Northern Beaches Secondary College
Manly Campus

2023

Higher School Certificate Trial Examination

Mathematics Standard

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using non-erasable black pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

Total Marks: 100

Section I – 15 marks (pages 2 - 10)

- Attempt Questions 1–15
- Allow about 25 minutes for this section.

Section II – 85 marks (pages 12 - 36)

- Attempt Questions 16 – 42
- Allow about 2 hours and 5 minutes for this section.

Section I

15 marks

Attempt Questions 1-15

Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1-15

Question 1

A school principal wants to know if the students are happy with the school.

He asked the opinion of every fifth student as they left the school in the afternoon.

What type of sampling did the principal do?

A. Census
B. Random
C. Stratified
D. Systematic

Question 2

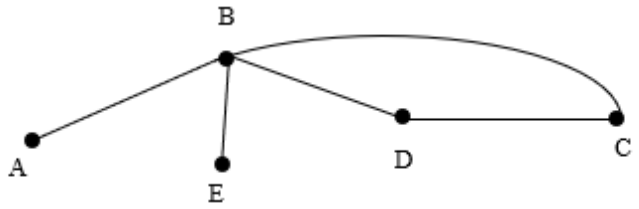
Maya is in Cape Town, which is 2 hours ahead of Coordinated Universal Time (UTC+2).Sarah is in Sydney, which is 10 hours ahead of Coordinated Universal Time (UTC+10).

When it is 3 pm in Cape Town, what time is it in Sydney?

A. 5 am
B. 7 am
C. 9 pm
D. 11pm

Question 3

Five cities are connected by this network diagram.



What is the total sum of degrees in the network?

A. 5
B. 6
C. 8
D. 10

Question 4

A patient is given 2.4 litres of a saline fluid over an 8-hour period. The fluid is delivered by an intravenous drip at a rate of 20 drops per millilitre.

What is the rate at which the fluid is delivered?

A. 10 drops/min
B. 12 drops/min
C. 100 drops/min
D. 120 drops/min

Question 5

Cal bought a new scooter that was marked at a discount.

He received 30% discount off the original price of the scooter as well as a $120 trade-in for his old scooter.

Cal then paid $720 for the new scooter. What was the original price of the new scooter?

A. $384
B. $600
C. $840
D. $1200

Question 6

The length of a desk is measured to be 2.61 metres correct to the nearest centimetre.

What is the absolute error of the length of the desk?

A. 0.005 *m*
B. 0.001 *m*
C. 0.05 *m*
D. 0.01 *m*

Question 7

The gravitational force (F) on an orbital satellite is given by the following formula.

$$F = \frac{mv^2}{r}$$

A formula for r is?

A. $r = \sqrt{\frac{m}{vF}}$

B. $r = \sqrt{\frac{mv}{F}}$

C. $r = \frac{mv^2}{F}$

D. $r = Fmv^2$

Question 8

Piper and Charlotte start from the same point, O, on a property. Piper walks 2 km on a bearing of $048°$. Charlotte also walks for 2 km and stops due South of where Piper has stopped.

What is the bearing of O from Charlotte?

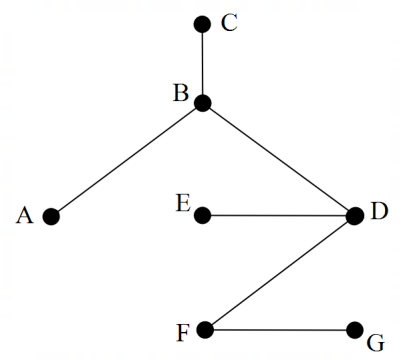
A. 048°
B. 132°
C. 228°
D. 312°

Question 9

Livinia is considering a network which connects vertices A through G and draws a minimum spanning tree for the network.

Which of the following could be the minimum spanning tree?

A.



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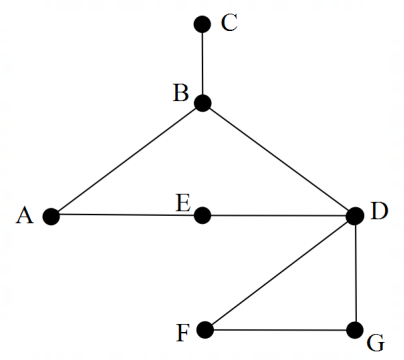
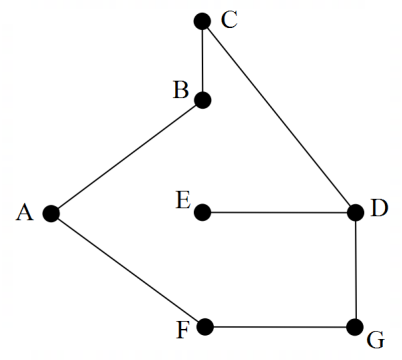
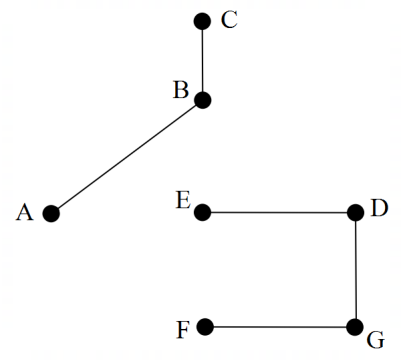


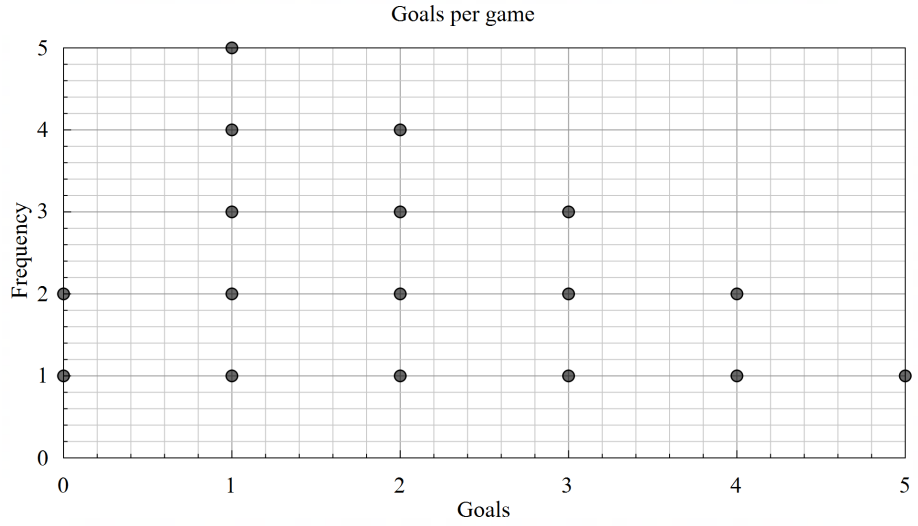
C.



D.



Question 10 A number of goals scored by a football team in each of 17 games is shown in the dot plot drawn below.



In their 18th game, the team scores 1 goal.

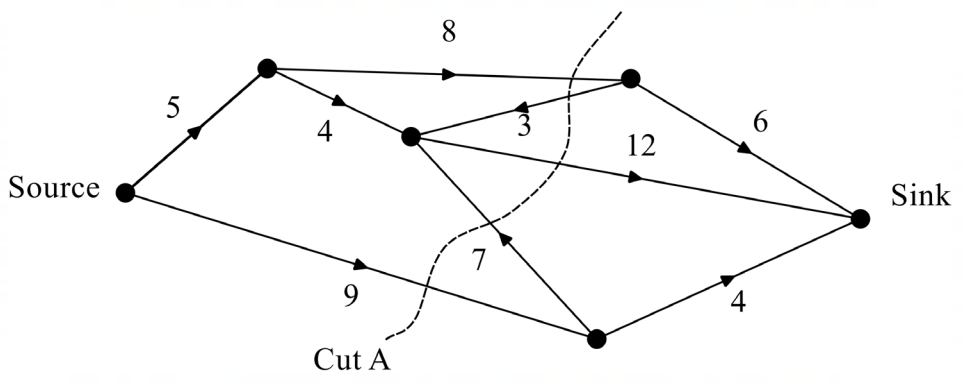
Which of the range, mean, median and mode will change after the 18th game is added to the dot plot?

A. Mean
B. Range
C. Median
D. Mode

Question 11 The z-score of Tom in a Science examination is -2. If Tom scored 34% in the Science examination, whilst the mean score is 50%, what is the standard deviation of scores?

A. 4%
B. 8%
C. 12%
D. 16%

Question 12 The network below shows the flow of water through a series of pipes.The edge numbers, in litres per minute, indicate the maximum flow through each pipe.



What is the capacity of cut A, in litres per minute?

A. 12
B. 19
C. 29
D. 39

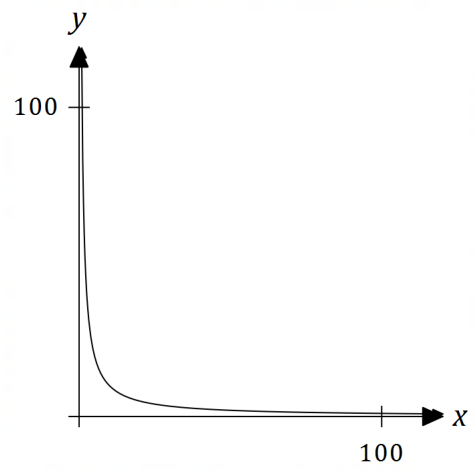
Question 13 Spencer borrows $560 000 to start his business.
His monthly repayments are $2815.94 over 25 years.

