SYDNEY GRAMMAR SCHOOL

CANDIDATE NUMBER

2023 Trial HSC Examination

Form VI Mathematics Standard 2

Tuesday 8th August 2023

8:40am

General Instructions

- Reading time — 10 minutes
- Working time — 2 hours 30 minutes
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 100

Section I (15 marks) Questions 1 – 15

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (85 marks) Questions 16 – 41

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.

Collection

- Write your candidate number on this page and on the multiple choice sheet.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 13 pupils

Writer: BR

Section I

15 marks
Attempt Questions 1-15
Allow about 25 minutes for this section
Use the multiple-choice answer sheet for Questions 1-15

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1. The distance between Blacktown and Darlinghurst is 50 km. Liam travels from Blacktown to Darlinghurst at an average speed of 75 km/h.

   How long does it take Liam to complete the journey?

   (A) 40 minutes

   (B) 45 minutes

   (C) 67 minutes

   (D) 90 minutes

2. Ethan opens a bank account and deposits \$5000 into it. Interest is paid at 4·5% per annum, compounding annually.

   Assuming no further deposits or withdrawals are made, what will be the balance in the account at the end of two years?

   (A) \$5450.00

   (B) \$5460.13

   (C) \$10 450.00

   (D) \$10 512.50

3. The distribution of heights of adult males in Colombia is known to follow a normal distribution with a mean of 170 cm and a standard deviation of 5 cm.

   What percentage of adult males in Colombia would be expected to have a height between 160 cm and 180 cm?

   (A) Approximately 68%

   (B) Approximately 95%

   (C) Approximately 99·7%

   (D) It cannot be determined based on the given information

4. Which compass bearing is the same as a true bearing of 140°T?

   (A) S40°E

   (B) S40°W

   (C) S50°E

   (D) S50°W

5. The Coordinated Universal Time (UTC) of Sydney is +10 hours and the UTC of New York is −4 hours.

   When the time in New York is 9 pm, Friday, what is the time in Sydney?

   (A) 11 am, Thursday

   (B) 3 pm, Friday

   (C) 11 am, Saturday

   (D) 3 pm, Saturday

6. In a study examining the relationship between hours of training for competitive swimming and competition swimming times, the researcher calculated the correlation coefficient to be −0·85.

   What can be inferred from this value?

   (A) There is a strong positive linear relationship between hours of training and competition times.

   (B) There is a strong negative linear relationship between hours of training and competition times.

   (C) There is a weak negative linear relationship between hours of training and competition times.

   (D) There is no relationship between hours of training and competition times.

7. Noah's foot is measured to be 25·0 cm, correct to one decimal place.

   What is the percentage error in this measurement?

   (A) 0·2%

   (B) 0·4%

   (C) 2%

   (D) 4%

8. Suppose $y = -3 - 7x$.

   When the value of x increases by 2, by how much does the y value decrease?

   (A) 2

   (B) 3

   (C) 7

   (D) 14

9. Olivia is paid \$22.55 per hour, as well as a meal allowance of \$15.30 per day.

   What are Olivia's total earnings if she works 8 hours per day for 5 days?

   (A) \$195.70

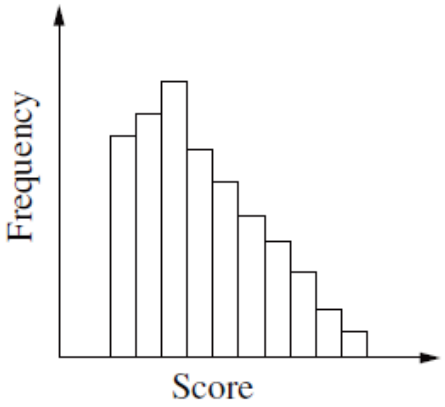
   (B) \$917.30

   (C) \$978.50

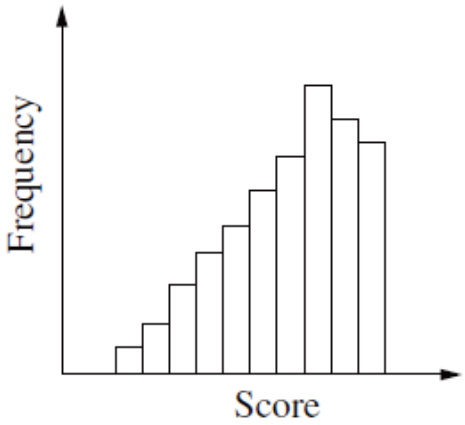
   (D) \$1514.00

10. Which histogram best represents a dataset that is positively skewed?

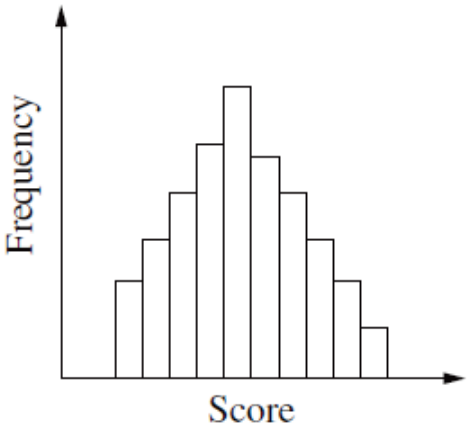
(A)



(B)



(C)



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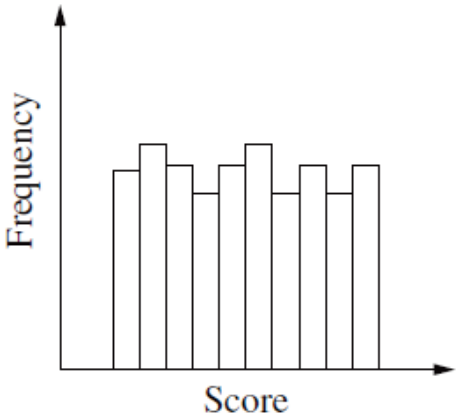
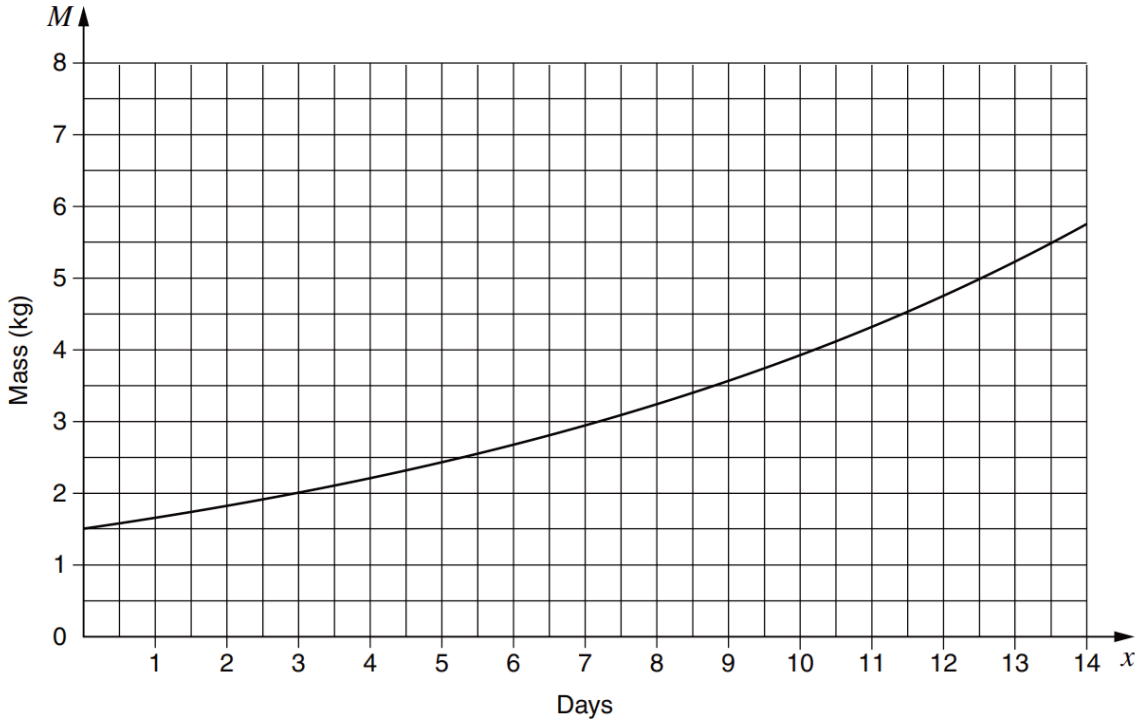


11. The mass M kg of a baby pig at age x days is given by $M = A(1{\cdot}1)^x$ where A is a constant. The graph of this equation is shown below.



What is the value of A?

(A) $-\dfrac{3}{2}$

(B) $-\dfrac{2}{3}$

(C) $\dfrac{2}{3}$

(D) $\dfrac{3}{2}$

12. Which of the following correctly expresses y as the subject of the formula $2x - 5y - 3 = 0$?

(A) $y = \dfrac{2}{5}x - 3$

(B) $y = \dfrac{2}{5}x + 3$

(C) $y = \dfrac{2x - 3}{5}$

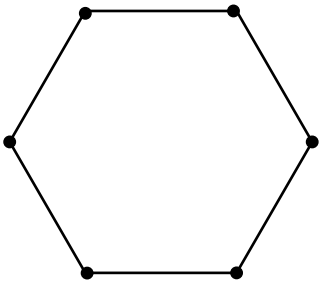
(D) $y = \dfrac{2x + 3}{5}$

13. The Boston Brawlers and the Atlanta Lightning have drawn each other in a wrestling competition.

    The Boston Brawlers and the Atlanta Lightning have three wrestlers each. Each wrestler of the Boston Brawlers must wrestle each wrestler of the Atlanta Lightning.

    Which of the following network diagrams could represent the wrestling matches to take place?

(A)



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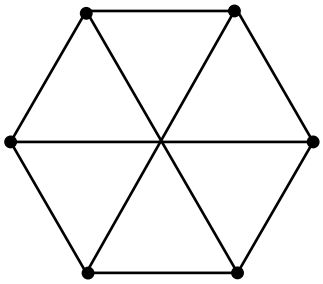
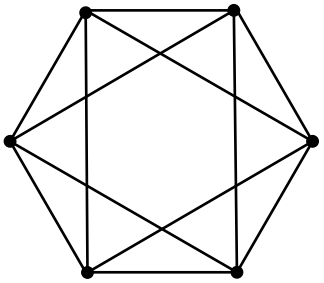
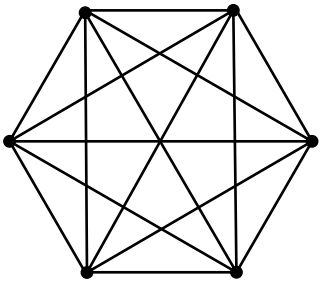


(C)



(D)



14. In twelve years, the future value of an investment will be \$250 000. The interest rate is 6% per annum, compounded half-yearly.

