 9 calls Q_U is 5th which is a 5.
Interquartile range = 5 – 2 = 3.

(b) 1

A high score is an outlier if it is greater than
$Q_3 + 1.5$
So 12 is an outlier.

Question 36

Measurement = 2.7cm from Parkes to Dubbo (answers from 2.5 to 2.7)
1cm = 25km
So distance =2.7 x 25km = 67.5 radius (62.5 for 2.5 cm)

$A = \pi r^2$

$= \pi \times 67.5^2$

$= 14314 \text{ km}^2$

(12272 if using 2.5)

2
Allow for different answers in measuring, due to printing differences,

Question 37

(a) (i) Passes through (40, 480) and the origin, so

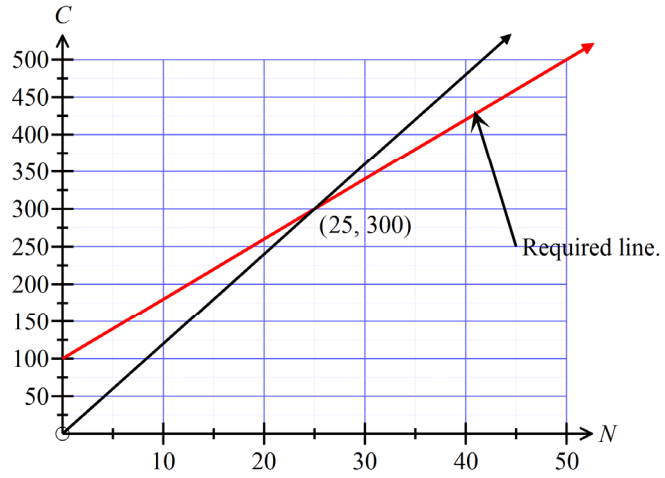
$$Gradient = m = \frac{480}{40} = 12$$

C intercept = 0

Equation is $C = 12N$ — 1

(b) (ii) $Gradient = 8$ and C intercept is 100.

Plot the intercept and use gradient to obtian a second point. — 2



(c) (iii) Break even point is (25, 300) which means that when 25 widgets are sold the income equals the expenses, so if less than 25 widgets are sold, a loss is made, but for more, a profit is made. — 1

Question 38

(a) Total residents = 100

P(no prob and male) $= \frac{35}{100} = \frac{7}{20}$ — 1

(b) Total females = 49

P(no prob given female) $= \frac{25}{49}$ — 1

Question 39

(a) $BAC_{FEMALE} = \dfrac{10N - 7.5H}{5.5M}$

$= \dfrac{10 \times 4 - 7.5 \times 2.5}{5.5 \times 45}$

$= 0.08585$

$= 0.086$ (2 d p)

Which is more than 0.08 — 1

(b) $\text{Time} = \dfrac{BAC}{0.015}$

$= \dfrac{0.086}{0.015}$

$= 5.73333$

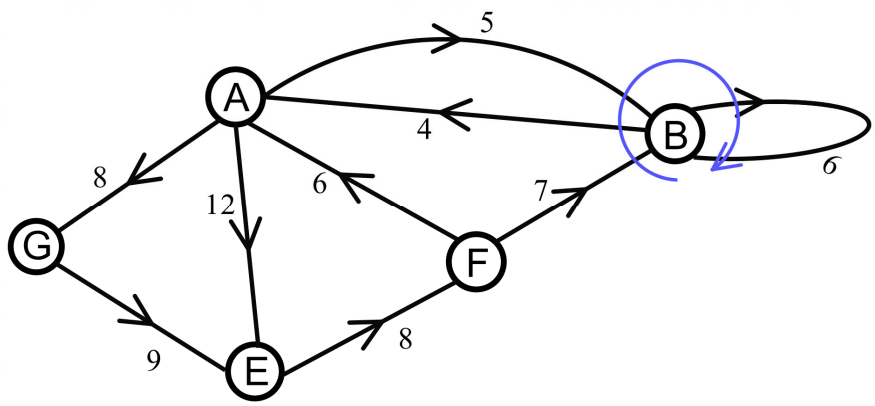
$= 6$ hours (nearest hour) — 1

Question 40

(a) (i) From calculator Mean = 46% and SD = 9%. — 2

(b) (ii) The evening half hour class has an SD of 15% so has the greatest variability. — 1