What amount of interest will Spencer pay over the term of the loan?

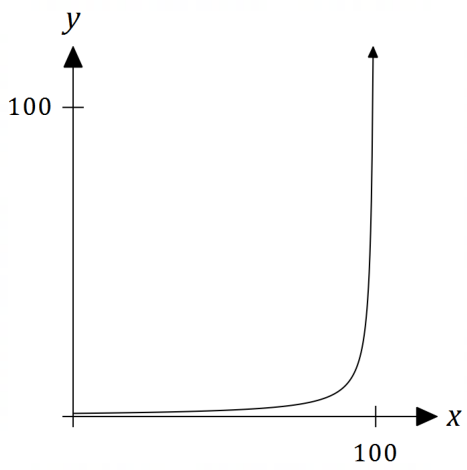
A. $ 22 400
B. $ 70 399
C. $ 284 782
D. $ 844 782

Question 14 Which of the following could be the graph of $y = \frac{100}{x}$?

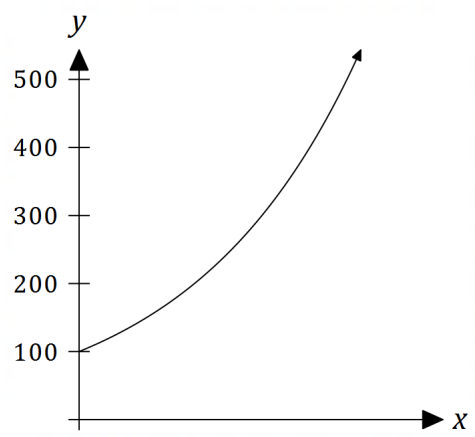
A.



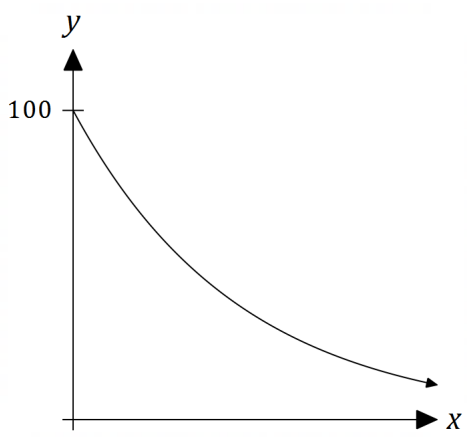
B.



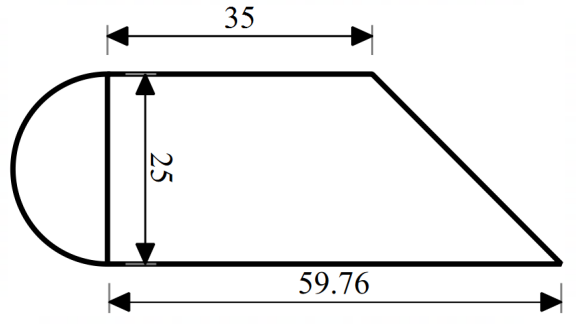
C.



D.



Question 15 Calculate the area of the shape below, rounded to the nearest whole number.



A. 715

B. 1050

C. 1430

D. 2166

End of Multiple Choice Questions

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Section II Answer Booklet

Section II

85 marks

Attempt All Questions – 16 to 42

Allow about 2 hours and 5 minutes for this section.

Instructions

- Answer the questions in the spaces provided.
  These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet.
  If you use this space, clearly indicate which question you are answering.

Question 16 (2 marks)

What is the average fuel consumption of a scooter that uses 8 litres of fuel to travel 182 km?

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Give your answer in L/100 km, correct to one decimal place.

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Question 17 (2 marks)

If it is 2:30 pm on Tuesday in Auckland (UTC $+11$), what time and day is it in Montreal (UTC -4)?

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Question 18 (2 marks)

Solve the equation $2x-5=\frac{2x-7}{3}$.

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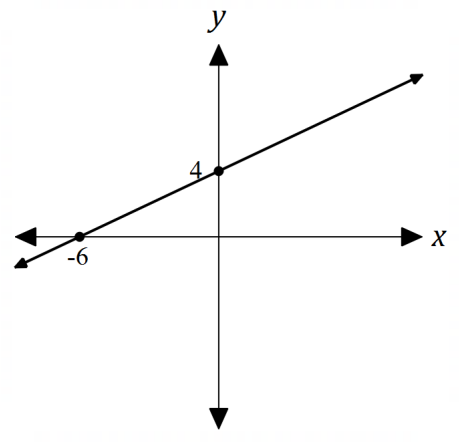
Question 19 (3 marks)

a) Find the equation of the line shown on the axes below. **2**



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b) Calculate the point of intersection of the line above with the line $x = 9$ **1**

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Question 20 (3 marks)

Rhea is driving on a highway where the legal speed limit is 110 km/h. She notices danger in the distance and immediately reacts to apply the brakes in order to avoid a collision.

If Rhea's reaction time is 1.5 seconds and she travels 50 metres in that time, use calculations to determine whether Rhea was driving within the legal limit.

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Question 21 (2 marks)

Three years ago, a couple bought 3000 shares in a business at $2.80 a share. The current value of the investment is $10 200.

Calculate the percentage increase in the couple's investment.
Give your answer as a percentage correct to 1 decimal place.

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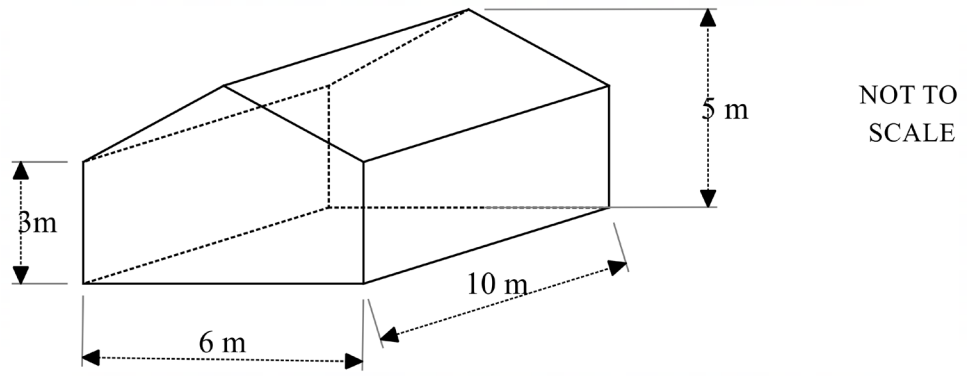
Question 22 (3 marks)

A barn is to be constructed according to the diagram below.



NOT TO SCALE

Calculate the volume of the barn, in cubic metres. **3**

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Question 23 (4 marks)

Jesse arrives at a party at 9 pm and has 4 standard drinks in the next 3 hours. She weighs 62 kg and is a P-plater with a zero BAC limit.
The formula to estimate a female's BAC is:

$$BAC_{female} = \frac{10N - 7.5H}{5.5M}$$

where N is the number of standard drinks consumed,

H is the number of hours drinking and

M is the person's mass in kilograms.

The formula to estimate the time (t hours) taken for a person to reach a BAC of zero is:

$$t = \frac{BAC}{0.015}.$$

What is the earliest time at which Jesse can legally drive herself home? **4**
Justify your answer by calculations.

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Question 24 (3 marks)

The frequency f of radio waves is inversely proportional to the wavelength w.
If a wave has a wavelength of 300 metres and a frequency of 1 600 hertz, what is the wavelength for a frequency of 1 000 hertz? **3**

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Question 25 (3 marks)

Find the two possible values of $\angle PQR$ if the area of triangle PQR is $27\ \text{cm}^2$.

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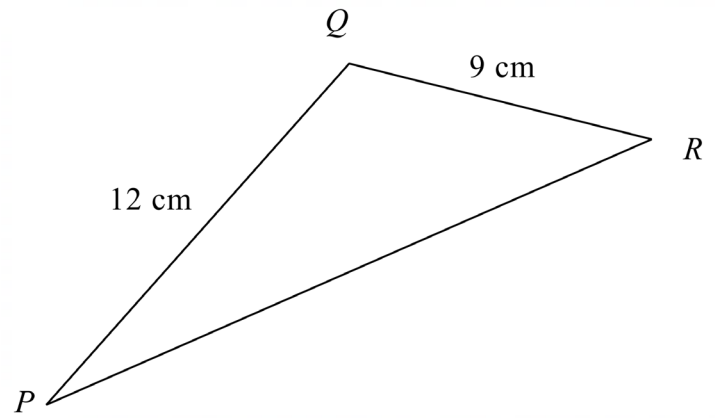

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Question 26 (3 marks)

The following method was used to estimate the current population of pygmy possums in a national park.

- 15 pygmy possums were caught, tagged and released.
- Six months later, 40 pygmy possums were caught, of which 3 had been tagged.

It is estimated that the population of pygmy possums will decrease at a rate of 10% per annum for the next five years.

What is the expected population of pygmy possums in the national park in five-years' time?

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Question 27 (3 marks)

The following table shows the Australian income tax rate.

Taxable Income	**Tax on this Income**
0 - $18 200	Nil
$18 201 - $45 000	19 cents for each $1 over $18 200
$45 001 - $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 - $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

Source credit: Australian Taxation Office (ATO)

At the end of the last financial year, Lincoln was required to pay income tax of $25 416.