    Which equation will give the present value (PV) of the investment?

(A) $PV = \dfrac{250\,000}{(1 + 0{\cdot}06)^{24}}$

(B) $PV = \dfrac{250\,000}{(1 + 0{\cdot}03)^{24}}$

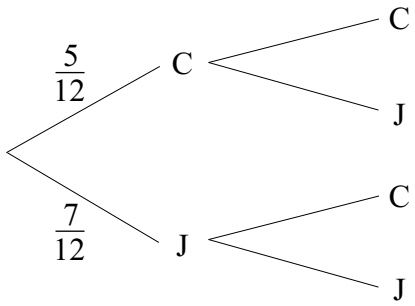
(C) $PV = \dfrac{250\,000}{(1 + 0{\cdot}06)^{12}}$

(D) $PV = \dfrac{250\,000}{(1 + 0{\cdot}03)^{12}}$

15. There are twelve doughnuts in a box. Five have chocolate filling (C) and seven have jam filling (J).

    Emma randomly chooses a doughnut from the box and eats it. Sophia then randomly chooses and eats one of the remaining doughnuts.

    A partially completed probability tree is shown below.



    What is the probability that Emma and Sophia choose doughnuts with different filling?

    (A) $\frac{35}{144}$

    (B) $\frac{35}{132}$

    (C) $\frac{35}{72}$

    (D) $\frac{35}{66}$

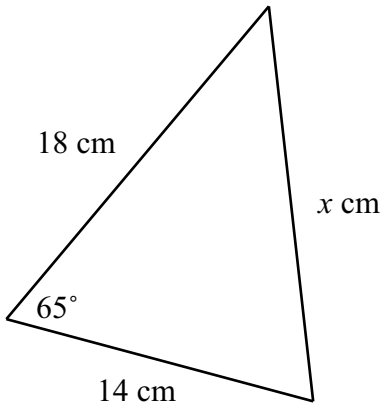
End of Section I

The paper continues in the next section

Section II

85 marks
Attempt Questions 16-41
Allow about 2 hours and 5 minutes for this section

QUESTION SIXTEEN (3 marks) **Marks** 3



The diagram above shows a triangle with sides of length x cm, 18 cm and 14 cm and an angle of 65°.

Use the cosine rule to calculate the value of x. Give your answer correct to two significant figures.

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QUESTION SEVENTEEN (4 marks) **Marks**

The following income tax table is used to calculate William's tax payable.

Taxable income	Tax on this income
\$0 – \$18 200	Nil
\$18 201 – \$45 000	21c for each \$1 over \$18 200
\$45 001 – \$120 000	\$5658 plus 34.5c for each \$1 over \$45 000
\$120 001 – \$180 000	\$31 503 plus 39c for each \$1 over \$120 000
\$180 001 and over	\$54 903 plus 47c for each \$1 over \$180 000

William earns a salary of \$98 767 and claims a tax deduction of \$2805 for work-related expenses.

(a) Calculate William's taxable income. 1

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(b) What is William's tax payable? 2

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(c) The Medicare levy is a tax to fund Medicare. It is 2% of taxable income. Calculate the amount of William's Medicare levy. 1

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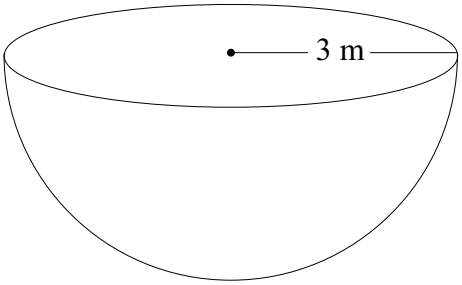
QUESTION EIGHTEEN (2 marks) **Marks**

The volume V of a sphere is given by the formula [2]

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.

A tank consists of the bottom half of a sphere of radius 3 metres, as shown below.



Find the volume of the tank in cubic metres. Give your answer correct to one decimal place.

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QUESTION NINETEEN (2 marks) **Marks**

The marks in a test were normally distributed. The mean mark was 70 and the standard deviation was 10. [2]

What percentage of marks were higher than 80?

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QUESTION TWENTY (4 marks) **Marks**

Alex, Ben and Caleb are in the school basketball team. In a recent game, Alex scored 24 points, Ben scored 20 points and Caleb scored 32 points.

(a) What is the ratio of Alex's to Ben's to Caleb's points scored? Give your answer in simplest form. 2

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(b) In this game, the ratio of the total number of points scored by Alex, Ben and Caleb to the total number of points scored by the whole team is 19 : 26. 2

How many points were scored by the whole team?

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QUESTION TWENTY-ONE (2 marks) **Marks**

The average inflation rate over the year from the start of January 2022 to the start of January 2023 was 6%. 2

After one year, at the start of January 2023, the cost of an air fryer was \$98.

Calculate the cost of the air fryer at the start of January 2022, assuming that the only change in the cost of the air fryer was due to inflation. Give your answer correct to the nearest cent.

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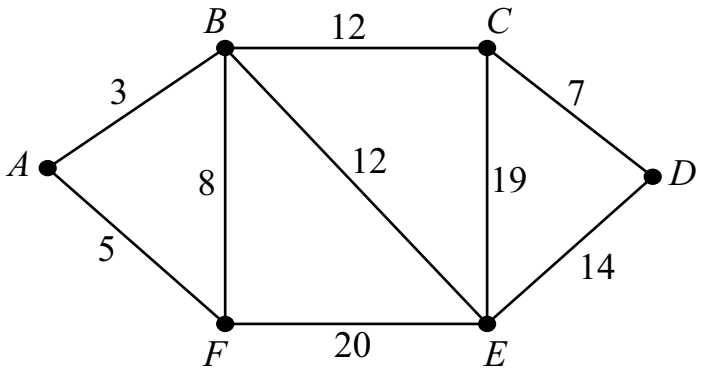
QUESTION TWENTY-TWO (3 marks) **Marks**

A network is shown below.



(a) What is the total weight of the path $AFBCD$? 1

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(b) Draw the minimum spanning tree for the network in the space below. 2

QUESTION TWENTY-THREE (3 marks) **Marks**

The formula $C = 23n + b$ is used to calculate the cost of producing Alpha Zone mobile phones, where C is the cost in dollars, n is the number of mobile phones produced and b is a fixed cost in dollars.

(a) Find the cost when 2133 Alpha Zone mobile phones are produced and the fixed cost is \$23 940. 1

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(b) Some Alpha Zone mobile phones have some extra features added and are called Alpha Zone Xtra mobile phones. The formula to calculate the production cost for Alpha Zone Xtra mobile phones is 2

$$C = 23n + an + 23\,940\,,$$

where a is the additional cost in dollars per Alpha Zone Xtra mobile phone produced.

Find the number of Alpha Zone Xtra mobile phones produced if the additional cost is \$8 per mobile phone and the total production cost is \$104 044.

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QUESTION TWENTY-FOUR (2 marks) **Marks**

The fuel consumption for a car is 7·6 litres per 100 kilometres. On a road trip, the car travels a distance of 1450 km and the fuel cost is \$2.02 per litre. [2]

What is the total fuel cost for this trip? Give your answer correct to the nearest cent.

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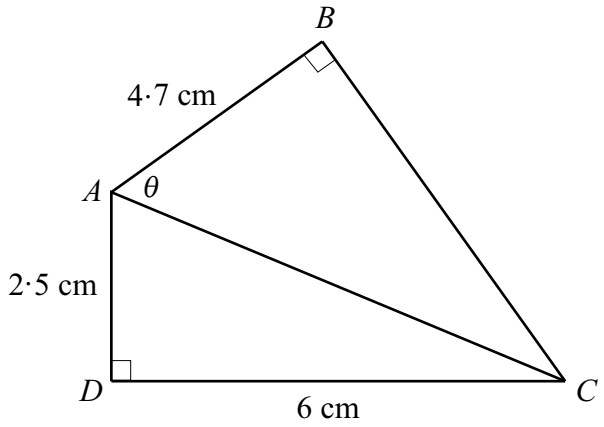
QUESTION TWENTY-FIVE (3 marks) **Marks**

Two right-angled triangles, ABC and ADC, are shown below. [3]



Calculate the size of angle θ. Give your answer correct to the nearest minute.

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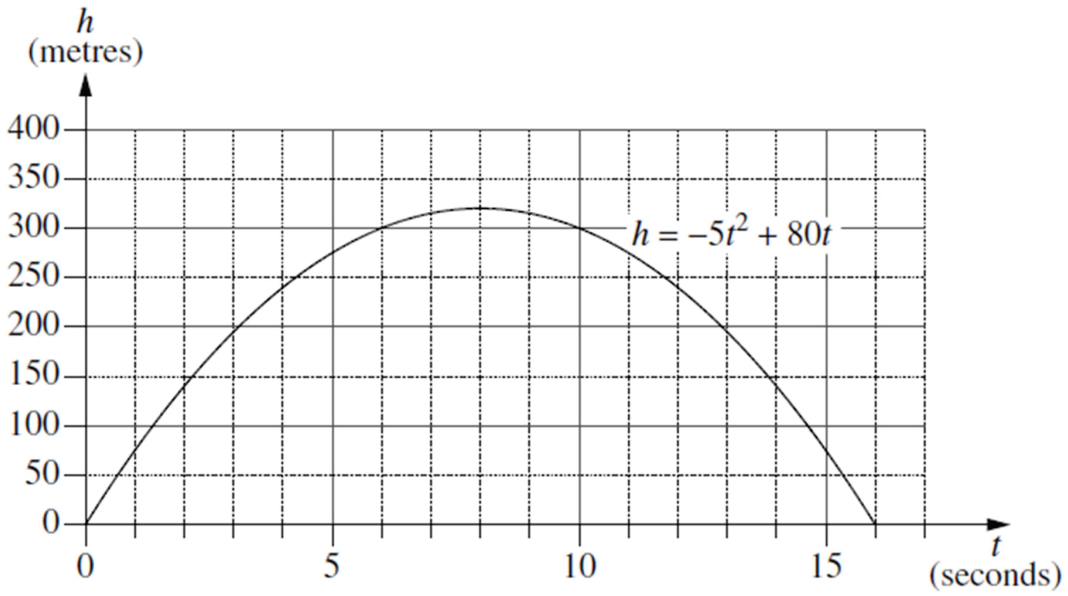
QUESTION TWENTY-SIX (2 marks) **Marks**

An object is projected vertically into the air. Its height, h metres, above the ground after t seconds is given by $h = -5t^2 + 80t$. [2]

A graph of this non-linear function is shown below.



