Neap

Trial Examination 2022

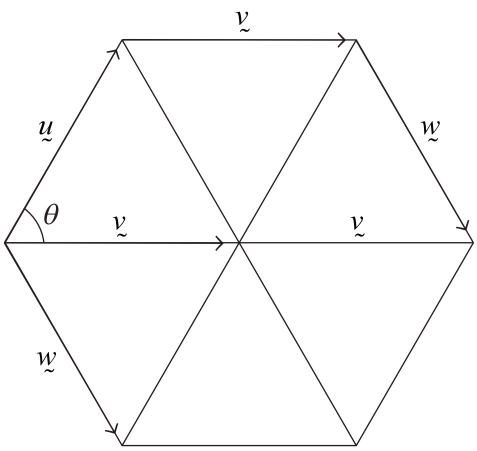
HSC Year 12 Mathematics Extension 1

Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **C** $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{\tan\theta+\tan\frac{\pi}{3}}{1-\tan\theta\tan\frac{\pi}{3}}$ $=\frac{\frac{1}{3}+\sqrt{3}}{1-\frac{\sqrt{3}}{3}}\times\frac{3}{3}$ $=\frac{1+3\sqrt{3}}{3-\sqrt{3}}$ Alternatively: $\tan\theta=\frac{1}{3}$ $\theta=\tan^{-1}\left(\frac{1}{3}\right)$ $=0.32175\ldots$ Going through each option to check gives: Option **A**: $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{\sqrt{3}+3}{3\sqrt{3}-1}$ $\theta+\frac{\pi}{3}=\tan^{-1}\left(\frac{\sqrt{3}+3}{3\sqrt{3}-1}\right)$ $\theta=-0.20184\ldots$ Option **B**: $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{\sqrt{3}-3}{3\sqrt{3}+1}$ $\theta+\frac{\pi}{3}=\tan^{-1}\left(\frac{\sqrt{3}-3}{3\sqrt{3}+1}\right)$ $\theta=-1.24904\ldots$ Option **C**: $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{1+3\sqrt{3}}{3-\sqrt{3}}$ $\theta+\frac{\pi}{3}=\tan^{-1}\left(\frac{1+3\sqrt{3}}{3-\sqrt{3}}\right)$ $\theta=0.32175\ldots$ (continues on next page)	ME–T2 Further Trigonometric Identities ME11–1 Band E2

Answer and explanation	**Syllabus content, outcomes and targeted performance bands**
(continued) Option **D**: $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{1-3\sqrt{3}}{3+\sqrt{3}}$ $\theta+\frac{\pi}{3}=\tan^{-1}\left(\frac{1-3\sqrt{3}}{3+\sqrt{3}}\right)$ $\theta=-1.77265\ldots$ Therefore, **C** is correct.	
Question 2 **A** Let $u=(\ln x)^2$. $\therefore du=2(\ln x)\times\frac{1}{x}dx$ *Note: Although the limits are the same for each option, the limits should always be changed when using the substitution method.* When $x=e,\ u=(\ln e)^2$ $=1$ When $x=e^2,\ u=(\ln e^2)^2$ $=4$ Rewriting the integral gives: $\int_e^{e^2}\frac{(\ln x)^3}{x}dx=\frac{1}{2}\int_e^{e^2}(\ln x)^2\frac{2(\ln x)}{x}dx$ $\therefore \frac{1}{2}\int_1^4 u\ du$	ME–C2 Further Calculus Skills ME12–1 Bands E2–E3
Question 3 **C** $\int\frac{4}{\sqrt{9-x^2}}dx=4\int\frac{1}{\sqrt{(3)^2-x^2}}dx$ $=4\sin^{-1}\left(\frac{x}{3}\right)+c$	ME–C2 Further Calculus Skills ME12–1 Bands E2–E3

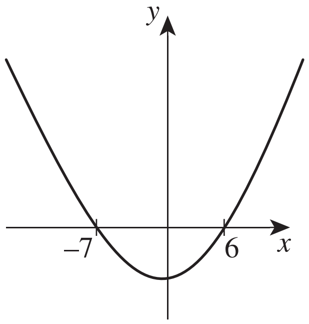
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 4 **B**  As shown on the diagram through vector addition, $\underset{\sim}{u}+\underset{\sim}{v}+\underset{\sim}{w}=2\underset{\sim}{v}$. $\therefore \underset{\sim}{u}\cdot\left(\underset{\sim}{u}+\underset{\sim}{v}+\underset{\sim}{w}\right)=\underset{\sim}{u}\cdot 2\underset{\sim}{v}$ $\cos\theta=\dfrac{\underset{\sim}{u}\cdot 2\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert 2\underset{\sim}{v}\rvert}$ $\therefore \underset{\sim}{u}\cdot 2\underset{\sim}{v}=\lvert\underset{\sim}{u}\rvert\lvert 2\underset{\sim}{v}\rvert\cos\theta$ Since all the triangles are equilateral, $\theta=\dfrac{\pi}{3}$, $\lvert\underset{\sim}{u}\rvert=4$ and $\lvert 2\underset{\sim}{v}\rvert=8$. $\therefore \underset{\sim}{u}\cdot 2\underset{\sim}{v}=4\times 8\times\cos\dfrac{\pi}{3}$ $=16$	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3