Calculate Lincoln's taxable income. **3**

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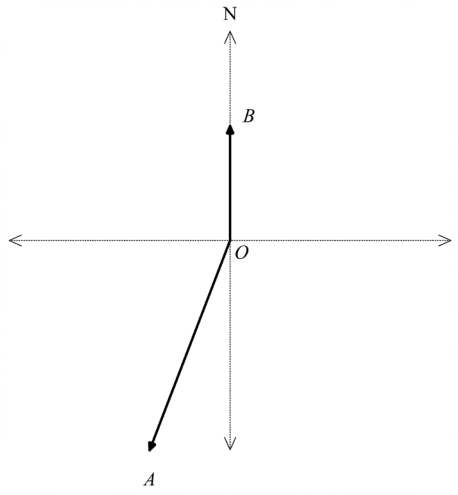
Question 28 (5 marks)

Two ships leave a harbour, O, shown in the diagram below. Ship A sails 95 km in a direction of $208^\circ T$, while ship B sails due north for 48 km.



a) Show that obtuse $\angle AOB = 152^\circ$

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b) Calculate the distance between the two ships, correct to one decimal place.

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c) What is the bearing of ship A from ship B, to the nearest degree?

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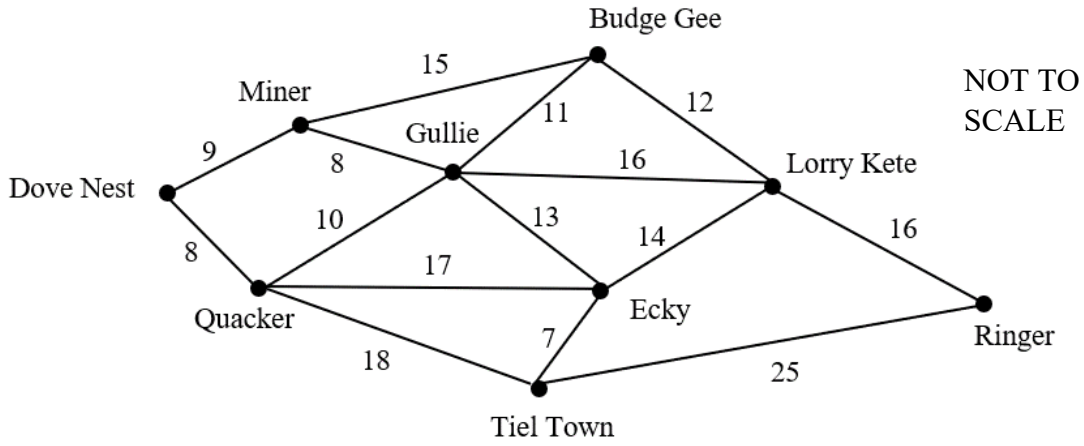
Question 29 (4 marks)

The Green Council wants to improve the NBN connection cables to various suburbs. The network diagram shows the distance between suburbs in kilometres.



a) Sketch the minimum spanning tree. **3**

b) Find the length of the minimum spanning tree that connects all the suburbs. **1**

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Question 30 (3 marks)

Shirlene owns a company producing and selling backpacks for \$80 each.

The income function is $y = 80x$, where x is the number of backpacks sold.

The cost of producing these backpacks includes a set-up cost of \$4500 and additional costs of \$30 per backpack.

a) Write the cost function in the form $y = mx + b$ **1**

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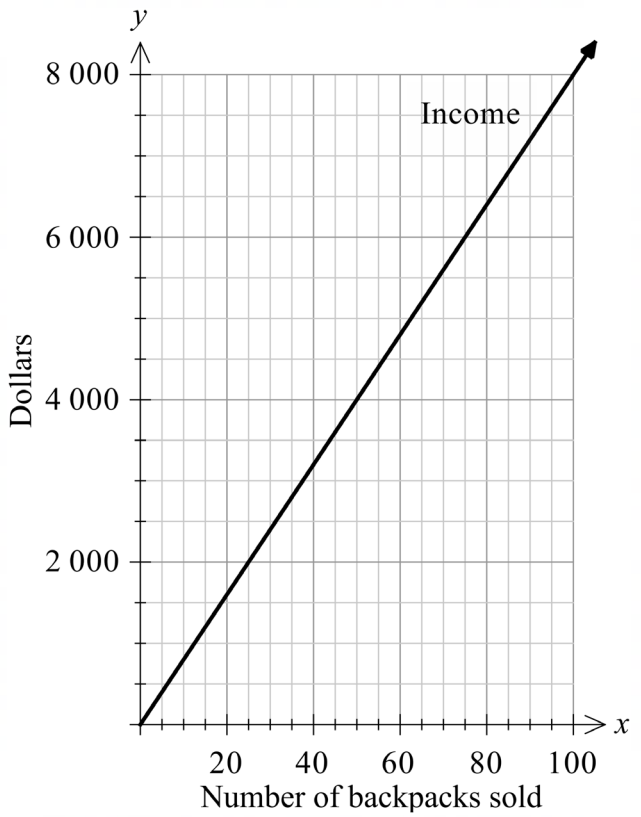
b) Draw the cost function on the set of axes below. **1**



c) Hence, or otherwise, determine Shirlene's Break-even point? **1**

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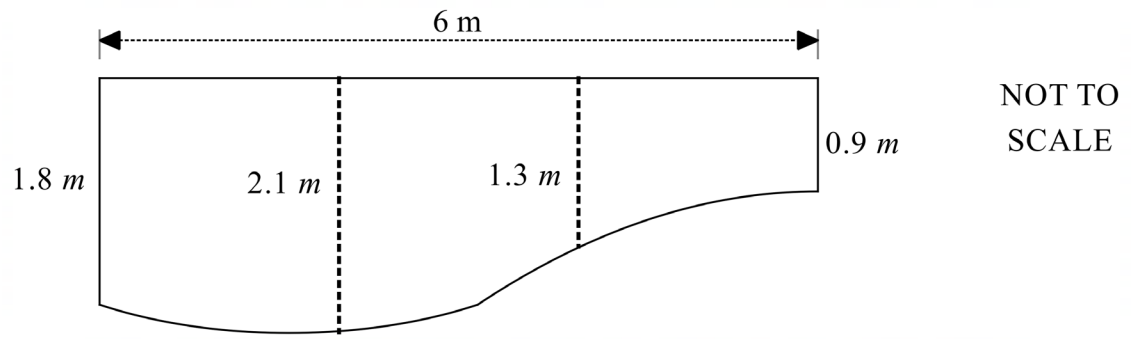
Question 31 (4 marks)

The uniform cross-section of a 12 metre long backyard pool is shown below.



Use three applications of the trapezoidal rule to determine the approximate capacity of the pool, in litres. **4**

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Question 32 (2 marks)

Explain the difference between a cycle and a path. **2**

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Question 33 (2 marks)

The formula below can be used to calculate the required dosages of medicine for children aged 1 – 12 years.

$$D = \frac{yA}{y + 12}$$

where D = dosage for children aged 1–12 years
y = age of child (in years)
A = adult dosage

What dosage should be given to a 32-month-old child if each adult dosage is 30 mL? Give your answer correct to 2 significant figures. **2**

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Question 34 (3 marks)

Ike has a bag that contains 5 red, and 8 black balls. Two balls are drawn randomly from the bag.

a) Find the probability that the first ball drawn is red. **1**

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b) Find the probability that both balls drawn are the same colour. **2**

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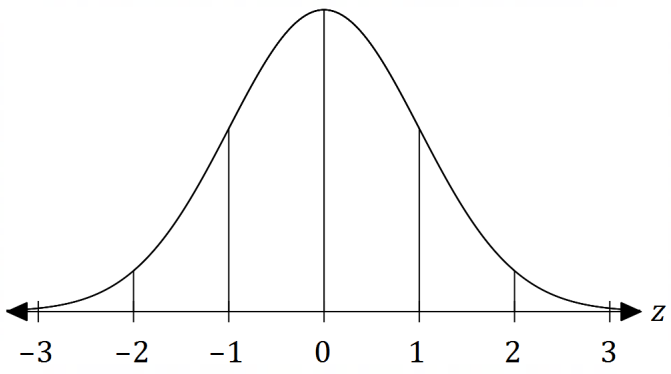
Question 35 (5 marks)

The weights of newborn babies are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.1 pounds.

A normal distribution curve is shown below, where the vertical lines represent z-scores from -3 to 3.



a) A baby is considered underweight if the baby's birthweight has a z-score that is less than -1.8. **Shade** the area under the normal distribution curve representing the percentage of babies that are born underweight. **1**

b) Below what birthweight, in pounds, is a baby considered to be underweight? Give your answer to one decimal place. **2**

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c) If the area under the normal distribution curve below $z = 1.8$ is equal to 0.9641, how many babies in 1,000 would be expected to be born underweight? **2**

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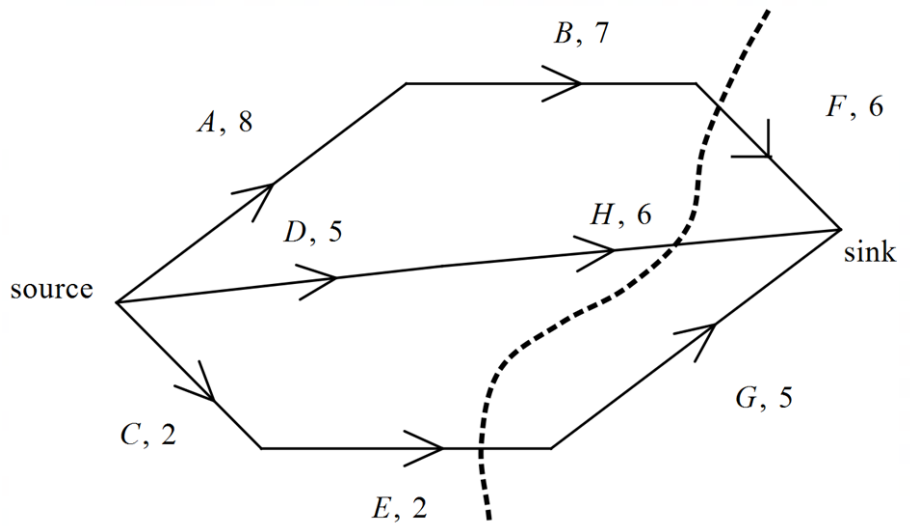
Question 36 (5 marks)

The flow of water through a series of pipes is shown in the network diagram below.



a) Determine the capacity of the cut. **1**

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b) Determine whether the given cut is the minimum cut of the diagram, by showing the maximum flow on the diagram. **2**

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c) The local council would like to increase the flow of water in the town.
Give 2 suggestions as to how the water flow can be increased.