Using the graph and a calculation, find the maximum height of the object above the ground.

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QUESTION TWENTY-SEVEN (3 marks) **Marks**

Charlotte has a credit card on which interest at 18% per annum, compounded daily, is charged on the amount owing. [3]

At the beginning of the month, Charlotte owes \$680 on her credit card. She makes no other purchases using the credit card, but fifteen days later, she repays \$350. Assuming that interest is charged for the fifteen days, calculate the amount owing on the credit card immediately after the \$350 payment is made. Give your answer correct to the nearest cent.

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QUESTION TWENTY-EIGHT (2 marks) **Marks**

The correlation coefficient between the time since a client joins Joe Delaney's body building program and their mass is 0·45. A client is confident that after 5 months their weight would have significantly reduced. Discuss two problems with this interpretation. [2]

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QUESTION TWENTY-NINE (3 marks) **Marks**

Henry invests \$22 000 in a savings account. Interest is paid at a fixed monthly rate. At the end of each month, after the monthly interest is added, Henry makes a deposit of \$700. 3

Henry has created a spreadsheet to show the activity in his savings account. The details for the first 6 months are shown below.

Month	Amount in account at beginning of month	Monthly interest	Deposit	Amount in account at the end of month
1	22 000.00	55.00	700	22 755.00
2	22 755.00	56.89	700	23 511.89
3	23 511.89	58.78	700	24 270.67
4	24 270.67	60.68	700	25 031.35
5	25 031.35	62.58	700	25 793.93
6	25 793.93	64.48	700	26 558.41
7			700	

By finding the monthly rate of interest, complete the final row above for the 7th month.

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QUESTION THIRTY (4 marks) **Marks**

Mia believes that the time it takes for an ice cube to melt (M minutes) varies inversely with the room temperature (T°C). Mia observes that at a room temperature of 20°C it takes 9 minutes for an ice cube to melt.

(a) Find the equation relating M and T. 2

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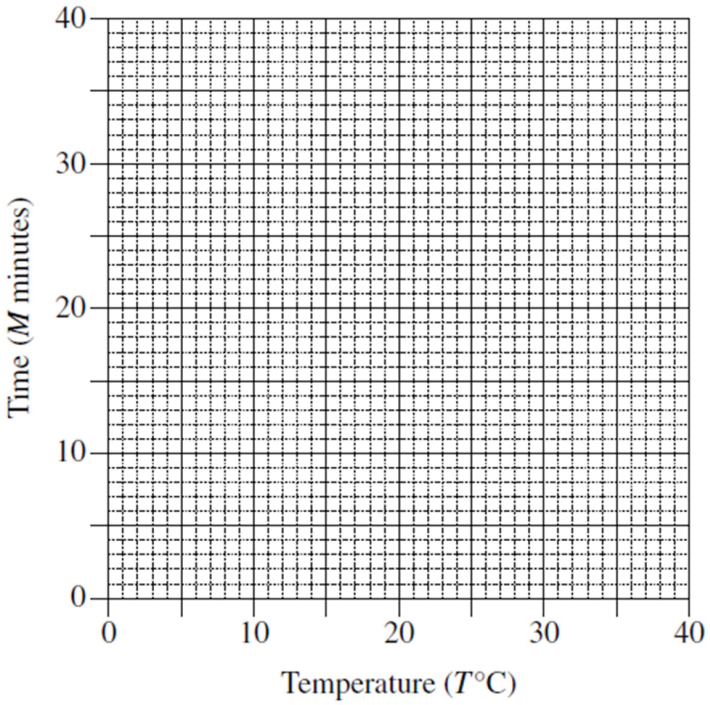
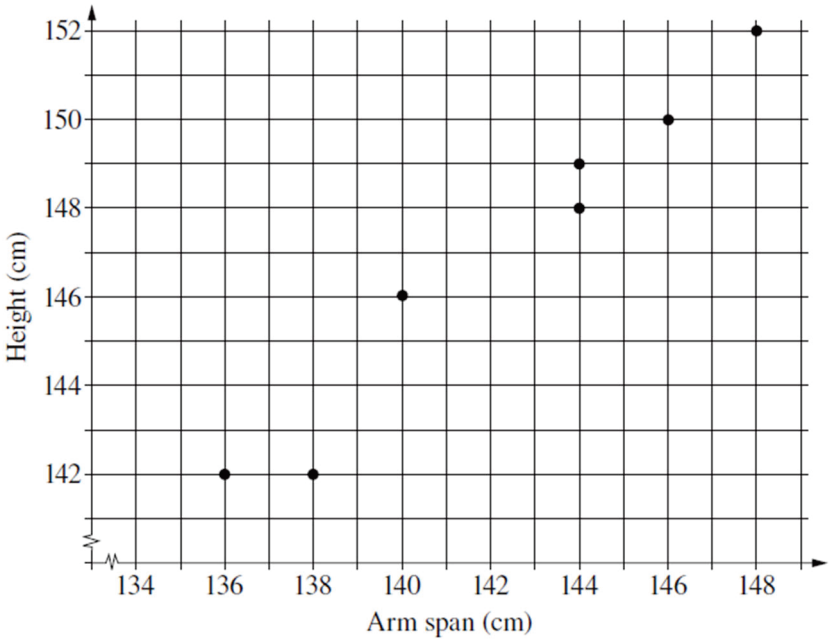
(b) By first completing this table of values below, graph the relationship between temperature and time from $T = 5$°C to $T = 30$°C. 2

T	5	15	30
M			



QUESTION THIRTY-ONE (3 marks) **Marks**

A set of bivariate data is collected by measuring the height and arm span of seven children. The graph below shows a scatterplot of these measurements.



(a) Calculate Pearson's correlation coefficient for the data. Give your answer correct to two decimal places. [1]

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(b) Find the equation of the least-squares regression line. Give numbers correct to three significant figures. [2]

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QUESTION THIRTY-TWO (4 marks) **Marks**

Daniel inherits \$50 000 and invests it in an account earning interest at a rate of 0·6% per month. At the end of each month, immediately after the interest has been paid, Daniel withdraws \$700.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1{\cdot}006) - 700\,,$$

where $n = 1, 2, 3, \ldots$ and $A_0 = 50\,000$.

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. 2

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(b) Calculate the amount of interest earned in the first three months. 2

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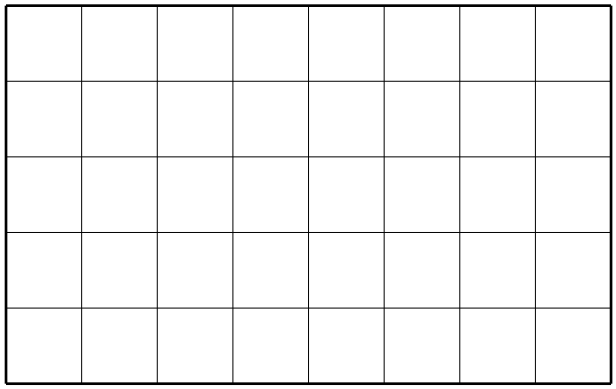
QUESTION THIRTY-THREE (4 marks) **Marks**

A rectangular sportsground has been drawn to scale on a 1 cm by 1 cm grid as shown below. The scale used is 1 : 2500. [4]



Owen took 15 minutes to walk around the perimeter of this sportsground.

What was Owen's average speed in kilometres per hour?

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QUESTION THIRTY-FOUR (4 marks) **Marks**

The table below shows the future value of an annuity of \$1.

Future values of an annuity of \$1

Years	Interest Rate per Annum			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

Harper is saving for a trip and estimates she will need \$20 000. She opens an account earning 4% per annum, compounded annually.

(a) How much does Harper need to deposit every year if she wishes to have enough money for the trip in 5 years time? 2

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(b) How much interest will Harper earn on her investment over the 5 years? Give your answer correct to the nearest dollar. 2

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QUESTION THIRTY-FIVE (3 marks) **Marks**

Eighty tickets are sold in a raffle with two prizes. Matthew buys five tickets.

(a) By drawing a tree diagram, or otherwise, find the probability that Matthew wins both prizes. 2

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(b) Find the probability that Matthew wins at least one prize. 1

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QUESTION THIRTY-SIX (7 marks) **Marks**

The Intelligence Quotient (IQ) scores for adults in City A are normally distributed with a mean of 105 and a standard deviation of 12.

The IQ scores for adults in City B are normally distributed with a mean of 114 and a standard deviation of 15.

(a) Jack is an adult who lives in City A and has an IQ score of 129. 2

What percentage of the adults in this city have an IQ score higher than Jack's?

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(b) There are 500 000 adults living in City B. 2

Calculate the number of adults in City B that would be expected to have an IQ score lower than Jack's.

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QUESTION THIRTY-SIX (Continued)

(c) Lucas, an adult who lives in City A, moves to City B. The z-score corresponding to his IQ score in City A is the same as the z-score corresponding to his IQ score in City B. 3

By first forming an equation, calculate Lucas' IQ score.

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QUESTION THIRTY-SEVEN (3 marks) **Marks**

The mean of three numbers is 255. If two of the numbers are 102 and 256, find the third number. 3

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QUESTION THIRTY-EIGHT (4 marks) **Marks**

A project requires activities A to F to be completed. The activity chart shows the immediate prerequisite(s) and duration for each activity.