Question 5 **A** **A** is correct. This option is reached through a process of elimination. When $x = 1$ and $y = 1$, the gradient is 0. Therefore, we can eliminate **B** and **C**. When $x = -1$ and $y = 1$, the gradient is negative. Therefore, we can eliminate **D**, leaving **A** as the only viable option.	ME–C3 Applications of Calculus ME12–4 Bands E2–E3

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 6 D For points of intersection, let $f(x) = g(x)$. $x^3 - 2x^2 + 3 = x^3 + 3x^2 - 2$ $5x^2 = 5$ $x^2 = 1$ $\therefore x = -1,\ 1$ $\text{area} = \int_{-1}^{1} \left(x^3 - 2x^2 + 3\right) - \left(x^3 + 3x^2 - 2\right)\, dx$ $= \int_{-1}^{1} x^3 - 2x^2 + 3 - x^3 - 3x^2 + 2\, dx$ $= \int_{-1}^{1} -5x^2 + 5\, dx$	ME–C3 Applications of Calculus ME12–1 Bands E2–E3
Question 7 D $2\cos x - 3\sin x = R\cos(x + \theta)$ $= R\cos x\cos\theta - R\sin x\sin\theta$ Equating both sides: $2\cos x = R\cos x\cos\theta$ and $3\sin x = R\sin x\sin\theta$ Therefore: $R\cos\theta = 2 \quad (1)$ $R\sin\theta = 3 \quad (2)$ $\frac{(2)}{(1)}$ $\therefore \tan\theta = \frac{3}{2}$	ME–T3 Trigonometric Equations ME12–3 Bands E2–E3
Question 8 C The 'worst-case scenario' is if five students receive an A grade, five students receive a B grade, five students receive a C grade, five students receive a D grade and five students receive an E grade. The next student must receive a grade of A, B, C, D, or E, so that they will be the sixth student to receive that grade. $\therefore 5 \times 5 + 1 = 26$ students	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3

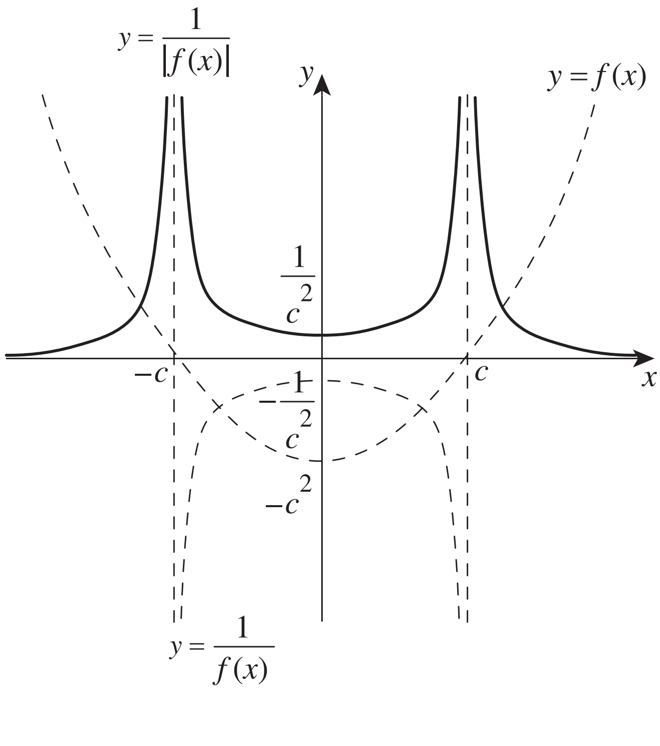
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **B** For $\underset{\sim}{p}$ and $\underset{\sim}{q}$ to be parallel: $\underset{\sim}{p} = k\underset{\sim}{q}$, where k is a constant. $\begin{pmatrix} t-8 \\ 6 \end{pmatrix} = k\begin{pmatrix} 3 \\ 2t \end{pmatrix}$ $t-8 = 3k \quad (1)$ $6 = 2kt \quad (2)$ From (2): $k = \frac{3}{t} \quad (3)$ Substituting (3) into (1): $t-8 = 3\left(\frac{3}{t}\right)$ $t^2 - 8t = 9$ $t^2 - 8t - 9 = 0$ $(t-9)(t+1) = 0$ $t = -1, 9$	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3
Question 10 **C** 5 glasses: $\binom{9}{5}$ 4 glasses, 1 no glasses: $\binom{9}{4}\times\binom{3}{1}$ 3 glasses, 2 no glasses: $\binom{9}{3}\times\binom{3}{2}$ $\text{total} = \binom{9}{5}+\binom{9}{4}\times\binom{3}{1}+\binom{9}{3}\times\binom{3}{2}$ $= 756$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a) $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$ $P(3) = 0$ and $P'(3) = 0$ $P(3) = 8(3)^4 - 38(3)^3 + 9(3)^2 + 3a + b$ $0 = 648 - 1026 + 81 + 3a + b$ $3a + b = 297 \quad (1)$ $P'(x) = 32x^3 - 114x^2 + 18x + a$ $P'(3) = 32(3)^3 - 114(3)^2 + 18(3) + a$ $0 = 864 - 1026 + 54 + a$ $a = 108$ Substitute $a = 108$ into (1): $3(108) + b = 297$ $b = -27$	ME–F2 Polynomials ME11–1 Bands E2–E3 • Provides the correct solution . . . . . .3 — • Substitutes $x = 3$ into $P(x)$. AND • Substitutes $x = 3$ into $P'(x)$. . . . . .2 — • Substitutes $x = 3$ into $P(x)$. OR • Substitutes $x = 3$ into $P'(x)$. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $\binom{5n+3}{5n+1} = \frac{(5n+3)!}{(5n+3-(5n+1))!(5n+1)!}$ $= \frac{(5n+3)!}{(2)!(5n+1)!}$ $= \frac{(5n+3)(5n+2)(5n+1)!}{2(5n+1)!}$ $= \frac{(5n+3)(5n+2)}{2}$ $\frac{(5n+3)(5n+2)}{2} \geq 528$ $25n^2 + 25n + 6 \geq 1056$ $n^2 + n - 42 \geq 0$ $(n-6)(n+7) \geq 0$ Sketching the graph of $(n - 6)(n + 7) \geq 0$ gives:  Reading from the graph, the sections where $y \geq 0$ occur when $x \leq -7$ and $x \geq 6$. Therefore, $n \leq -7$, $n \geq 6$. As n is a positive integer, $n \geq 6$.	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Provides the correct solution . . . . . .3 • Uses the formula for $\binom{n}{k}$ AND simplifies to the correct quadratic . . . . .2 • Uses the formula for $\binom{n}{k}$ AND makes some progress toward simplifying the correct quadratic 1

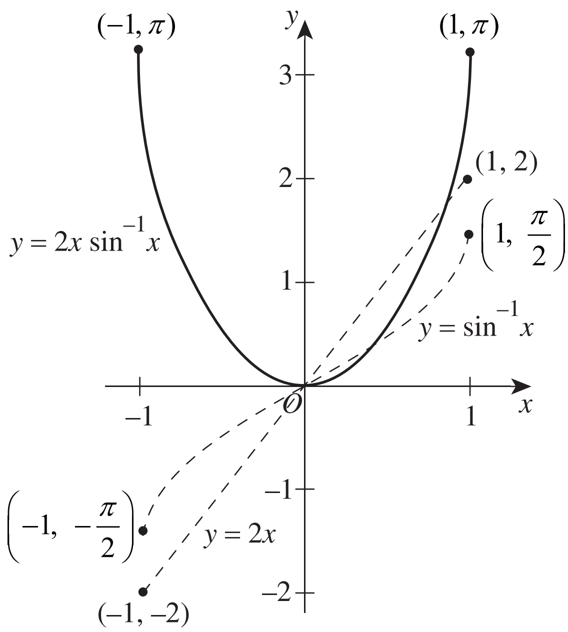
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) (i) Volume of a cylinder: $V = \pi r^2 h$ $= \pi \times 0.4^2 \times h$ $= 0.16\pi h$ $\frac{dV}{dh} = 0.16\pi$ Using the chain rule: $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $k\sqrt{h} = 0.16\pi \times \frac{dh}{dt}$ $0.16\pi \times \frac{1}{\sqrt{h}} \times \frac{dh}{dt} = k$ $0.16\pi \int \frac{1}{\sqrt{h}}\, dh = \int k\, dt$ $0.16\pi \int h^{-\frac{1}{2}}\, dh = kt + c$ $0.16\pi \left[\frac{h^{\frac{1}{2}}}{\frac{1}{2}} \right] = kt + c$ $0.32\pi\sqrt{h} = kt + c$ When $t = 0$, $h = 1$. $0.32\pi\sqrt{1} = c$ $c = 0.32\pi$ $0.32\pi\sqrt{h} = kt + 0.32\pi$ When $t = 20$, $h = 0.36$. $0.32\pi\sqrt{0.36} = 20k + 0.32\pi$ $0.192\pi = 20k + 0.32\pi$ $20k = -0.128\pi$ $k = -\frac{4}{625}\pi$	ME–C1 Rates of Change ME–C3 Applications of Calculus ME11–4, 12–4 Bands E2–E4 • Provides the correct solution . . . . . .3 • Uses the chain rule, integrates and finds the values of the constant . . . . .2 • Differentiates the volume formula and attempts to use the chain rule 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $0.32\pi\sqrt{h} = -\frac{4}{625}\pi t + 0.32\pi$ When $h = 0$: $0.32\pi\sqrt{0} = -\frac{4}{625}\pi t + 0.32\pi$ $-\frac{4}{625}\pi t + 0.32\pi = 0$ $\frac{4}{625}\pi t = 0.32\pi$ $t = 50$ minutes *Note: Consequential on answer to* ***Question 11(c)(i).***	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Provides the correct solution 1
(d) $T_7 = \binom{9}{6}(2x)^6(-p)^3$ $= \binom{9}{6} \times 64x^6 \times -p^3$ $= -5376p^3x^3$ $-5376p^3 = -672\ 000$ $p^3 = 125$ $p = 5$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Provides the correct solution . . . . . . 2 • Provides the correct solution of T_7 1
(e) 	ME–F1 Further Work with Functions ME11–1, 11–7 Bands E2–E3 • Provides the correct solution . . . . . . 3 • Draws the graph of $\frac{1}{\|f(x)\|}$ without turning points OR without asymptotes. OR • Draws the graph of $\frac{1}{f(x)}$ with turning points AND asymptotes . . . . 2 • Draws the graph of $\frac{1}{f(x)}$ without turning points OR without asymptotes 1