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Question 37 (3 marks)

Cian has a credit card with no interest-free period and started in July with a zero balance.

Interest is compounded daily, at a daily rate of 0.042%.

Interest is charged on the date of purchase but not the date the account is paid.

In July, Cian made only one purchase on the 12th of July for $640. Cian paid the account in full on the 1st of August.

Calculate the total interest he was charged for July.

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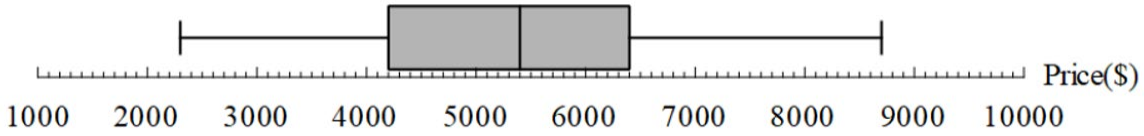
Question 38 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



a) Find the interquartile range. **1**

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b) Determine if there are any outliers. (Reasons and working required) **2**

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It is given that the mean painting value is \$5500 dollars. Four paintings of respective prices \$3400, \$3500, \$5700 and \$6800 are now donated to an art gallery.

c) Find the mean and median of the prices of the remaining paintings in the warehouse. **2**

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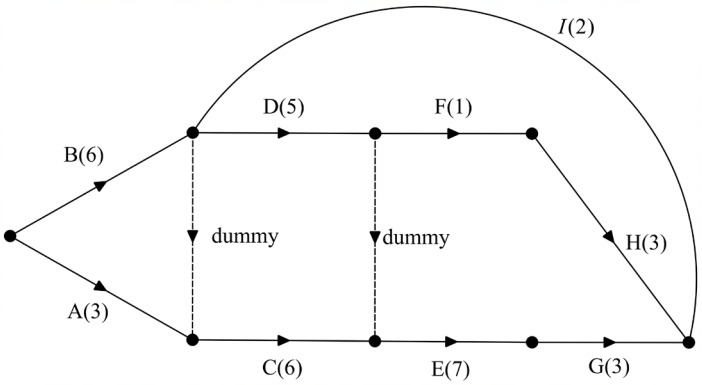
Question 39 (5 marks)

A project is represented by the activity network shown below.



a) Complete the precedence table shown below. **2**

Activity	Duration (weeks)	Immediate predecessors
A	3	–
B	6	–
C	6	
D	5	*B*
E	7	
F	1	*D*
G	3	*E*
H	3	
I	2	*B*

b) Determine the critical path for this project. **3**

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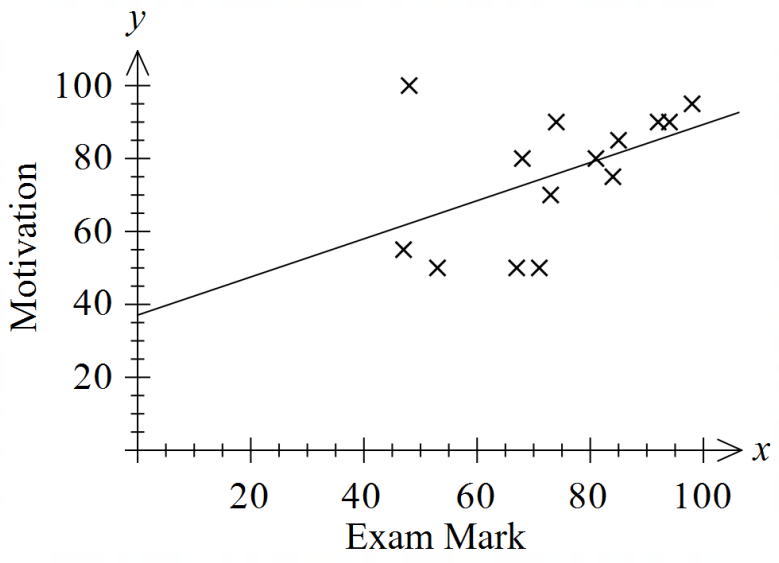
Question 40 (3 marks)

The table below shows the results of a small study of students. They were asked to rank their motivation to do well in the subject and later sat an exam in that subject.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*
Mark	47	48	53	67	68	71	73	74	81	84	85	92	94	98
Motivation	55	100	50	50	80	50	70	90	80	75	85	90	90	95

A scatterplot of the data is given below.



a) Calculate Pearson's correlation coefficient, r, for this data set. **1**

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b) Describe the relationship and it's strength between examination mark and motivation. **2**

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Question 41 (3 marks)

A photocopier was purchased for $25 000.

During the first three years, the photocopier depreciated at 12% per annum using the declining-balance method.

During the fourth year, the photocopier is then depreciated at 1.4 cents/page printed.

At the end of the fourth year, the photocopier is valued at $14 236.80.

How many pages were printed in the fourth year? **3**

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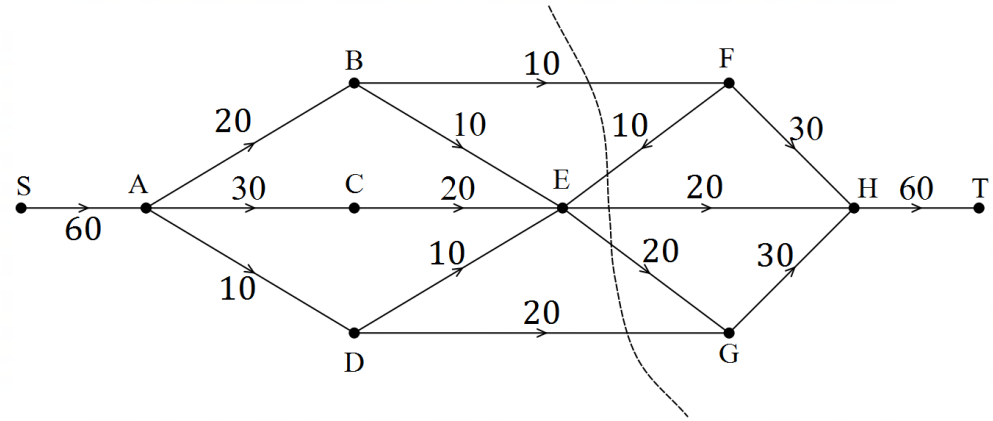
Question 42 (5 marks)

The flow diagram shows the capacity of corridors in an exhibit. The entrance (source) is S and the exit (sink) is T.



a) Find the capacity of the cut represented by the dashed line. **1**

.....

.....

.....

b) The minimum cut is known to include the corridor connecting exhibits A and B. Draw the minimum cut and find the maximum flow of the network. **2**

.....

.....

.....

.....

c) Suggest only one change which would increase the maximum flow of the network to 60 people. **2**

.....

.....

.....

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End of Paper

Northern Beaches Secondary College
Manly Campus

2023

Higher School Certificate Trial Examination

Mathematics Standard

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using non-erasable black pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

Total Marks: 100

Section I – 15 marks (pages 2 - 10)

- Attempt Questions 1–15
- Allow about 25 minutes for this section.

Section II – 85 marks (pages 12 - 36)

- Attempt Questions 16 – 42
- Allow about 2 hours and 5 minutes for this section.

Section I

15 marks

Attempt Questions 1-15

Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1-15

Question 1

A school principal wants to know if the students are happy with the school.

He asked the opinion of every fifth student as they left the school in the afternoon.

What type of sampling did the principal do?

A. Census
B. Random
C. Stratified
D. Systematic

Question 2

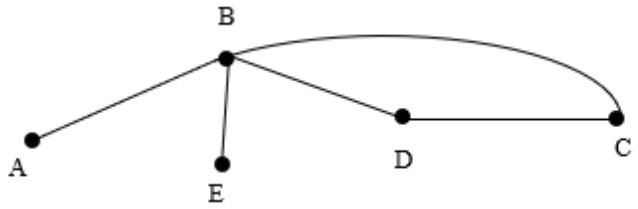
Maya is in Cape Town, which is 2 hours ahead of Coordinated Universal Time (UTC+2).Sarah is in Sydney, which is 10 hours ahead of Coordinated Universal Time (UTC+10).

When it is 3 pm in Cape Town, what time is it in Sydney?

A. 5 am
B. 7 am
C. 9 pm
D. 11pm

Question 3

Five cities are connected by this network diagram.



What is the total sum of degrees in the network?

A. 5
B. 6
C. 8
D. 10

Question 4

A patient is given 2.4 litres of a saline fluid over an 8-hour period. The fluid is delivered by an intravenous drip at a rate of 20 drops per millilitre.

What is the rate at which the fluid is delivered?