Activity	Immediate prerequisite(s)	Duration in hours
A	——	3
B	*A*	8
C	*A*	4
D	*B*	3
E	*C, D*	5
F	*E*	2

(a) Draw a network diagram showing the EST and the LST for each activity. 2

(b) Write down the minimum time for the project to be completed. 1

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(c) Determine the float time of the non-critical activity. 1

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QUESTION THIRTY-NINE (3 marks) **Marks**

The formula below is used to calculate an estimate for blood alcohol content (BAC) for females: 3

$$BAC_{Female} = \frac{10N - 7{\cdot}5H}{5{\cdot}5M}$$

The number of hours required for a person to reach zero BAC after they stop consuming alcohol is given by the following formula:

$$\text{Time} = \frac{BAC}{0{\cdot}015}$$

Abigail weighs 63 kg. She consumed 5 standard drinks between 7:45 pm and 1:00 am the following day. She then stopped drinking alcohol.

Using the given formulae, calculate the time in the morning when Abigail's BAC should reach zero.

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QUESTION FORTY (5 marks) **Marks**

A large group of people undertook a survey regarding the number of hours of sleep they had in a certain week. The results of the survey were normally distributed with 2·5% of the people indicating that they had less than 42 hours of sleep per week and 2·5% of the people indicating that they had more than 54 hours of sleep per week.

(a) Determine the mean number of hours of sleep per week of the group. 1

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(b) What was the standard deviation? 1

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(c) A person from the group has a z-score of $-2{\cdot}5$. How many hours of sleep per week do they have? 1

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(d) A person is selected randomly from the group. Determine the probability as a percentage that they had between 51 and 57 hours of sleep per week. 2

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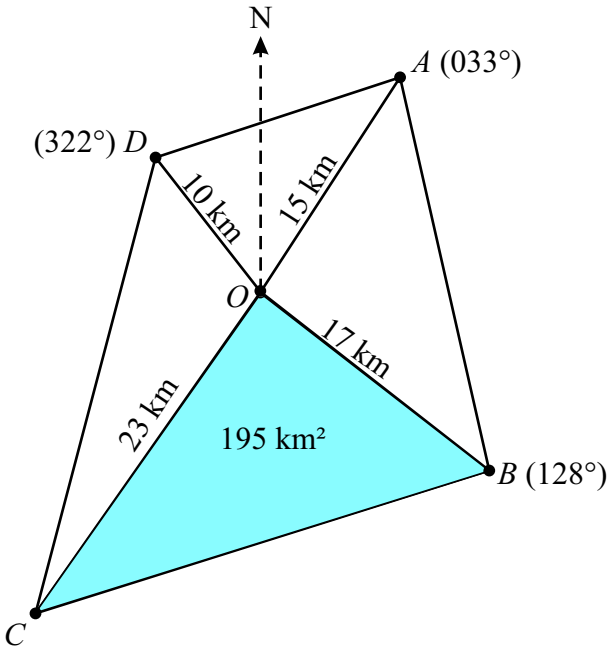
QUESTION FORTY-ONE (3 marks) **Marks**

A compass radial survey shows the positions of four towns A, B, C and D relative to point O. 3



The area of triangle BOC is $195\,\text{km}^2$.

Calculate the bearing of town C from point O. Give your answer correct to the nearest degree.

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END OF PAPER

SYDNEY GRAMMAR SCHOOL

SOLUTION
CANDIDATE NUMBER

2023 **Trial HSC Examination**

Form VI Mathematics Standard 2

Tuesday 8th August 2023

8:40am

General Instructions

- Reading time — 10 minutes
- Working time — 2 hours 30 minutes
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 100

Section I (15 marks) Questions 1 – 15

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (85 marks) Questions 16 – 41

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.

Collection

- Write your candidate number on this page and on the multiple choice sheet.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 13 pupils

Writer: BR

Section I

15 marks
Attempt Questions 1-15
Allow about 25 minutes for this section
Use the multiple-choice answer sheet for Questions 1-15

---

1. The distance between Blacktown and Darlinghurst is 50 km. Liam travels from Blacktown to Darlinghurst at an average speed of 75 km/h.

   How long does it take Liam to complete the journey?

   (A) 40 minutes
   (B) 45 minutes
   (C) 67 minutes
   (D) 90 minutes

   $S = \frac{d}{t}$
   $t = \frac{d}{S}$
   $= \frac{50}{75} \times 60$

2. Ethan opens a bank account and deposits \$5000 into it. Interest is paid at 4·5% per annum, compounding annually.

   Assuming no further deposits or withdrawals are made, what will be the balance in the account at the end of two years?

   (A) \$5450.00
   (B) \$5460.13
   (C) \$10 450.00
   (D) \$10 512.50

   $5000(1 + 0.045)^2$

3. The distribution of heights of adult males in Colombia is known to follow a normal distribution with a mean of 170 cm and a standard deviation of 5 cm.

   What percentage of adult males in Colombia would be expected to have a height between 160 cm and 180 cm?

   (A) Approximately 68%
   (B) Approximately 95%
   (C) Approximately 99·7%
   (D) It cannot be determined based on the given information

   -2 to 2

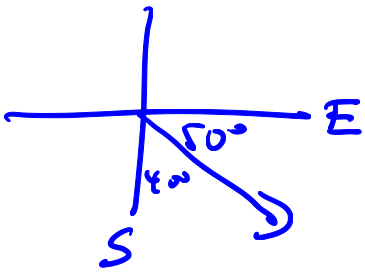
4. Which compass bearing is the same as a true bearing of 140°T?

   (A) S40°E

   (B) S40°W

   (C) S50°E

   (D) S50°W



5. The Coordinated Universal Time (UTC) of Sydney is +10 hours and the UTC of New York is −4 hours.

   When the time in New York is 9 pm, Friday, what is the time in Sydney?

   (A) 11 am, Thursday

   (B) 3 pm, Friday

   (C) 1 am, Saturday

   (D) 3 pm, Saturday

NY → 14hrs → SYD
9pm F 1am S

6. In a study examining the relationship between hours of training for competitive swimming and competition swimming times, the researcher calculated the correlation coefficient to be −0·85.

   What can be inferred from this value?

   (A) There is a strong positive linear relationship between hours of training and competition times.

   (B) There is a strong negative linear relationship between hours of training and competition times.

   (C) There is a weak negative linear relationship between hours of training and competition times.

   (D) There is no relationship between hours of training and competition times.

7. Noah's foot is measured to be 25·0 cm, correct to one decimal place.

   What is the percentage error in this measurement?

   (A) 0·2%

   (B) 0·4%

   (C) 2%

   (D) 4%

$\frac{0.05}{25} \times 100\%$

8. Suppose $y = -3 - 7x$.

   When the value of x increases by 2, by how much does the y value decrease?

   (A) 2
   (B) 3
   (C) 7
   (D) 14

$x = 1, y = -10$
$x = 3, y = -24$

9. Olivia is paid \$22.55 per hour, as well as a meal allowance of \$15.30 per day.

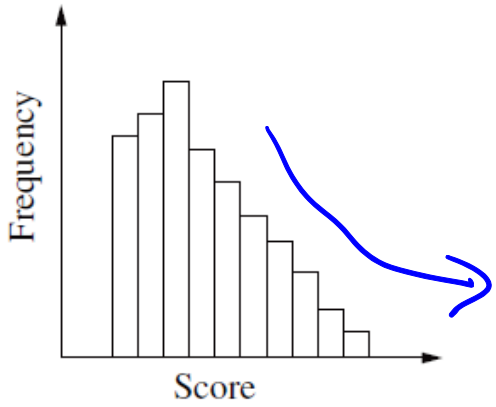
   What are Olivia's total earnings if she works 8 hours per day for 5 days?

   (A) \$195.70
   (B) \$917.30
   (C) \$978.50
   (D) \$1514.00

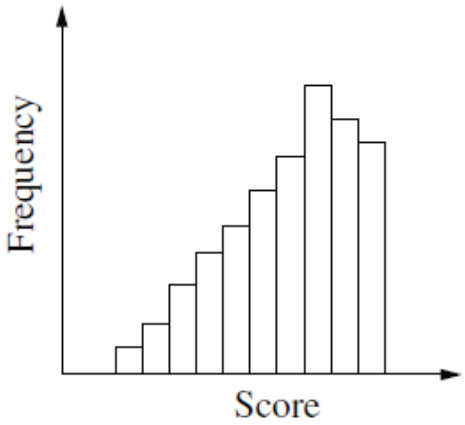
$(22.55(8) + 15.30) \times 5$

10. Which histogram best represents a dataset that is positively skewed?

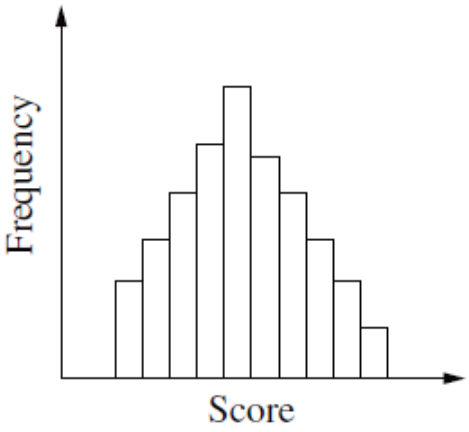
(A)



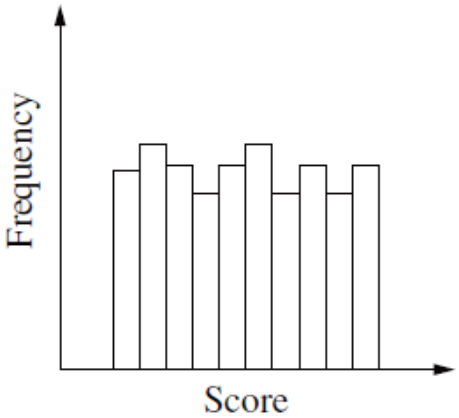
(B)