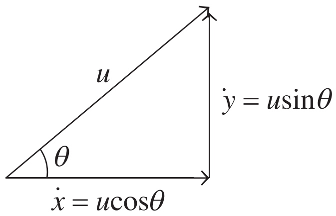
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 12	
(a) Step 1: Proving the statement is true for $n = 1$ gives: $\text{LHS} = 1 \times 2^{-(1-1)}$ $= 1$ $\text{RHS} = \dfrac{2^{1+1} - 1 - 2}{2^{1-1}}$ $= \dfrac{4 - 1 - 2}{1}$ $= 1$ LHS = RHS Therefore, the statement is true for $n = 1$. Step 2: Assuming the statement is true for $n = k$ gives: $1 + 2 \times 2^{-1} + 3 \times 2^{-2} + 4 \times 2^{-3} + \ldots + k \times 2^{-(k-1)} = \dfrac{2^{k+1} - k - 2}{2^{k-1}}$ Step 3: Proving that the statement is true for $n = k + 1$ gives: $1 + 2 \times 2^{-1} + 3 \times 2^{-2} + 4 \times 2^{-3} + \ldots + k \times 2^{-(k-1)} + (k+1) \times 2^{-k} = \dfrac{2^{k+2} - (k+1) - 2}{2^k}$ $= \dfrac{2^{k+2} - k - 3}{2^k}$ $\text{LHS} = 1 + 2 \times 2^{-1} + 3 \times 2^{-2} + 4 \times 2^{-3} + \ldots + k \times 2^{-(k-1)} + (k+1) \times 2^{-k}$ $= \dfrac{2^{k+1} - k - 2}{2^{k-1}} + (k+1) \times 2^{-k}$ (by assumption) $= \dfrac{2^{k+1} - k - 2}{2^{k-1}} + \dfrac{k+1}{2^k}$ $= \dfrac{2^{k+2} - 2k - 4}{2^k} + \dfrac{k+1}{2^k}$ $= \dfrac{2^{k+2} - 2k - 4 + k + 1}{2^k}$ $= \dfrac{2^{k+2} - k - 3}{2^k}$ = RHS If $n = k$ is true, then $n = k + 1$ is true. Therefore, by mathematical induction, the statement is true for $n \geq 1$.	ME–P1 Proof by Mathematical Induction ME12–1 Bands E2–E3 • Provides the correct proof for all steps. .3 • Provides the correct proof for step 1. AND • Makes some progress using the assumption for step 3 . . . . .2 • Provides the correct proof for step 1 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) (i) $\frac{d}{dx}(-\cot x)=\frac{d}{dx}\left(-\frac{\cos x}{\sin x}\right)$ Let $u=-\cos x$ and $v=\sin x$. $\frac{du}{dx}=\sin x$ $\frac{dv}{dx}=\cos x$ Using the quotient rule gives: $\frac{d}{dx}\left(-\frac{\cos x}{\sin x}\right)=\frac{(\sin x)(\sin x)-(-\cos x)(\cos x)}{(\sin x)^2}$ $=\frac{\sin^2 x+\cos^2 x}{\sin^2 x}$ $=\frac{1}{\sin^2 x}$	ME–C2 Further Calculus Skills ME12–1 Bands E2–E3 • Provides the correct solution . . . . . .2 • Attempts to the use quotient rule OR equivalent merit 1
(ii) Let $x=4\sin\theta$ and $dx=4\cos\theta\, d\theta$. When $x=2$, $4\sin\theta=2$. $\sin\theta=\frac{1}{2}$ $\theta=\frac{\pi}{6}$ When $x=2\sqrt{3},\ 4\sin\theta=2\sqrt{3}$. $\sin\theta=\frac{\sqrt{3}}{2}$ $\theta=\frac{\pi}{3}$ $\int_{2}^{2\sqrt{3}}\frac{1}{x^2\sqrt{16-x^2}}dx$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{4\cos\theta}{(4\sin\theta)^2\sqrt{16-(4\sin\theta)^2}}d\theta$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{4\cos\theta}{16\sin^2\theta\sqrt{16-16\sin^2\theta}}d\theta$ (continues on next page)	ME–C2 Further Calculus Skills ME12–1 Bands E2–E3 • Provides the correct solution . . . . . .4 • Finds the complete integrand in terms of θ. . . . . .3 • Finds $\frac{dx}{d\theta}$. AND • Changes the limits . . . . .2 • Finds $\frac{dx}{d\theta}$. OR • Changes the limits 