A. 10 drops/min
B. 12 drops/min
C. 100 drops/min
D. 120 drops/min

Question 5

Cal bought a new scooter that was marked at a discount.

He received 30% discount off the original price of the scooter as well as a $120 trade-in for his old scooter.

Cal then paid $720 for the new scooter. What was the original price of the new scooter?

A. $384
B. $600
C. $840
D. $1200

Question 6

The length of a desk is measured to be 2.61 metres correct to the nearest centimetre.

What is the absolute error of the length of the desk?

A. 0.005 *m*
B. 0.001 *m*
C. 0.05 *m*
D. 0.01 *m*

Question 7

The gravitational force (F) on an orbital satellite is given by the following formula.

$$F = \frac{mv^2}{r}$$

A formula for r is?

A. $r = \sqrt{\frac{m}{vF}}$

B. $r = \sqrt{\frac{mv}{F}}$

C. $r = \frac{mv^2}{F}$

D. $r = Fmv^2$

Question 8

Piper and Charlotte start from the same point, O, on a property. Piper walks 2 km on a bearing of $048°$. Charlotte also walks for 2 km and stops due South of where Piper has stopped.

What is the bearing of O from Charlotte?

A. 048°

B. 132°

C. 228°

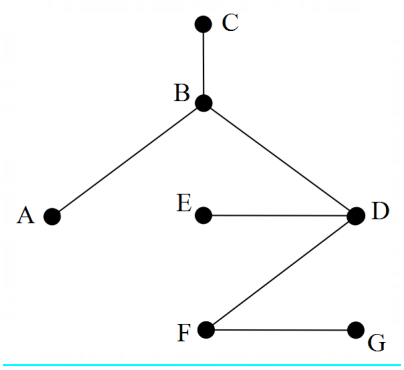
D. 312°

Question 9

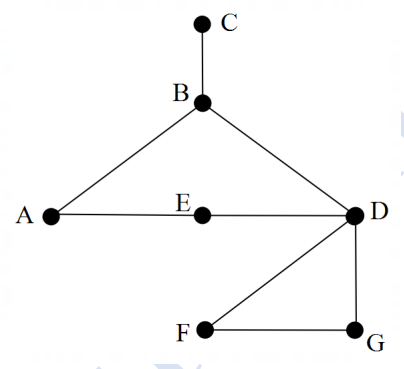
Livinia is considering a network which connects vertices A through G and draws a minimum spanning tree for the network.

Which of the following could be the minimum spanning tree?

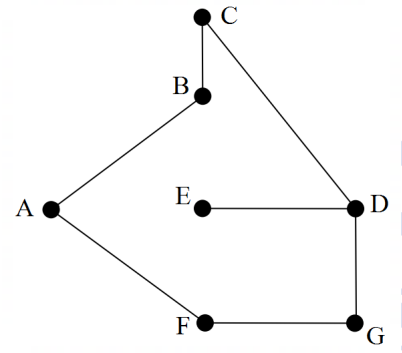
A.



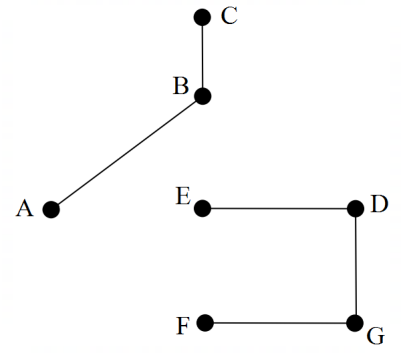
B.



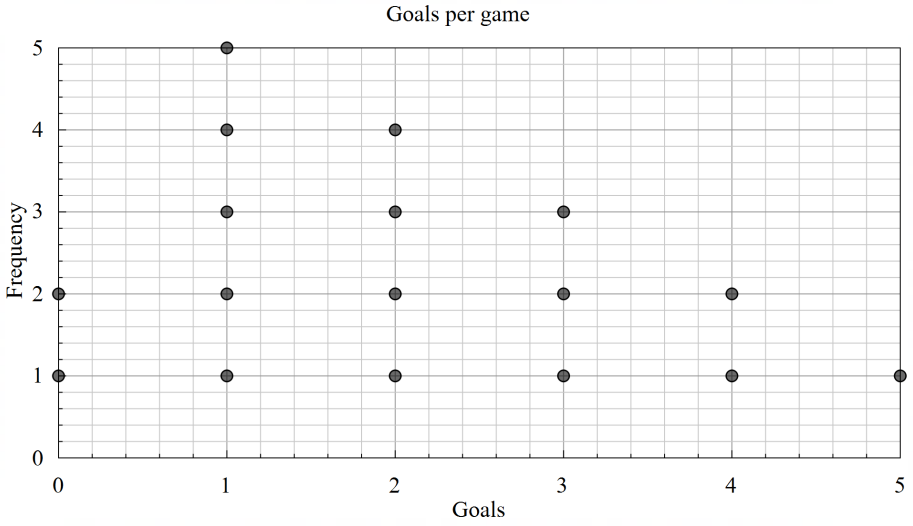
C.



D.



Question 10 A number of goals scored by a football team in each of 17 games is shown in the dot plot drawn below.



In their 18th game, the team scores 1 goal.

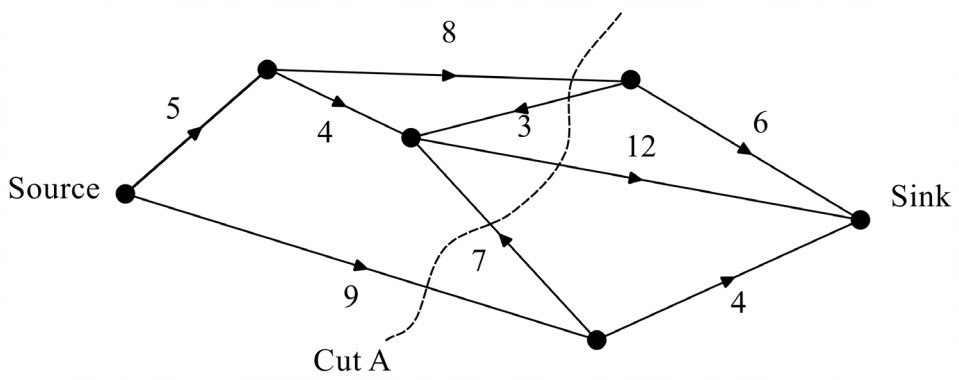
Which of the range, mean, median and mode will change after the 18th game is added to the dot plot?

A. Mean
B. Range
C. Median
D. Mode

Question 11 The z-score of Tom in a Science examination is -2. If Tom scored 34% in the Science examination, whilst the mean score is 50%, what is the standard deviation of scores?

A. 4%
B. 8%
C. 12%
D. 16%

Question 12 The network below shows the flow of water through a series of pipes.The edge numbers, in litres per minute, indicate the maximum flow through each pipe.



What is the capacity of cut A, in litres per minute?

A. 12
B. 19
C. 29
D. 39

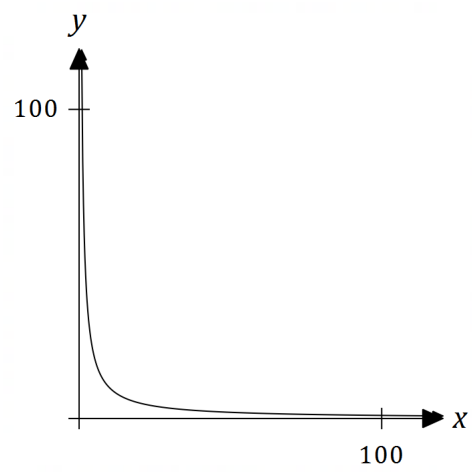
Question 13 Spencer borrows $560 000 to start his business. His monthly repayments are $2815.94 over 25 years.

What amount of interest will Spencer pay over the term of the loan?

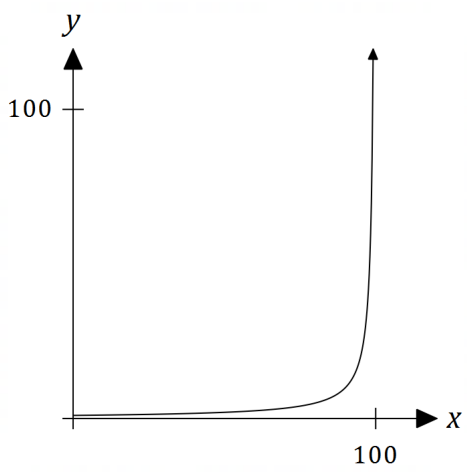
A. $ 22 400
B. $ 70 399
C. $ 284 782
D. $ 844 782

Question 14 Which of the following could be the graph of $y = \frac{100}{x}$?

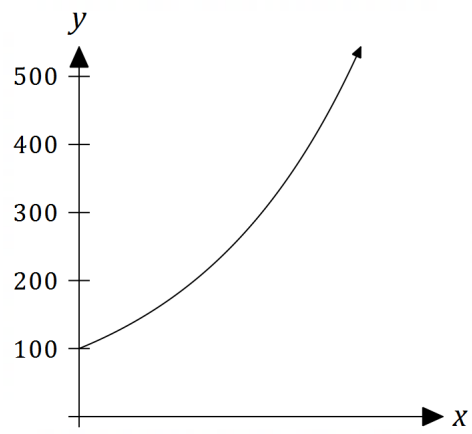
A.