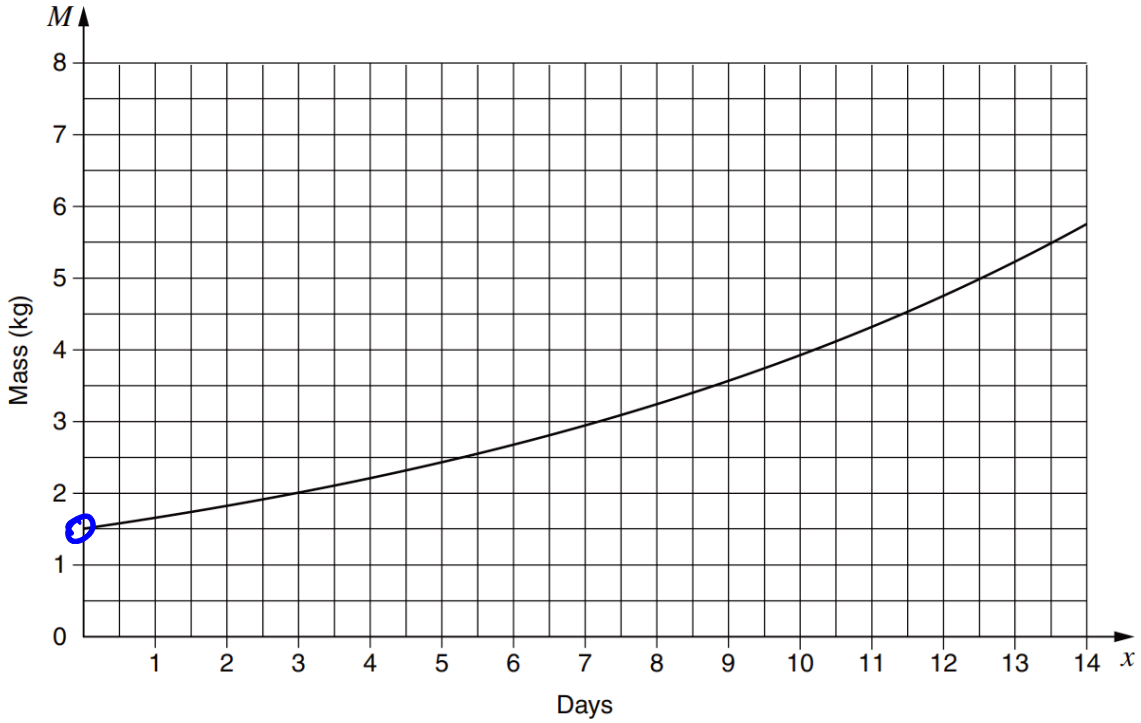
(C)



(D)



11. The mass M kg of a baby pig at age x days is given by $M = A(1{\cdot}1)^x$ where A is a constant. The graph of this equation is shown below.



What is the value of A?

(A) $-\frac{3}{2}$

(B) $-\frac{2}{3}$

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

12. Which of the following correctly expresses y as the subject of the formula $2x - 5y - 3 = 0$?

(A) $y = \frac{2}{5}x - 3$

(B) $y = \frac{2}{5}x + 3$

(C) $y = \frac{2x - 3}{5}$

(D) $y = \frac{2x + 3}{5}$

$5y = 2x - 3$

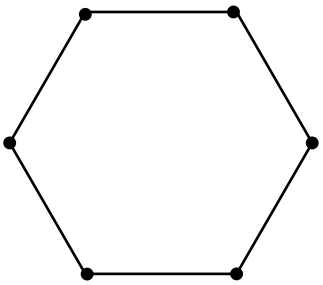
$y = \frac{2x - 3}{5}$

13. The Boston Brawlers and the Atlanta Lightning have drawn each other in a wrestling competition.

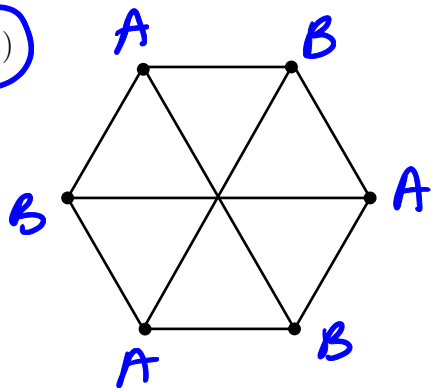
    The Boston Brawlers and the Atlanta Lightning have three wrestlers each. Each wrestler of the Boston Brawlers must wrestle each wrestler of the Atlanta Lightning.

    Which of the following network diagrams could represent the wrestling matches to take place?

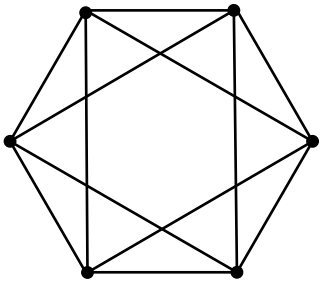
(A)



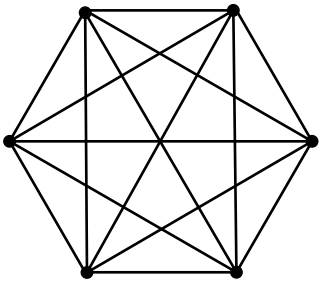
(B)



(C)



(D)



14. In twelve years, the future value of an investment will be \$250 000. The interest rate is 6% per annum, compounded half-yearly.

    Which equation will give the present value (PV) of the investment?

(A) $PV = \dfrac{250\,000}{(1+0{\cdot}06)^{24}}$

(B) $PV = \dfrac{250\,000}{(1+0{\cdot}03)^{24}}$

$250\,000 = PV(1+0{\cdot}03)^{24}$

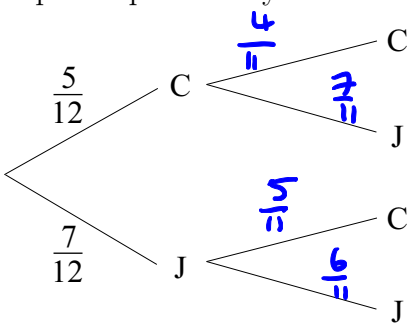
(C) $PV = \dfrac{250\,000}{(1+0{\cdot}06)^{12}}$

(D) $PV = \dfrac{250\,000}{(1+0{\cdot}03)^{12}}$

15. There are twelve doughnuts in a box. Five have chocolate filling (C) and seven have jam filling (J).

    Emma randomly chooses a doughnut from the box and eats it. Sophia then randomly chooses and eats one of the remaining doughnuts.

    A partially completed probability tree is shown below.



    What is the probability that Emma and Sophia choose doughnuts with different filling?

    (A) $\frac{35}{144}$

    (B) $\frac{35}{132}$

    (C) $\frac{35}{72}$

    (D) $\frac{35}{66}$

$$P(CJ) + P(JC)$$
$$= \frac{5}{12} \cdot \frac{7}{11} + \frac{7}{12} \cdot \frac{5}{11}$$

End of Section I

The paper continues in the next section

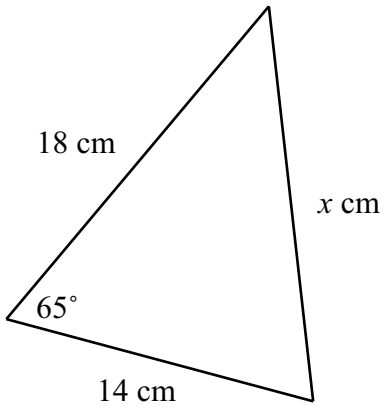
Section II

85 marks
Attempt Questions 16-41
Allow about 2 hours and 5 minutes for this section

QUESTION SIXTEEN (3 marks)

Marks

3



The diagram above shows a triangle with sides of length x cm, 18 cm and 14 cm and an angle of 65°.

Use the cosine rule to calculate the value of x. Give your answer correct to two significant figures.

$x^2 = 18^2 + 14^2 - 2(18)(14)\cos 65^\circ$ ✓

$= 307.00\ldots$ ✓

$\therefore x \doteq 18$ cm ✓

QUESTION SEVENTEEN (4 marks) **Marks**

The following income tax table is used to calculate William's tax payable.

Taxable income	Tax on this income
\$0 – \$18 200	Nil
\$18 201 – \$45 000	21c for each \$1 over \$18 200
\$45 001 – \$120 000	\$5658 plus 34.5c for each \$1 over \$45 000
\$120 001 – \$180 000	\$31 503 plus 39c for each \$1 over \$120 000
\$180 001 and over	\$54 903 plus 47c for each \$1 over \$180 000

William earns a salary of \$98 767 and claims a tax deduction of \$2805 for work-related expenses.

(a) Calculate William's taxable income. 1

$98767 - 2805$

$= \$95\,962$ ✓

(b) What is William's tax payable? 2

$5658 + (0.345)(95962 - 45000)$ ✓

$= \$23\,239.89$ ✓

(c) The Medicare levy is a tax to fund Medicare. It is 2% of taxable income. Calculate the amount of William's Medicare levy. 1

$(0.02)(95962)$

$= \$1919.24$ ✓

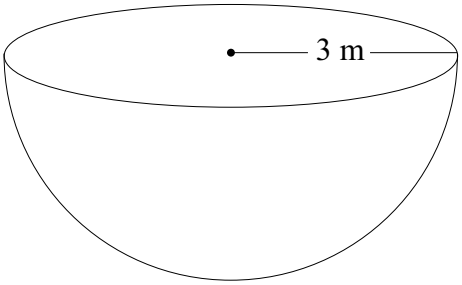
QUESTION EIGHTEEN (2 marks) **Marks**

The volume V of a sphere is given by the formula [2]

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.

A tank consists of the bottom half of a sphere of radius 3 metres, as shown below.



Find the volume of the tank in cubic metres. Give your answer correct to one decimal place.

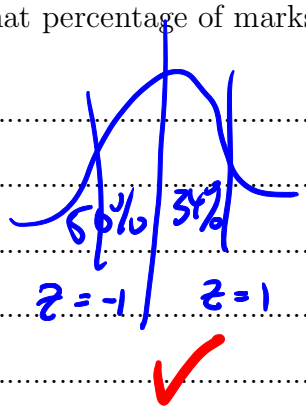
$V = \frac{1}{2} \cdot \frac{4}{3}\pi(3)^3$ ✓

$\doteq 56.5 \text{ m}^3$ ✓

QUESTION NINETEEN (2 marks) **Marks**

The marks in a test were normally distributed. The mean mark was 70 and the standard deviation was 10. [2]

What percentage of marks were higher than 80?



✓

$100 - 50 - 34 = 16\%$ ✓

QUESTION TWENTY (4 marks) **Marks**

Alex, Ben and Caleb are in the school basketball team. In a recent game, Alex scored 24 points, Ben scored 20 points and Caleb scored 32 points.

(a) What is the ratio of Alex's to Ben's to Caleb's points scored? Give your answer in simplest form. [2]

$24 : 20 : 32$ ✓

$= 6 : 5 : 8$ ✓

(b) In this game, the ratio of the total number of points scored by Alex, Ben and Caleb to the total number of points scored by the whole team is 19 : 26. [2]

How many points were scored by the whole team?