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) (continued) $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{16\sin^2\theta \times 4\sqrt{1-\sin^2\theta}} d\theta$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{16\sin^2\theta \times 4\sqrt{\cos^2\theta}} d\theta$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{16\sin^2\theta \times 4\cos\theta} d\theta$ $= \frac{1}{16}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^2\theta} d\theta$ $= \frac{1}{16}\left[-\cot\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ (using part (i)) $= -\frac{1}{16}\left[\frac{1}{\tan\theta}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= -\frac{1}{16}\left(\frac{1}{\tan\frac{\pi}{3}} - \frac{1}{\tan\frac{\pi}{6}}\right)$ $= -\frac{1}{16}\left(\frac{1}{\sqrt{3}} - \sqrt{3}\right)$ $= -\frac{1}{16}\left(\frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{3}\right)$ $= \frac{2\sqrt{3}}{48}$ $= \frac{\sqrt{3}}{24}$ *Note: Consequential on answer to **Question 12(b)(i)**.*	

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) (i) $f(x) = 2x\sin^{-1}x$ $f(-x) = 2(-x)\sin^{-1}(-x)$ $= -2x\sin^{-1}(-x)$ Since the function is odd, $\sin^{-1}(-x) = -\sin^{-1}x$. $f(-x) = 2x\sin^{-1}x$ $= f(x)$ Therefore, $f(x)$ is an even function.	ME–F1 Further Work with Functions ME–T1 Inverse Trigonometric Functions ME11–1, 11–3 Bands E2–E3 • Provides the correct solution 1
(ii) 	ME–F1 Further Work with Functions ME–T1 Inverse Trigonometric Functions ME11–1, 11–3 Bands E2–E3 • Provides the correct solution . . . . . . 2 • Sketches the graph. OR • Provides the coordinates of the endpoints 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) (i) $E(\hat{p}) = E\left(\frac{X}{n}\right)$ $= \frac{E(X)}{n}$ $= \frac{np}{n}$ $= p$ $= 0.03$ $\sigma(\hat{p}) = \sigma\left(\frac{X}{n}\right)$ $= \frac{\sigma(X)}{n}$ $= \frac{\sqrt{npq}}{n}$ $= \frac{\sqrt{250 \times 0.03 \times 0.97}}{250}$ $= 0.0108$	ME–S1 The Binomial Distribution ME12–5 Bands E2–E3 • Finds the mean AND standard deviation 1
(ii) For the z-score: $z = \frac{0.02 - 0.03}{0.0108}$ $= -0.9269$ ≈ -0.93 Using the table: $P(Z < -0.93) = 1 - P(Z < 0.93)$ $= 1 - 0.8238$ $= 0.1762$ Therefore, the probability that 2% of the noodle packets in the sample weigh less than 82 grams is 0.1762. *Note: Consequential on answer to* ***Question 12(d)(i)****.*	ME–S1 The Binomial Distribution ME12–5 Bands E2–E3 • Provides the correct solution . . . . . . 2 • Uses the z-score table 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 13	
(a) (i)  $\ddot{x} = 0, \dot{x} = C_1$ When $t = 0$, $\dot{x} = u\cos\theta$. $C_1 = u\cos\theta$ $\dot{x} = u\cos\theta$ $x = ut\cos\theta + C_2$ When $t = 0$, $x = 0$. $C_2 = 0$ $x = ut\cos\theta$ $\ddot{y} = -10$ $\dot{y} = -10t + C_3$ When $t = 0$, $\dot{y} = u\sin\theta$. $C_3 = u\sin\theta$ $\dot{y} = -10t + u\sin\theta$ $y = -5t^2 + ut\sin\theta + C_4$ When $t = 0$, $y = 147$. $C_4 = 147$ $y = -5t^2 + ut\sin\theta + 147$	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3 • Provides the correct solution . . . . . .3 • Finds the equations of motion in either the horizontal OR vertical direction AND makes some progress in the other direction . . . . .2 • Makes some progress in deriving the horizontal OR vertical equations of motion 1