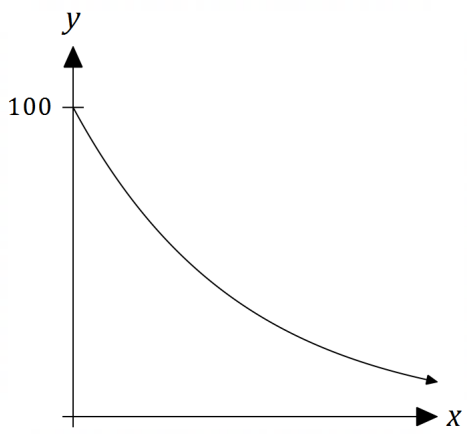
B.



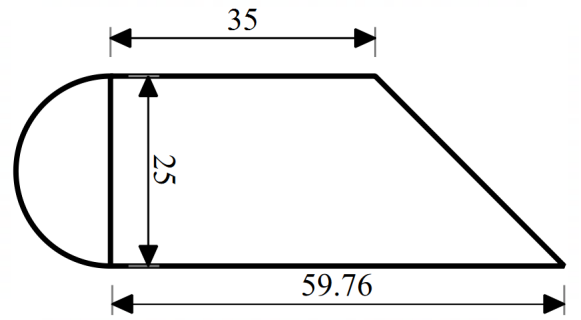
C.



D.



Question 15 Calculate the area of the shape below, rounded to the nearest whole number.



A. 715

B. 1050

C. 1430

D. 2166

End of Multiple Choice Questions

Northern Beaches Secondary College
Manly Campus

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Higher School Certificate Trial Examination

Mathematics Standard

Section II Answer Booklet

Section II

85 marks

Attempt All Questions – 16 to 42

Allow about 2 hours and 5 minutes for this section.

Instructions

- Answer the questions in the spaces provided.
  These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet.
  If you use this space, clearly indicate which question you are answering.

Question 16 (2 marks)

What is the average fuel consumption of a scooter that uses 8 litres of fuel to travel 182 km?

2

Give your answer in L/100 km, correct to one decimal place.

$$100 \times \frac{8}{182} = 4.4\,\text{L/100 km}$$

Question 17 (2 marks)

If it is 2:30 pm on Tuesday in Auckland (UTC $+11$), what time and day is it in Montreal (UTC -4)? **2**

Montreal is behind Auckland

2:30pm – 15hrs = 11:30am Monday

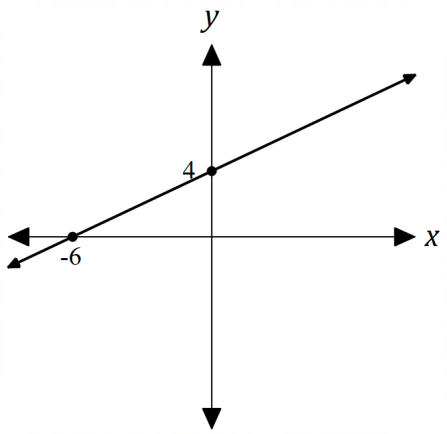
Question 18 (2 marks)

Solve the equation $2x - 5 = \frac{2x - 7}{3}$. **2**

$$\begin{aligned} 6x - 15 &= 2x - 7 \\ 6x - 2x &= 15 - 7 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

Question 19 (3 marks)

a) Find the equation of the line shown on the axes below. **2**



$c = 4$

$m = \frac{4}{6} = \frac{2}{3}$ **1**

$y = mx + c$

$y = \frac{2}{3}x + 4$

b) Calculate the point of intersection of the line above with the line $x = 9$

$y = \frac{2}{3}(9) + 4 = 10$

$(9,10)$

Question 20 (3 marks)

Rhea is driving on a highway where the legal speed limit is 110 km/h. She notices danger in the distance and immediately reacts to apply the brakes in order to avoid a collision.

If Rhea's reaction time is 1.5 seconds and she travels 50 metres in that time, use calculations to determine whether Rhea was driving within the legal limit.

3

$$s = \frac{d}{t}$$

$$s = \frac{50}{1.5}$$

$$s = 33.3 \text{ m/s}$$

Rhea's speed $\frac{33.3 \times 3600}{1000} = 120$ km/h

Therefore, Rhea was over the speed limit as she was doing 120 km/h.

Question 21 (2 marks)

Three years ago, a couple bought 3000 shares in a business at $2.80 a share. The current value of the investment is $10 200.

Calculate the percentage increase in the couple's investment.
Give your answer as a percentage correct to 1 decimal place.

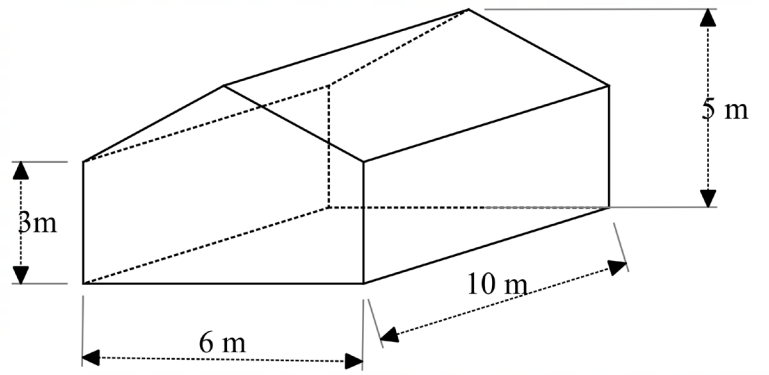
2

$3000 \times 2.8 = 8400$

$$\frac{10200 - 8400}{8400} \times 100\% \approx 21.4\%$$

Question 22 (3 marks)

A barn is to be constructed according to the diagram below.



NOT TO SCALE

Calculate the volume of the barn, in cubic metres. **3**

Front face area:

$3\times 6+\frac{1}{2}\times(5-3)\times 6=24\text{ m}^2$

Volume is $24\times 10=240\,\text{m}^3$

Question 23 (4 marks)

Jesse arrives at a party at 9 pm and has 4 standard drinks in the next 3 hours. She weighs 62 kg and is a P-plater with a zero BAC limit.
The formula to estimate a female's BAC is:

$$BAC_{female} = \frac{10N - 7.5H}{5.5M}$$

where N is the number of standard drinks consumed,

H is the number of hours drinking and

M is the person's mass in kilograms.

The formula to estimate the time (t hours) taken for a person to reach a BAC of zero is:

$$t = \frac{BAC}{0.015}.$$

What is the earliest time at which Jesse can legally drive herself home? Justify your answer by calculations. **4**

$BAC = \frac{4 \times 10 - 7.5 \times 3}{5.5 \times 62} \approx 0.0513...$

$t = \frac{0.0513...}{0.015} = 3.42...$ hours

3.42... hours is 3 hours 25 minutes

So Jesse can drive again at 3:25 am

Question 24 (3 marks)

The frequency f of radio waves is inversely proportional to the wavelength w.
If a wave has a wavelength of 300 metres and a frequency of 1 600 hertz, what is the wavelength for a frequency of 1 000 hertz? **3**

$w = \frac{k}{f}$

$Substitute\ w = 300\ and\ f = 1600$

$300 = \frac{k}{1600}$

$k = 480\ 000$

$w = \frac{480000}{f}$

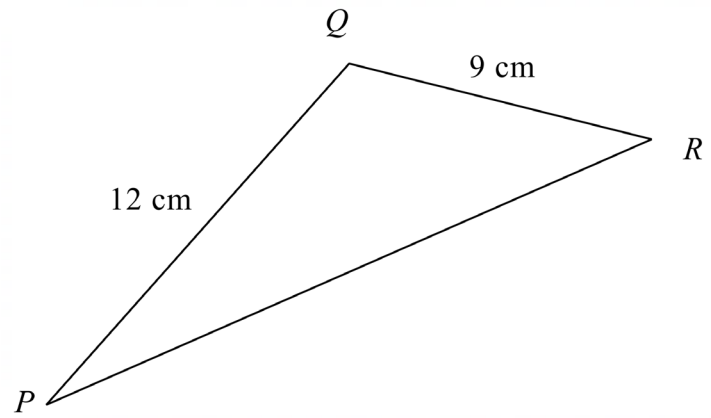
$Substitute\ f = 1000$

$w = \frac{480000}{1000} = 480m$

Question 25 (3 marks)

Find the two possible values of $\angle PQR$ if the area of triangle PQR is 27 cm^2.

3



NOT TO SCALE

$$27 = \frac{1}{2} \times 12 \times 9 \times \sin(\angle PQR)$$

$$27 = 54 \sin(\angle PQR)$$

$$\sin(\angle PQR) = \frac{27}{54} = \frac{1}{2}$$

$$\therefore \angle PQR = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ \text{ or } 180^\circ - 30^\circ = 150^\circ$$

Question 26 (3 marks)

The following method was used to estimate the current population of pygmy possums in a national park.

- 15 pygmy possums were caught, tagged and released.
- Six months later, 40 pygmy possums were caught, of which 3 had been tagged.

It is estimated that the population of pygmy possums will decrease at a rate of 10% per annum for the next five years.

What is the expected population of pygmy possums in the national park in five-years' time?

3

Estimated current population: $\frac{15}{x} = \frac{3}{40} \Rightarrow x = \frac{15 \times 40}{3} = 200$

Estimated future population: $200 \times (1 - 10\%)^5 \approx 118$

Question 27 (3 marks)

The following table shows the Australian income tax rate.