Question 41

(a) 

The circle around B passes through 5 edges, so the degree is 5 — 1

(b) **Incorrect**
It is not disconnected as every vertex can be reached from any other vertex.
It has only one loop, not 2, which is at B with weight 6.
Correct
It is a directed network — 1

Question 42

(a) (i) 6% p.a. interest = 0.5% per month. — 1

So $r = 0.0050$

Ten years is 120 months, so $N = 120$

From table the factor is 90.0735.

So $90.0735 \times \text{Monthly repayment} = 65000$

$$\text{Monthly repayment} = \frac{65000}{90.0735}$$

$$= \$721.63$$

(b) (ii) Total repaid (ten years) = $721.63 \times 120 = \$86\,595.60$ — 2

Nine years is 108 months, so $N = 108$

From table the factor is 83.2934.

So $83.2934 \times \text{Monthly repayment} = 65000$

$$\text{Monthly repayment} = \frac{65000}{83.2934}$$

$$= \$780.37$$

Total repaid (nine years) = $780.37 \times 108 = \$84\,279.96$

Amount of interest saved = \$86 595.60 - \$84 279.96

= \$2 315.64

Question 43

(a) — 2

Each trapezoid has a height of $\frac{8}{4} = 2$ metres

$$\text{Area} = \frac{2}{2}(4 + 5) + \frac{2}{2}(5 + 7) + \frac{2}{2}(7 + 4) + \frac{2}{2}(4 + 5)$$

$$= 9 + 12 + 11 + 9$$

$$= 41 \text{ m}^2$$

(b) — 1

Since depth is the same, it is a prism.