 TEN_Y12_MExt1_SB_2022

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) When $t = T$, $y = 0$. $-5T^2 + uT\sin\theta + 147 = 0$ $uT\sin\theta = 5T^2 - 147 \quad (1)$ When $t = T$, $v = 4u$. $v = \sqrt{\dot{x}^2 + \dot{y}^2}$ $4u = \sqrt{(u\cos\theta)^2 + (-10T + u\sin\theta)^2}$ $16u^2 = u^2\cos^2\theta + 100T^2 - 20uT\sin\theta + u^2\sin^2\theta$ $= u^2(\cos^2\theta + \sin^2\theta) + 100T^2 - 20uT\sin\theta$ $= u^2 + 100T^2 - 20uT\sin\theta$ $(\text{as } \cos^2\theta + \sin^2\theta = 1)$ $15u^2 = 100T^2 - 20uT\sin\theta \quad (2)$ Substituting (1) into (2): $15u^2 = 100T^2 - 20(5T^2 - 147)$ $= 100T^2 - 100T^2 + 2940$ $= 2940$ $u^2 = 196$ $u = 14 \ (\text{as } u > 0)$ *Note: Consequential on answer to* ***Question 13(a)(i)***.	ME–V1 Introduction to Vectors ME12–2 Bands E2–E4 • Provides the correct solution . . . . . .3 ——— • Obtains the equation $uT\sin\theta = 5T^2 - 147$. AND • Obtains the equation $15u^2 = 100T^2 - 20uT\sin\theta$ . . . . . . .2 ——— • Obtains the equation $uT\sin\theta = 5T^2 - 147$. OR • Obtains the equation $15u^2 = 100T^2 - 20uT\sin\theta$ 1
(iii) From (2): $15u^2 = 100T^2 - 20uT\sin\theta$ Substituting $u = 14$: $15(14)^2 = 100T^2 - 20(14)T\sin\theta$ $2940 = 100T^2 - 280T\sin\theta$ $280T\sin\theta = 100T^2 - 2940$ $\sin\theta = \dfrac{100T^2 - 2940}{280T}$ $= \dfrac{5T^2 - 147}{14T}$ *Note: Consequential on answer to* ***Question 13(a)(ii)***.	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3 • Provides the correct solution . . . . . .2 ——— • Uses the initial velocity from part (a)(ii) AND makes some progress toward the solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(iv) Minimum value of T is when $\sin\theta = 0$ $\left(\text{as } 0 < \theta < \frac{\pi}{2}\right)$. $\frac{5T^2 - 147}{14T} = 0$ $5T^2 - 147 = 0$ $5T^2 = 147$ $T^2 = \frac{147}{5}$ $T = \pm\frac{\sqrt{147}}{\sqrt{5}}$ $= \pm\frac{\sqrt{735}}{5}$ $T > 0$, therefore, the minimum value is $\frac{\sqrt{735}}{5}$. Maximum value of T is when $\sin\theta = 1$. $\frac{5T^2 - 147}{14T} = 1$ $5T^2 - 147 = 14T$ $5T^2 - 14T - 147 = 0$ $T = \frac{14 \pm \sqrt{(-14)^2 - 4(5)(-147)}}{2(5)}$ $= -\frac{21}{5}, 7$ $T > 0$, therefore, the maximum value is 7. $\therefore \frac{\sqrt{735}}{5} < T < 7$	ME–T3 Trigonometric Equations ME12–3 Bands E2–E4 • Provides the correct solution . . . . . .3 • Attempts to solve for $\sin\theta = 0$ AND $\sin\theta = 1$. . . . . .2 • Solves for $\sin\theta = 0$. OR • Solves for $\sin\theta = 1$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) (i) $y = x\int f(x)dx$ Let $u = x$ and $v = \int f(x)dx$. $\frac{du}{dx} = 1$ $\frac{dv}{dx} = f(x)$ Using the product rule gives: $\frac{dy}{dx} = xf(x) + \int f(x)dx$ Multiplying by x gives: $x\frac{dy}{dx} = x^2 f(x) + x\int f(x)dx$ $x\frac{dy}{dx} = x^2 f(x) + y$ $x\frac{dy}{dx} - y = x^2 f(x)$	ME–C3 Applications of Calculus ME12–4 Bands E3–E4 • Provides the correct solution . . . . . .2 • Attempts to use the product rule OR equivalent merit 1