$\frac{24 + 20 + 32}{19} \times 26$ ✓

$= 104$ points ✓

QUESTION TWENTY-ONE (2 marks) **Marks**

The average inflation rate over the year from the start of January 2022 to the start of January 2023 was 6%. [2]

After one year, at the start of January 2023, the cost of an air fryer was \$98.

Calculate the cost of the air fryer at the start of January 2022, assuming that the only change in the cost of the air fryer was due to inflation. Give your answer correct to the nearest cent.

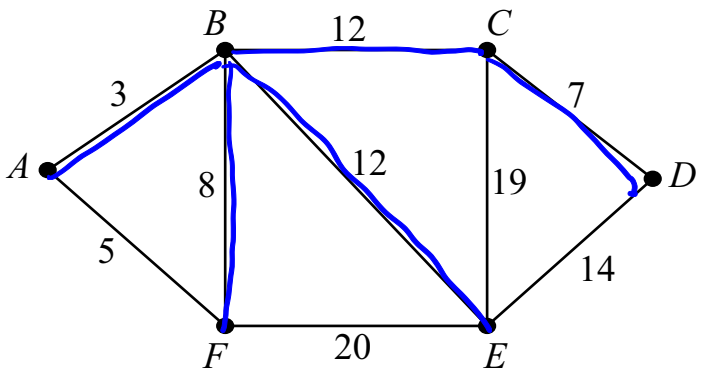
$x(1.06) = 98$

$x = \frac{98}{1.06}$ ✓

$= \$92.45$ ✓

QUESTION TWENTY-TWO (3 marks) **Marks**

A network is shown below.

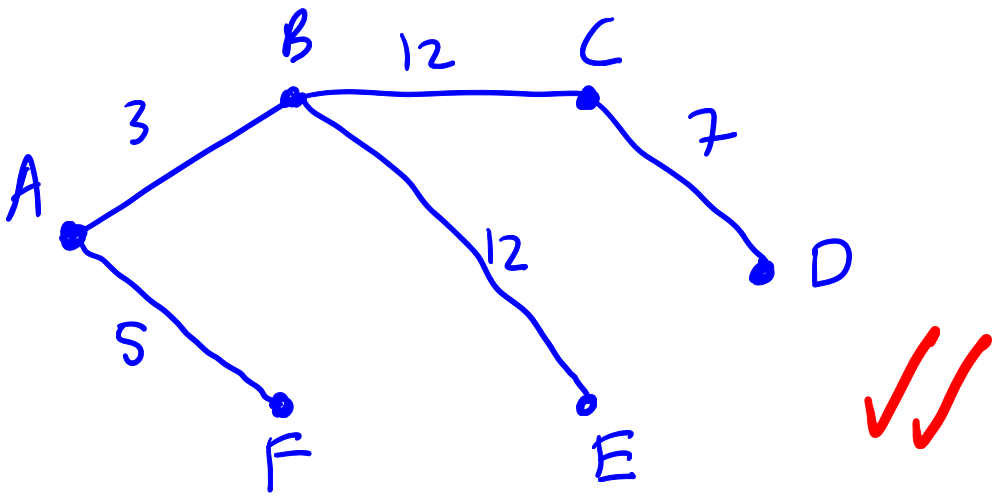


(a) What is the total weight of the path $AFBCD$? 1

5 + 8 + 12 + 7

= 32 ✓

(b) Draw the minimum spanning tree for the network in the space below. 2



✓✓

QUESTION TWENTY-THREE (3 marks) **Marks**

The formula $C = 23n + b$ is used to calculate the cost of producing Alpha Zone mobile phones, where C is the cost in dollars, n is the number of mobile phones produced and b is a fixed cost in dollars.

(a) Find the cost when 2133 Alpha Zone mobile phones are produced and the fixed cost is \$23 940. 1

$C = 23(2133) + 23940$ ✓

$= \$72\,999$

(b) Some Alpha Zone mobile phones have some extra features added and are called Alpha Zone Xtra mobile phones. The formula to calculate the production cost for Alpha Zone Xtra mobile phones is 2

$$C = 23n + an + 23\,940\,,$$

where a is the additional cost in dollars per Alpha Zone Xtra mobile phone produced.

Find the number of Alpha Zone Xtra mobile phones produced if the additional cost is \$8 per mobile phone and the total production cost is \$104 044.

$104\,044 = 23n + 8n + 23940$ ✓

$80\,104 = 31n$

$\therefore n = 2584$ ✓

QUESTION TWENTY-FOUR (2 marks) **Marks**

The fuel consumption for a car is 7·6 litres per 100 kilometres. On a road trip, the car travels a distance of 1450 km and the fuel cost is \$2.02 per litre. [2]

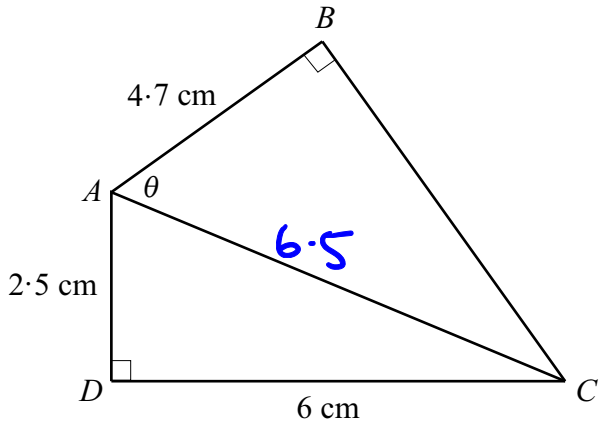
What is the total fuel cost for this trip? Give your answer correct to the nearest cent.

$\frac{1450}{100} \times 7{\cdot}6 \times 2{\cdot}02$ ✓

$= \$222{\cdot}60$ ✓

QUESTION TWENTY-FIVE (3 marks) **Marks**

Two right-angled triangles, ABC and ADC, are shown below. [3]



Calculate the size of angle θ. Give your answer correct to the nearest minute.

$AC^2 = (2{\cdot}5)^2 + (6)^2$

$\therefore AC = 6{\cdot}5$ ✓

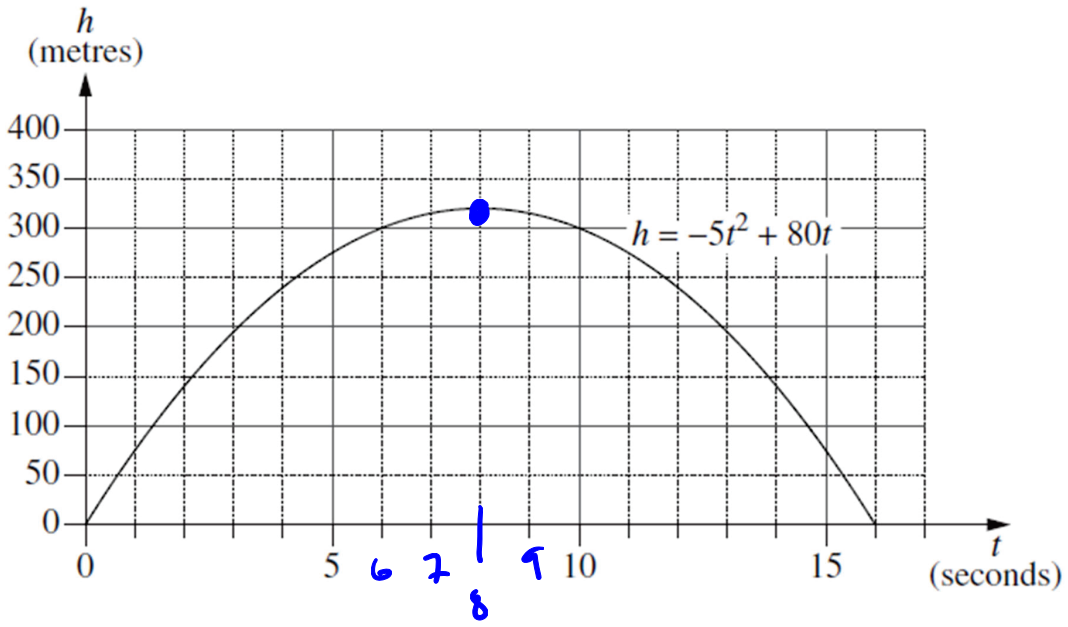
$\cos\theta = \frac{4{\cdot}7}{6{\cdot}5}$ ✓

$\therefore \theta = 43^\circ 41'$ ✓

QUESTION TWENTY-SIX (2 marks) **Marks**

An object is projected vertically into the air. Its height, h metres, above the ground after t seconds is given by $h = -5t^2 + 80t$. [2]

A graph of this non-linear function is shown below.



Using the graph and a calculation, find the maximum height of the object above the ground.

max height @ $t = 8$ ✓

$h = -5(8)^2 + 80(8)$

$= 320$ m ✓

QUESTION TWENTY-SEVEN (3 marks) **Marks**

Charlotte has a credit card on which interest at 18% per annum, compounded daily, is charged on the amount owing. [3]

At the beginning of the month, Charlotte owes \$680 on her credit card. She makes no other purchases using the credit card, but fifteen days later, she repays \$350. Assuming that interest is charged for the fifteen days, calculate the amount owing on the credit card immediately after the \$350 payment is made. Give your answer correct to the nearest cent.

$680\left(1+\frac{0.18}{365}\right)^{15}-350$ ✓ ✓

$= \$335.05$ ✓

QUESTION TWENTY-EIGHT (2 marks) **Marks**

The correlation coefficient between the time since a client joins Joe Delaney's body building program and their mass is 0·45. A client is confident that after 5 months their weight would have significantly reduced. Discuss two problems with this interpretation. [2]

* It is a positive value of r, so weight should increase not decrease. ✓

* The correlation is moderate/weak. That is, not strong. ✓

QUESTION TWENTY-NINE (3 marks) **Marks**

Henry invests \$22 000 in a savings account. Interest is paid at a fixed monthly rate. At the end of each month, after the monthly interest is added, Henry makes a deposit of \$700. 3

Henry has created a spreadsheet to show the activity in his savings account. The details for the first 6 months are shown below.

Month	Amount in account at beginning of month	Monthly interest	Deposit	Amount in account at the end of month
1	22 000.00	55.00	700	22 755.00
2	22 755.00	56.89	700	23 511.89
3	23 511.89	58.78	700	24 270.67
4	24 270.67	60.68	700	25 031.35
5	25 031.35	62.58	700	25 793.93
6	25 793.93	64.48	700	26 558.41
7	26558.41	66.40 ✓	700	27 324.81 ✓

By finding the monthly rate of interest, complete the final row above for the 7th month.