Taxable Income	**Tax on this Income**
0 - $18 200	Nil
$18 201 - $45 000	19 cents for each $1 over $18 200
$45 001 - $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 - $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

Source credit: Australian Taxation Office (ATO)

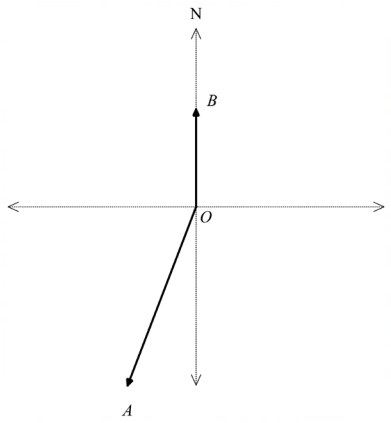
At the end of the last financial year, Lincoln was required to pay income tax of $25 416.

Calculate Lincoln's taxable income. **3**

$25416 = (x - 45000) \times 0.325 + 5092$
$20324 = (x - 45000) \times 0.325$
$62535.38 = x - 45000$
$x = \$107\ 535.38$
Lincoln's taxable income is $107 535.38

Question 28 (5 marks)

Two ships leave a harbour, O, shown in the diagram below. Ship A sails 95 km in a direction of $208^\circ T$, while ship B sails due north for 48 km.



M

a) Show that obtuse $\angle AOB = 152^\circ$

$< MOA = 208 - 180 = 28$

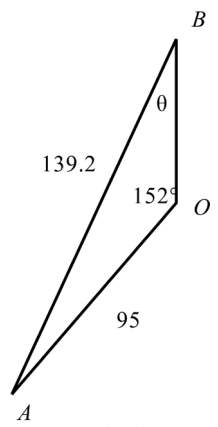
$\therefore Obtuse\ < AOB = 180 - 28 = 152°$ **1**

b) Calculate the distance between the two ships, correct to one decimal place.

$$AB^2 = 48^2 + 95^2 - 2 \times 48 \times 95 \times \cos 152^\circ$$
$$= 19381.482...$$
$$AB = \sqrt{19381.482...}$$
$$= 139.2 \text{ km}$$

2

c) What is the bearing of ship A from ship B, to the nearest degree?



$$\frac{\sin\theta}{95} = \frac{\sin 152^\circ}{139.2...}$$
$$\sin\theta = \frac{95\sin 152^\circ}{139.2...} = 0.320...$$
$$\theta = 18^\circ 41'$$

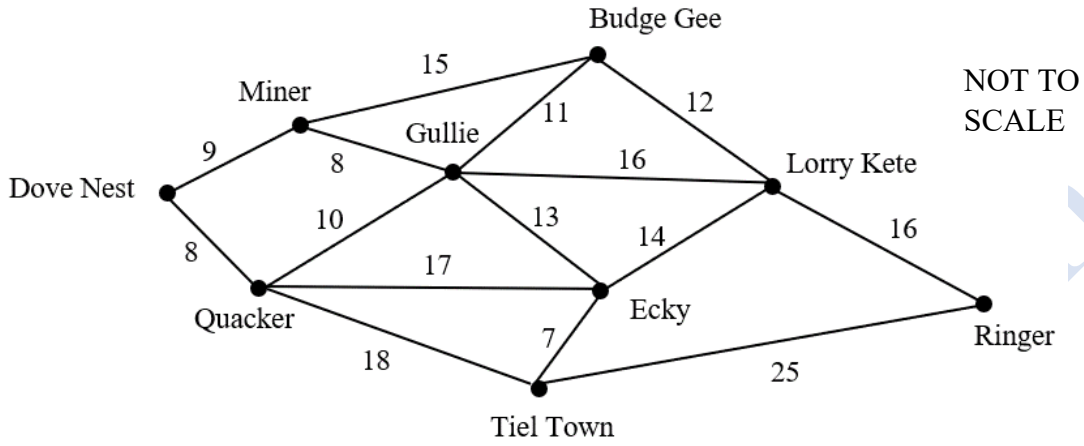
So A is $198^\circ 41'$ from B

Bearing 199° T

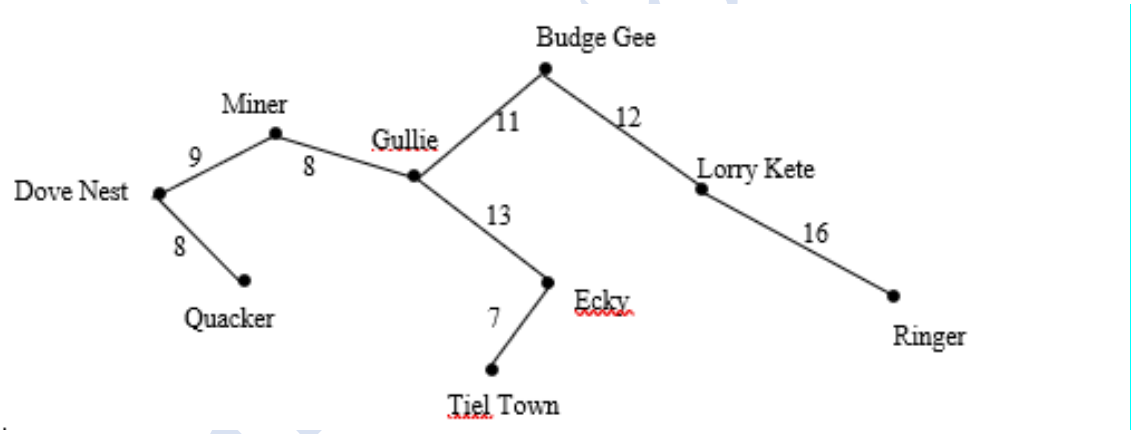
2

Question 29 (4 marks)

The Green Council wants to improve the NBN connection cables to various suburbs. The network diagram shows the distance between suburbs in kilometres.



a) Sketch the minimum spanning tree. **3**



b) Find the length of the minimum spanning tree that connects all the suburbs. **1**

$7 + 8 + 8 + 9 + 11 + 12 + 13 + 16 = 84\ km$

Question 30 (3 marks)

Shirlene owns a company producing and selling backpacks for $80 each.

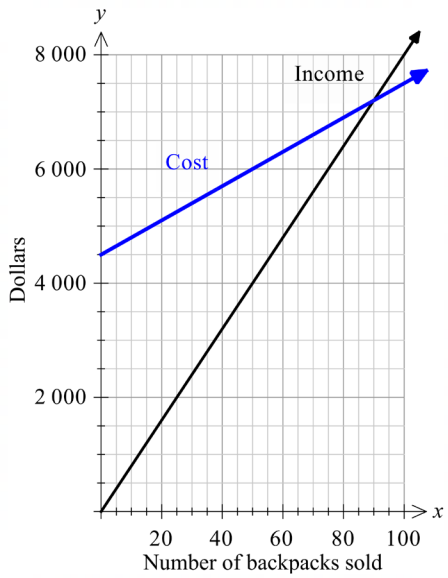
The income function is $y = 80x$, where x is the number of backpacks sold.

The cost of producing these backpacks includes a set-up cost of $4500 and additional costs of $30 per backpack.

a) Write the cost function in the form $y = mx + b$ **1**

$y = 30x + 4500$

b) Draw the cost function on the set of axes below. **1**



c) Hence, or otherwise, determine Shirlene's Break-even point? **1**

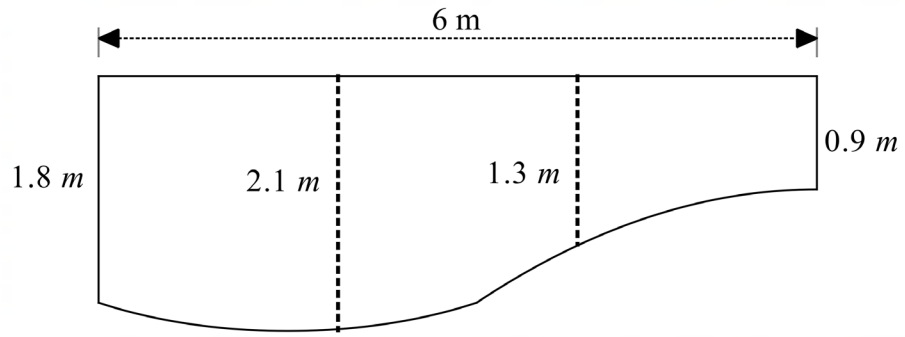
At intersection point, number of backpacks sold is 90.

Show working if using equations, or show marking on the graph.

Question 31

Question 32 (4 marks)

The uniform cross-section of a 12 metre long backyard pool is shown below.



NOT TO SCALE

Use three applications of the trapezoidal rule to determine the approximate capacity of the pool, in litres. **4**

$$A \approx \frac{2}{2}(1.8+2.1)+\frac{2}{2}(2.1+1.3)+\frac{2}{2}(1.3+0.9)$$
$$= 9.5\,\text{m}^2$$
$$V = 9.5 \times 12$$
$$= 114\ \text{m}^3$$

Hence the capacity is $114\,000\,L$ or $114\,\text{kL}$

Question 33 (2 marks)

Explain the difference between a cycle and a path. **2**

A cycle is a circuit that doesn't repeat any vertices whilst a path cannot have any repeated vertices or edges.