(ii) $x\frac{dy}{dx} - y = x^5$ $x^2 f(x) = x^5$ (using part (b)(i)) $f(x) = x^3$ $y = x\int f(x)dx$ $= x\int x^3 dx$ $= x \times \frac{x^4}{4} + c$ $= \frac{x^5}{4} + c$ When $x = 2$ and $y = 5$: $5 = \frac{2^5}{4} + c$ $c = -3$ $\therefore y = \frac{x^5}{4} - 3$ *Note: Consequential on answer to* ***Question 13(b)(i).***	ME–C3 Applications of Calculus ME12–4 Bands E3–E4 • Provides the correct solution . . . . . .2 • Finds $f(x)$ using part (b)(i) OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
(a) To find the y-intercept, let $x = 0$. $y = \sqrt{m - 3(0)}$ $= \sqrt{m}$ Rearranging to make x the subject gives: $y = \sqrt{m - 3x}$ $y^2 = m - 3x$ $3x = m - y^2$ $x = \frac{1}{3}(m - y^2)$ $\text{volume} = \pi \int_0^{\sqrt{m}} \left(\frac{1}{3}(m - y^2)\right)^2 dy$ $= \frac{\pi}{9} \int_0^{\sqrt{m}} (m - y^2)^2 dy$ $= \frac{\pi}{9} \int_0^{\sqrt{m}} m^2 - 2my^2 + y^4 dy$ $= \frac{\pi}{9} \left[m^2 y - \frac{2my^3}{3} + \frac{y^5}{5}\right]_0^{\sqrt{m}}$ $= \frac{\pi}{9} \left(m^2 \times \sqrt{m} - \frac{2m \times (\sqrt{m})^3}{3} + \frac{(\sqrt{m})^5}{5} - (0 - 0 + 0)\right)$ $= \frac{\pi}{9} \left(m^2 \sqrt{m} - \frac{2}{3} m^2 \sqrt{m} + \frac{1}{5} m^2 \sqrt{m}\right)$ $= \frac{\pi}{9} \times \frac{8}{15} m^2 \sqrt{m}$ $= \frac{8\pi}{135} m^2 \sqrt{m}$ $\frac{8\pi}{135} m^2 \sqrt{m} = \frac{5000\pi}{27}$ $m^2 \sqrt{m} = 3125$ $m^{\frac{5}{2}} = 3125$ $m = (3125)^{\frac{2}{5}}$ $= 25$	ME–C3 Applications of Calculus ME12–4 Bands E2–E4 • Provides the correct solution . . . . . . 4 • Finds the integrand for the volume of solid of revolution . . . . . 3 • Finds the y-intercept. AND • Rearranges the equation to make x the subject . . . . 2 • Finds the y-intercept. OR • Rearranges the equation to make x the subject 1

 TEN_Y12_MExt1_SB_2022

		Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b)	(i)	For an inverse function to exist, the function needs to be monotonically increasing or decreasing. Solving $f'(x)=0$ to find the minimum turning point gives: $f(x)=x-4x^{\frac{1}{2}}-1$ $f'(x)=1-2x^{-\frac{1}{2}}$ $=1-\frac{2}{\sqrt{x}}$ $1-\frac{2}{\sqrt{x}}=0$ $\sqrt{x}-2=0$ $\sqrt{x}=2$ $x=4$ $\therefore k=4$	ME–F1 Further Work with Functions ME11–1 Bands E3–E4 • Provides the correct solution . . . . . .2 • Attempts to find the minimum turning point OR equivalent merit. 1
	(ii)	$f(x)=x-4\sqrt{x}-1,\ x\geq 4$ For $f^{-1}(x)$: $x=y-4\sqrt{y}-1,\ y\geq 4$ Completing the square (in terms of $\sqrt{y}$) gives: $x=(y-4\sqrt{y}+4)-4-1$ $=(\sqrt{y}-2)^2-5$ $(\sqrt{y}-2)^2=x+5$ $\sqrt{y}-2=\pm\sqrt{x+5}$ $\sqrt{y}=2\pm\sqrt{x+5}$ $y=(2\pm\sqrt{x+5})^2$ Since $y\geq 4$: $\therefore f^{-1}(x)=(2+\sqrt{x+5})^2$ *Note: Consequential on answer to **Question 14(b)(i)**.