$\frac{55}{22\,000} \times 100\% = 0.25\%$ per month ✓

QUESTION THIRTY (4 marks) **Marks**

Mia believes that the time it takes for an ice cube to melt (M minutes) varies inversely with the room temperature (T°C). Mia observes that at a room temperature of 20°C it takes 9 minutes for an ice cube to melt.

(a) Find the equation relating M and T. 2

$M = \frac{k}{T}$ ✓

$9 = \frac{k}{20}$

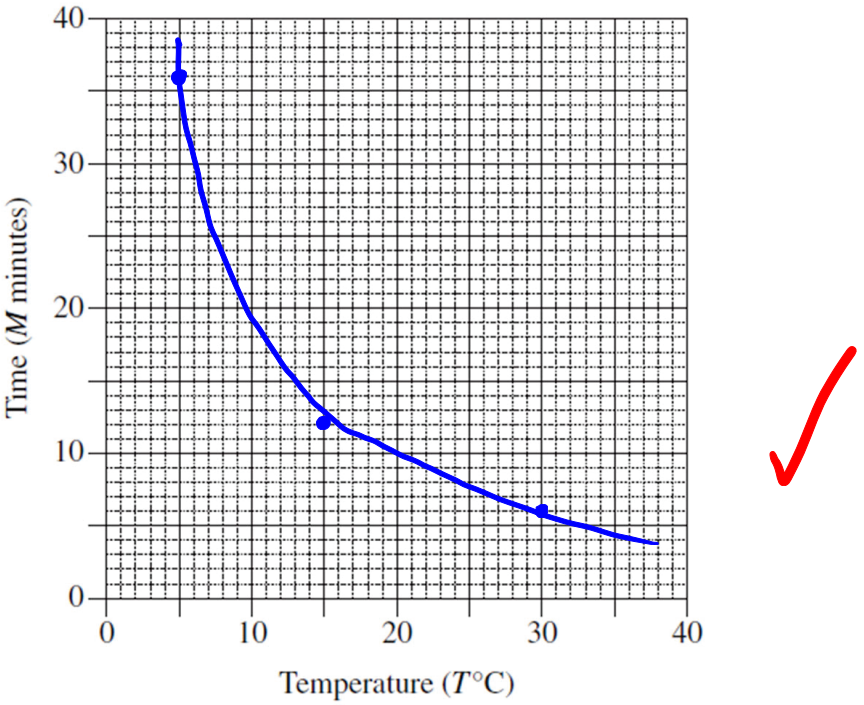
$k = 180$ ✓

$\therefore \quad M = \frac{180}{T}$

(b) By first completing this table of values below, graph the relationship between temperature and time from $T = 5$°C to $T = 30$°C. 2

T	5	15	30
M	36	12	6

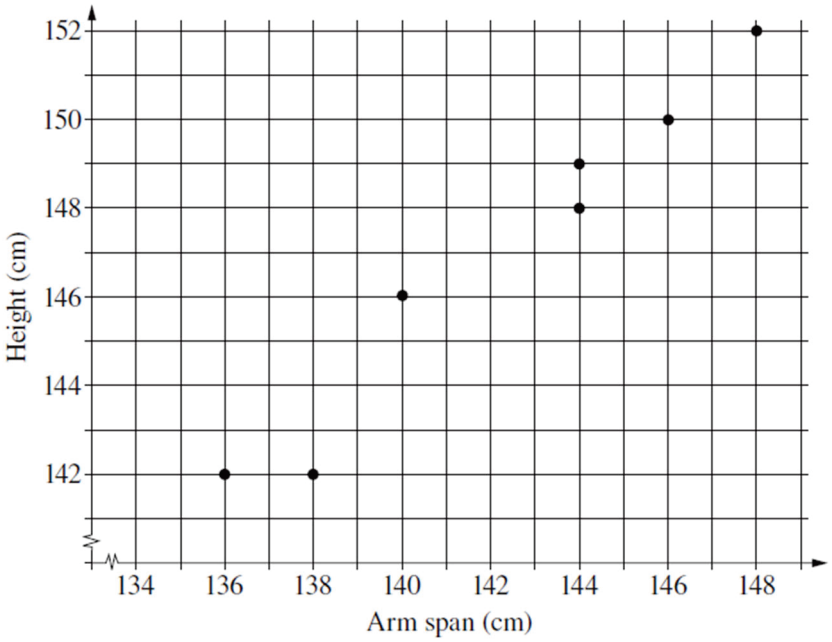
✓



✓

QUESTION THIRTY-ONE (3 marks) **Marks**

A set of bivariate data is collected by measuring the height and arm span of seven children. The graph below shows a scatterplot of these measurements.



(a) Calculate Pearson's correlation coefficient for the data. Give your answer correct to two decimal places. [1]

0·98 ✓

(b) Find the equation of the least-squares regression line. Give numbers correct to three significant figures. [2]

$a = 23.7$ ✓ $a + bx$

$b = 0.866$

$\therefore y = 23.7 + 0.866x$ ✓

QUESTION THIRTY-TWO (4 marks) **Marks**

Daniel inherits \$50 000 and invests it in an account earning interest at a rate of 0·6% per month. At the end of each month, immediately after the interest has been paid, Daniel withdraws \$700.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1{\cdot}006) - 700\,,$$

where $n = 1, 2, 3, \ldots$ and $A_0 = 50\,000$.

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. [2]

$A_0 = 50\,000$

$A_1 = 50\,000(1.006) - 700$ ✓

$= 49\,600$

$A_2 = 49\,600(1.006) - 700$

$= 49\,197.60$

$A_3 = 49\,197.60\,(1.006) - 700$

$= \$48\,792.79$ ✓

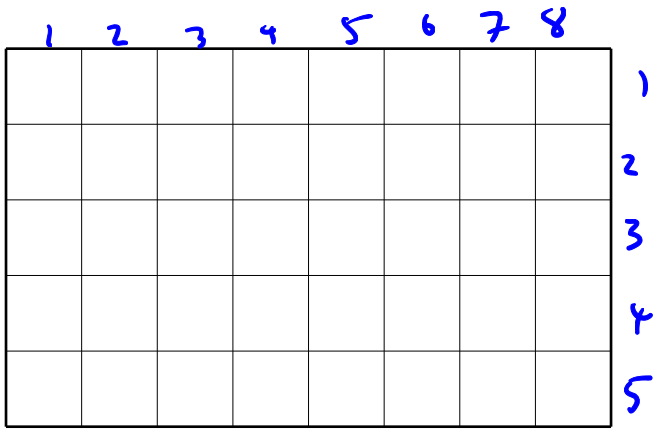
(b) Calculate the amount of interest earned in the first three months. [2]

$700 \times 3 - (50\,000 - 48\,792.79)$ ✓

$= \$892.79$ ✓

QUESTION THIRTY-THREE (4 marks) **Marks**

A rectangular sportsground has been drawn to scale on a 1 cm by 1 cm grid as shown below. The scale used is 1 : 2500. [4]



Owen took 15 minutes to walk around the perimeter of this sportsground.

What was Owen's average speed in kilometres per hour?

$8 \times 2 + 5 \times 2 = 26$ cm ✓

$26 \times 2500 = 65\,000$ cm

$= 0.65$ km ✓

$s = \frac{d}{t}$

$= \frac{0.65}{\frac{15}{60}}$ ✓

$= 0.65 \div \frac{15}{60}$

$= 2.6$ km/h ✓

QUESTION THIRTY-FOUR (4 marks) **Marks**

The table below shows the future value of an annuity of \$1.

Future values of an annuity of \$1

Years	Interest Rate per Annum			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

Harper is saving for a trip and estimates she will need \$20 000. She opens an account earning 4% per annum, compounded annually.

(a) How much does Harper need to deposit every year if she wishes to have enough money for the trip in 5 years time? 2

$x(5.416) = 20000$

$x = \frac{20000}{5.416}$ ✓

$= \$3692.76$ ✓

(b) How much interest will Harper earn on her investment over the 5 years? Give your answer correct to the nearest dollar. 2

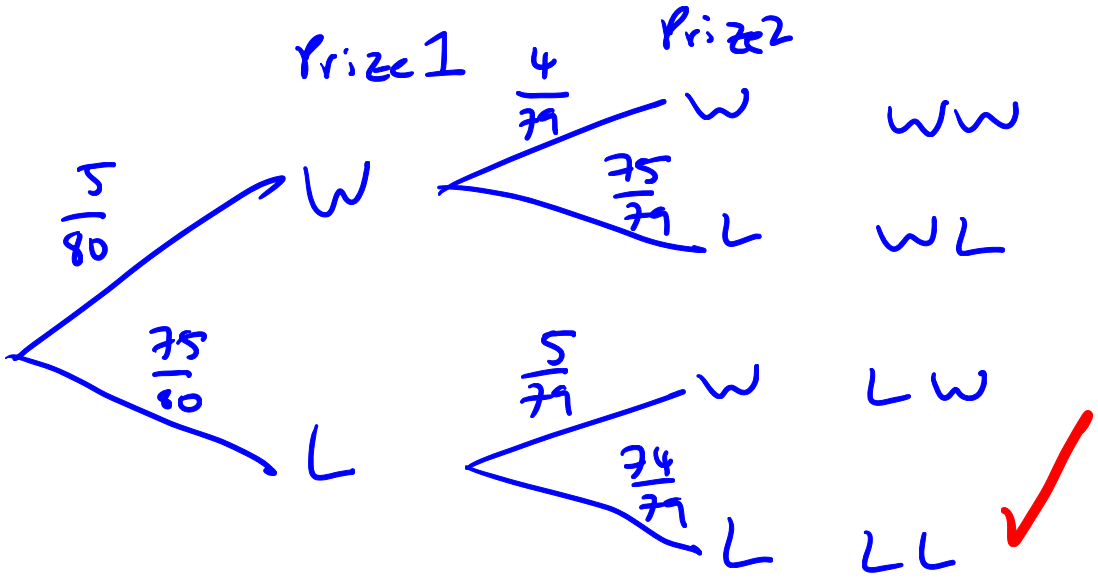
$20000 - 3692.76 \times 5$ ✓

$\doteq \$1536$ ✓

QUESTION THIRTY-FIVE (3 marks) **Marks**

Eighty tickets are sold in a raffle with two prizes. Matthew buys five tickets.