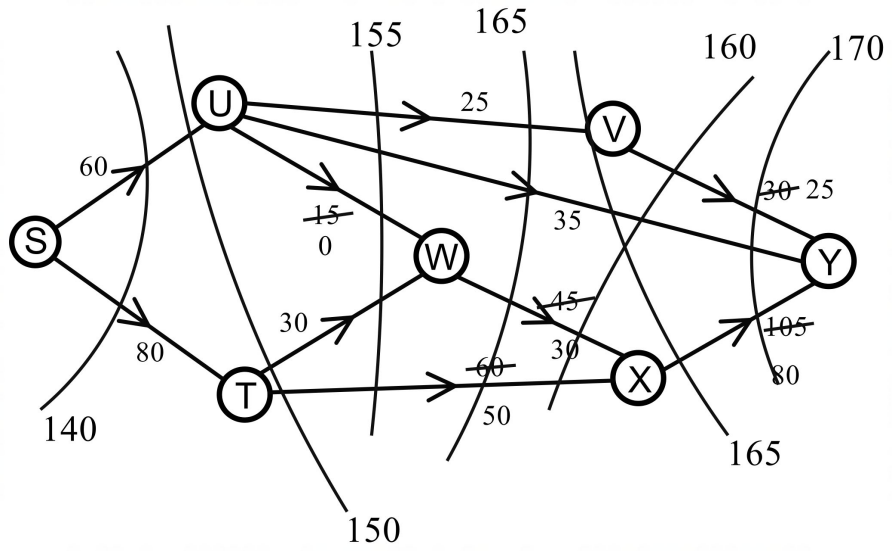
$V = A h$

$= 41 \times 1.5$

$= 61.5$ "m"^3

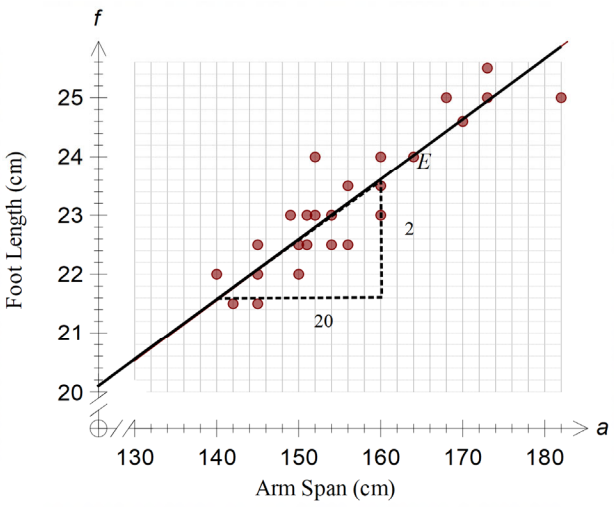
Capacity = 61.5×1000

= 61 500 litres

Question 44		
(a)	Only 25 can flow into V so the maximum outflow is 25, but the pipe has a capacity of 30 so the excess flow is 30 – 25 = 5 Gigalitres.	1
(b)	The values of the various cuts are shown before reducing the flow to the actual amounts. So the maximum flow is the minimum cut of 140 Gigalitres per hour If the cuts were done after reducing the flows, all cuts would be 140.  	2
(c)	The "theoretical" maximum flow from the diagram is 170 through the last 3 pipes. If UV were increased by 5 to 30 it would bring VY to maximum capacity To allow UV to receive the extra 5 and to bring UW to its capacity of 15, SU would need to be increased by 20 to 80 To allow TX to reach its capacity ST could be increased by 10 to 90 This achieves a maximum flow of 170	1

Question 45		
(a)	$\angle DOC = 260° - 210° = 50°$	1
(b)	$\angle AOB = 95°$ Area $= \frac{1}{2} ab \sin O$ $= \frac{1}{2} \times 250 \times 500 \times \sin 95°$ $= 62262$ $= 62\,000\ m^2$ $= 6.2\ ha$	1
(c)	$\angle BOC = 210 - 95 = 115°$ $BC^2 = 250^2 + 380^2 - 2 \times 250 \times 380 \times \cos 115°$ $= 287197$ $BC = 536\ m$	1
(d)	In ΔDOE $\cos \angle DOE = \frac{550^2 + 450^2 - 180^2}{2 \times 550 \times 450}$ $= 0.954747...$ $\angle DOE = \cos^{-1}(0.954747..)$ $= 17°$ (nearest degree) bearing $= 260° + 17°$ $= 277°$	2

Question 46

(a) 

Plot point E on the graph. — 1

(b) A correlation approaching 1 means that as one quantity increases, so does the other in a close linear relationship. — 1

(c) The line drawn on the graph. — 1

(d) Draw a right triangle
Rise = 23.6 – 21.6 = 2
Run = 160 – 140 = 20

$$\text{Gradient} = \frac{2}{20} = \frac{1}{10}$$

Y intercept = 20.1
Equation is

$$y = \frac{1}{10}x + 20.1$$

$$f = \frac{1}{10}a + 20.1$$

2
Answers could be slightly different depending on the line of best fit.

Question 47

(a) $\text{Daily interest rate} = \frac{18.5}{365} = 0.05\%$ per day — 1

(b) Amount owing on 1st July = 2500 – 1200 = 1300.
So $PV = 1300$
There are 31 days in July so $n = 31$
$r = 0.05 \div 100 = 0.0005$

$$FV = PV(1 + r)^n$$

$$\text{New balance} = 1300 \times (1.0005)^{31}$$
$$= \$1320.58$$

OR \$1320.30 if using the rounded value of 1.0005 — 2

(c) Interest = 1320.58 – 1300 = \$20.58 (or \$20.30) — 1