*	ME–F1 Further Work with Functions ME11–1 Bands E3–E4 • Provides the correct solution . . . . . .2 • Attempts to complete the square OR equivalent merit 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(iii)	For the minimum point on $f(x)$: $f(4) = 4 - 4\sqrt{4} - 1$ $= -5$ For $f(x)$: D: $x \geq 4$ and R: $y \geq -5$ For $f^{-1}(x)$: D: $x \geq -5$ and R: $y \geq 4$ *Note: Consequential on answer to **Question 14(b)(i)**.*	ME–F1 Further Work with Functions ME11–1 Bands E2–E3 • Provides the correct solution . . . . . .2 • Finds coordinates of the minimum turning point. OR • Finds the domain and range of $f(x)$. 1
(c) (i)	$(x + \tan\theta)(x - \cot\theta) = 0$ $x^2 - (\cot\theta)x + (\tan\theta)x - (\tan\theta)(\cot\theta) = 0$ $x^2 - (\cot\theta - \tan\theta)x - (\tan\theta)\left(\frac{1}{\tan\theta}\right) = 0$ $x^2 - \left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)x - 1 = 0$ $x^2 - \left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}\right)x - 1 = 0$ $x^2 - 2\left(\frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}\right)x - 1 = 0$ Using the double angle formulae gives: $x^2 - 2\left(\frac{\cos 2\theta}{\sin 2\theta}\right)x - 1 = 0$ $x^2 - 2(\cot 2\theta)x - 1 = 0$	ME–F2 Polynomials ME–T2 Further Trigonometric Identities ME11–1, 11–2 Bands E2–E3 • Provides the correct solution . . . . . .2 • Expands $(x + \tan\theta)(x - \cot\theta)$ AND makes some progress toward the solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $x = -\tan\theta$ and $x = \cot\theta$ are roots of $x^2 - 2(\cot 2\theta)x - 1 = 0$ Substitute $\theta = \frac{\pi}{8}$. So, $x = -\tan\left(\frac{\pi}{8}\right)$ and $x = \cot\left(\frac{\pi}{8}\right)$ are roots of: $x^2 - 2\left(\cot\left(\frac{\pi}{4}\right)\right)x - 1 = 0$ $x^2 - 2x - 1 = 0 \quad \left(\text{as } \cot\left(\frac{\pi}{4}\right) = 1\right)$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$ $= \frac{2 \pm \sqrt{8}}{2}$ $= \frac{2 \pm 2\sqrt{2}}{2}$ $= 1 \pm \sqrt{2}$ $\therefore -\tan\left(\frac{\pi}{8}\right) = 1 - \sqrt{2} \quad \left(\text{as } -\tan\left(\frac{\pi}{8}\right) < 0\right)$ and $\cot\left(\frac{\pi}{8}\right) = 1 + \sqrt{2}$ $\therefore \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$	ME–F2 Polynomials ME–T2 Further Trigonometric Identities ME11–1, 11–2, 11–7 Bands E3–E4 • Provides the correct solution . . . . . .2 • Substitutes $\theta = \frac{\pi}{8}$ into the equation and makes some progress 1
(iii) Substitute $\theta = \frac{\pi}{16}$. So, $x = -\tan\left(\frac{\pi}{16}\right)$ and $x = \cot\left(\frac{\pi}{16}\right)$ are roots of: $x^2 - 2\left(\cot\left(\frac{\pi}{8}\right)\right)x - 1 = 0$ Sum of roots: $-\tan\left(\frac{\pi}{16}\right) + \cot\left(\frac{\pi}{16}\right) = \frac{2\left(\cot\frac{\pi}{8}\right)}{1}$ $\cot\left(\frac{\pi}{16}\right) - \tan\left(\frac{\pi}{16}\right) = 2\left(1 + \sqrt{2}\right)$ (from part (c)(ii)) *Note: Consequential on answer to **Question 14(c)(ii)**.*	ME–F2 Polynomials ME–T2 Further Trigonometric Identities ME11–1, 11–2, 11–7 Bands E3–E4 • Provides the correct solution using part (c)(ii) 1