(a) By drawing a tree diagram, or otherwise, find the probability that Matthew wins both prizes. 2



$\frac{5}{80} \cdot \frac{4}{79} = \frac{1}{316}$ ✓

(b) Find the probability that Matthew wins at least one prize. 1

$P(\text{at least 1}) = 1 - P(LL)$

$= 1 - \frac{75}{80} \cdot \frac{74}{79}$

$= \frac{77}{632}$ ✓

QUESTION THIRTY-SIX (7 marks) **Marks**

The Intelligence Quotient (IQ) scores for adults in City A are normally distributed with a mean of 105 and a standard deviation of 12.

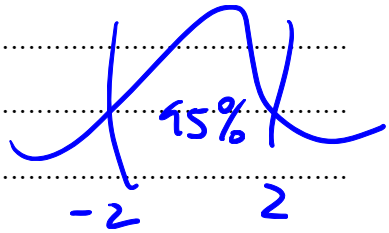
The IQ scores for adults in City B are normally distributed with a mean of 114 and a standard deviation of 15.

(a) Jack is an adult who lives in City A and has an IQ score of 129. 2

What percentage of the adults in this city have an IQ score higher than Jack's?

$z = \frac{129 - 105}{12}$

$= 2$ ✓



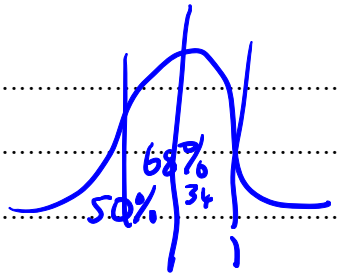
$\therefore$ 2.5% ✓ have a score higher than Jack's.

(b) There are 500 000 adults living in City B. 2

Calculate the number of adults in City B that would be expected to have an IQ score lower than Jack's.

$z = \frac{129 - 114}{15}$

$= 1$



$\therefore$ 84% ✓ would have a score lower

$\therefore$ $(0.84)(500000) = 420000$ adults. ✓

QUESTION THIRTY-SIX (Continued)

(c) Lucas, an adult who lives in City A, moves to City B. The z-score corresponding to his IQ score in City A is the same as the z-score corresponding to his IQ score in City B. [3]

By first forming an equation, calculate Lucas' IQ score.

$$\frac{x-105}{12} = \frac{x-114}{15} \checkmark$$

$$15x - 1575 = 12x - 1368 \checkmark$$

$$3x = 207$$

$$x = 69 \checkmark$$

$\therefore$ Lucas has an IQ of 69.

QUESTION THIRTY-SEVEN (3 marks) **Marks**

The mean of three numbers is 255. If two of the numbers are 102 and 256, find the third number. [3]

$$\frac{x + 102 + 256}{3} = 255 \checkmark$$

$$x + 102 + 256 = 765 \checkmark$$

$$x + 358 = 765$$

$$x = 407$$

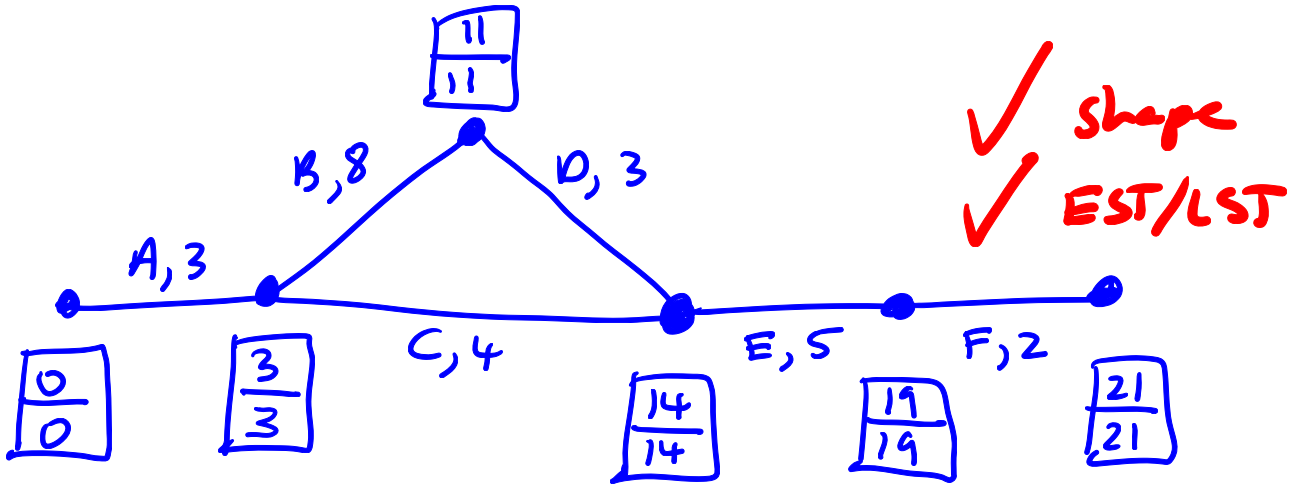
$\therefore$ The third number is 407. $\checkmark$

QUESTION THIRTY-EIGHT (4 marks) **Marks**

A project requires activities A to F to be completed. The activity chart shows the immediate prerequisite(s) and duration for each activity.

Activity	Immediate prerequisite(s)	Duration in hours
A	——	3
B	*A*	8
C	*A*	4
D	*B*	3
E	*C, D*	5
F	*E*	2

(a) Draw a network diagram showing the EST and the LST for each activity. 2



(b) Write down the minimum time for the project to be completed. 1

21 hours ✓

.....

(c) Determine the float time of the non-critical activity. 1

14 − 4 − 3 = 7 hours ✓

.....

.....

.....

QUESTION THIRTY-NINE (3 marks) **Marks**

The formula below is used to calculate an estimate for blood alcohol content (BAC) for females: [3]

$$BAC_{Female} = \frac{10N - 7{\cdot}5H}{5{\cdot}5M}$$

The number of hours required for a person to reach zero BAC after they stop consuming alcohol is given by the following formula:

$$\text{Time} = \frac{BAC}{0{\cdot}015}$$

Abigail weighs 63 kg. She consumed 5 standard drinks between 7:45 pm and 1:00 am the following day. She then stopped drinking alcohol.

Using the given formulae, calculate the time in the morning when Abigail's BAC should reach zero.

$BAC = \frac{10(5) - 7{\cdot}5(5{\cdot}25)}{5{\cdot}5(63)}$ ✓

$= 0{\cdot}03$

$\text{Time} = \frac{0{\cdot}03}{0{\cdot}015}$

$= 2$ hours ✓

∴ Reach zero @ 3am. ✓

QUESTION FORTY (5 marks) **Marks**

A large group of people undertook a survey regarding the number of hours of sleep they had in a certain week. The results of the survey were normally distributed with 2·5% of the people indicating that they had less than 42 hours of sleep per week and 2·5% of the people indicating that they had more than 54 hours of sleep per week.

(a) Determine the mean number of hours of sleep per week of the group. 1

$\frac{54+42}{2} = 48$ ✓

(b) What was the standard deviation? 1

$z = 2$ for 54

$\therefore \quad 6 = \frac{54-48}{2} = 3$ ✓

(c) A person from the group has a z-score of $-2{\cdot}5$. How many hours of sleep per week do they have? 1

$-2{\cdot}5 = \frac{x-48}{3}$

$x - 48 = -7{\cdot}5$

$x = 40{\cdot}5$ hours ✓

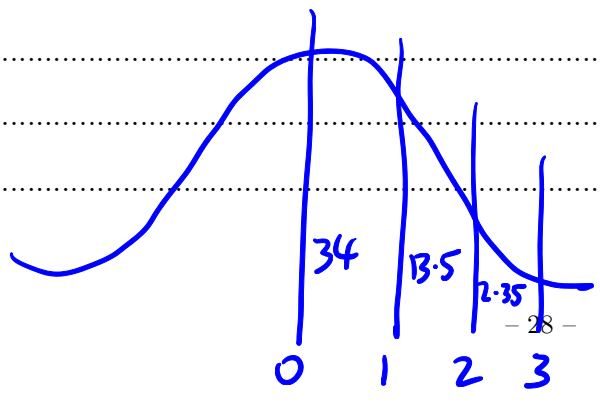
(d) A person is selected randomly from the group. Determine the probability as a percentage that they had between 51 and 57 hours of sleep per week. 2

51 $z = 1$

57 $z = 3$ ✓

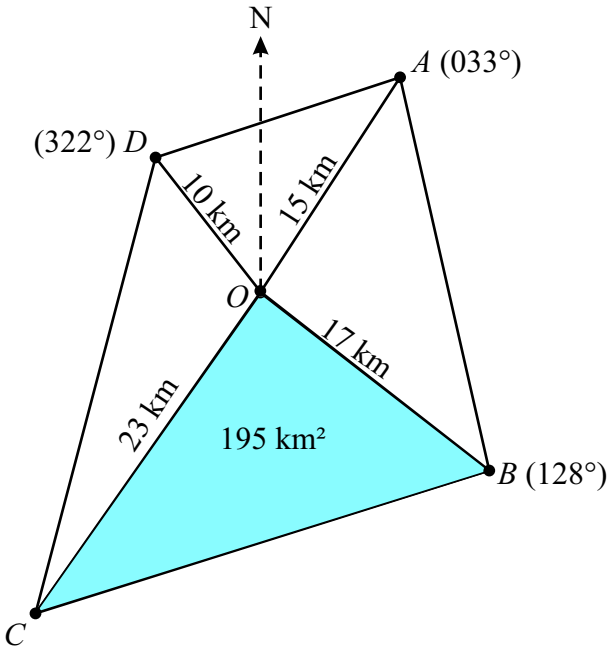
$13{\cdot}5\% + 2{\cdot}35\%$

$= 15{\cdot}85\%$ ✓



QUESTION FORTY-ONE (3 marks) **Marks**

A compass radial survey shows the positions of four towns A, B, C and D relative to point O. [3]



The area of triangle BOC is $195\,\text{km}^2$.

Calculate the bearing of town C from point O. Give your answer correct to the nearest degree.

$195 = \frac{1}{2}(23)(17)\sin\theta$ ✓

$\sin\theta = \frac{390}{391}$

$\therefore \theta = 86°$ ✓

$\therefore$ Bearing is 214°T ✓

END OF PAPER