Question 34 (2 marks)

The formula below can be used to calculate the required dosages of medicine for children aged 1 – 12 years.

$$D = \frac{yA}{y + 12}$$

where D = dosage for children aged 1–12 years
y = age of child (in years)
A = adult dosage

What dosage should be given to a 32-month-old child if each adult dosage is 30 mL? Give your answer correct to 2 significant figures. **2**

$$y = \frac{32}{12} = \frac{8}{3}$$

$$D = \frac{\frac{8}{3}(30)}{\frac{8}{3} + 12}$$

$$= 5.45\ldots$$

$$= 5.5mL$$

Question 35 (3 marks)

Ike has a bag that contains 5 red, and 8 black balls. Two balls are drawn randomly from the bag.

a) Find the probability that the first ball drawn is red. **1**

$$\frac{5}{13}$$

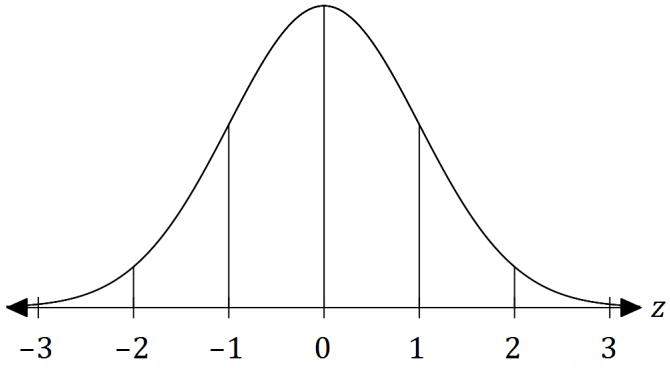
b) Find the probability that both balls drawn are the same colour. **2**

$$P(\text{same colour}) = P(\text{both red}) + P(\text{both black})$$
$$= \frac{5}{13} \times \frac{4}{12} + \frac{8}{13} \times \frac{7}{12}$$
$$= \frac{20 + 56}{156}$$
$$= \frac{19}{39}$$

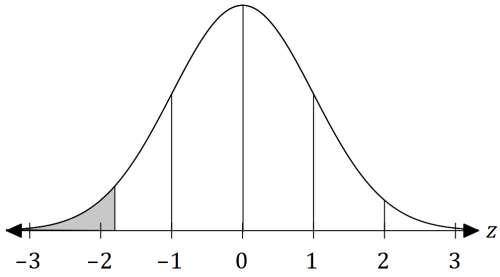
Question 36 (5 marks)

The weights of newborn babies are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.1 pounds.

A normal distribution curve is shown below, where the vertical lines represent z-scores from -3 to 3.



a) A baby is considered underweight if the baby's birthweight has a z-score that is less than -1.8. **Shade** the area under the normal distribution curve representing the percentage of babies that are born underweight. **1**



2

b) Below what birthweight, in pounds, is a baby considered to be underweight? Give your answer to one decimal place.

$$z = \frac{x - \mu}{\sigma}$$

$$-1.8 = \frac{x - 7.5}{1.1}$$

Multiply both sides by 1.1

$$-1.98 = x - 7.5$$

Add 7.5 to both sides

5.5 lbs

2

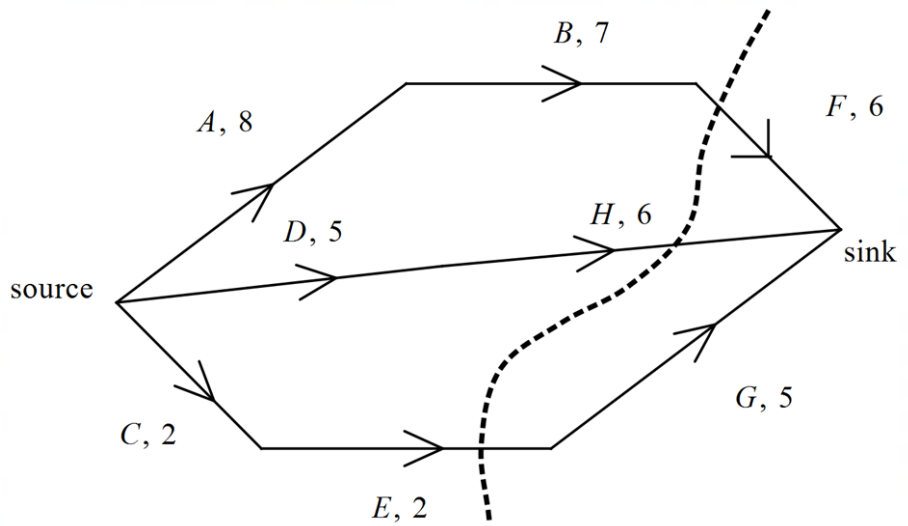
c) If the area under the normal distribution curve below $z = 1.8$ is equal to 0.9641, how many babies in 1,000 would be expected to be born underweight?

Area to the right of $z = 1.8$ is $1 - 0.9641 = 0.0359$

$1000 \times 0.0359 = 35.9 \approx 36$ babies.

Question 37 (5 marks)

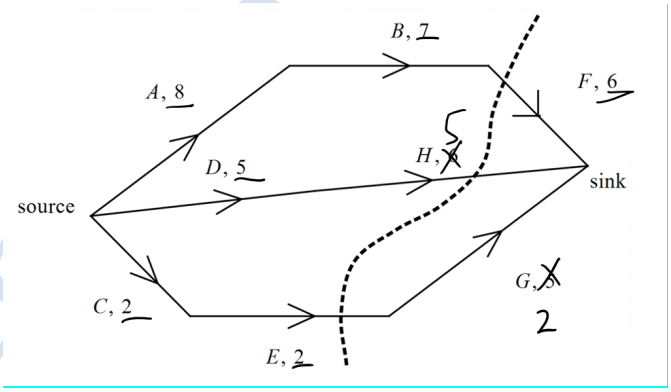
The flow of water through a series of pipes is shown in the network diagram below.



a) Determine the capacity of the cut. **1**

$6 + 6 + 2 = 14$

b) Determine whether the given cut is the minimum cut of the diagram, by showing the maximum flow on the diagram. **2**



2

$5 + 6 + 2 = 13$

Hence, the given cut is not the minimum cut as 13 is the maximum flow.

c) The local council would like to increase the flow of water in the town.
Give 2 suggestions as to how the water flow can be increased.

- *Increase the flow at D to 6.*
- *Increase the flow at C and E to 5.*

Question 38 (3 marks)

Cian has a credit card with no interest-free period and started in July with a zero balance.

Interest is compounded daily, at a daily rate of 0.042%.

Interest is charged on the date of purchase but not the date the account is paid.

In July, Cian made only one purchase on the 12th of July for $640. Cian paid the account in full on the 1st of August.

Calculate the total interest he was charged for July.

3

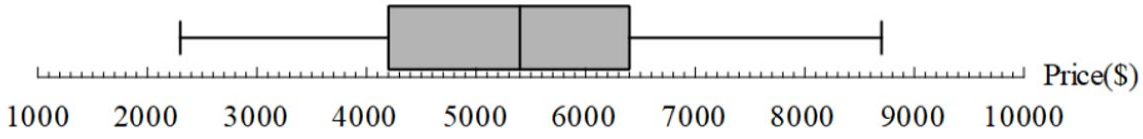
Interest is charged for 31-11=20 days

Interest is $640(1+0.00042)^{20} - 640 \approx 5.40$

So interest charged is $5.40

Question 39 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



a) Find the interquartile range. **1**

$IQR = 6400 - 4200 = 2200$

b) Determine if there are any outliers. (Reasons and working required) **2**

$Q_1 - 1.5 \times IQR = 4200 - 1.5 \times 2200 = 900$

which is below the minimum price. So no low outliers

$Q_3 + 1.5 \times IQR = 6400 + 1.5 \times 2200 = 9700$

which is above the maximum price. So no high outliers

It is given that the mean painting value is \$5500 dollars. Four paintings of respective prices \$3400, \$3500, \$5700 and \$6800 are now donated to an art gallery.

c) Find the mean and median of the prices of the remaining paintings in the warehouse. **2**

Median price is \$5400. Since 2 prices below and 2 prices above the median are removed, the median will be unchanged.

Prior to donation, total price of all paintings was $30 \times \$5500 = \$165\,000$.

After the donation, the total price of the remaining 26 paintings will be

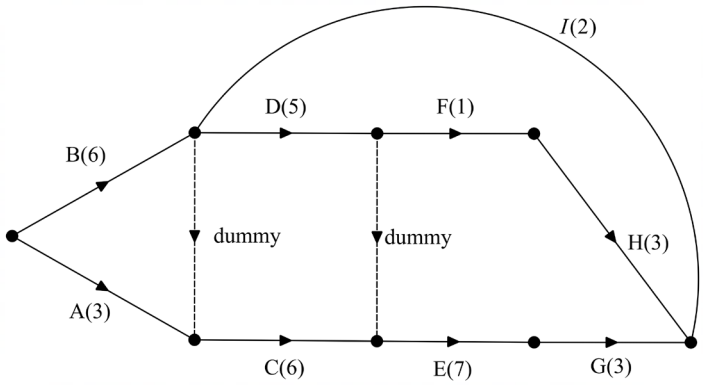
$\$165\,000 - \$3400 - \$3500 - \$5700 - \$6800 = \$145\,600$

The average is then $\dfrac{\$145\,600}{26} = \$5600$

No mark for the inclusion of the removed paintings as an error.

Question 40 (5 marks)

A project is represented by the activity network shown below.



a) Complete the precedence table shown below. **2**

Activity	Duration (weeks)	Immediate predecessors
A	3	–
B	6	–
C	6	
D	5	*B*
E	7	
F	1	*D*
G	3	*E*
H	3	
I	2	*B*

Activity	Duration (weeks)	Immediate predecessors
A	3	–
B	6	–
C	6	*A*, *B*
D	5	*B*
E	7	*C*, *D*
F	1	*D*
G	3	*E*
H	3	*F*
I	2	*B*

b) Determine the critical path for this project. **3**

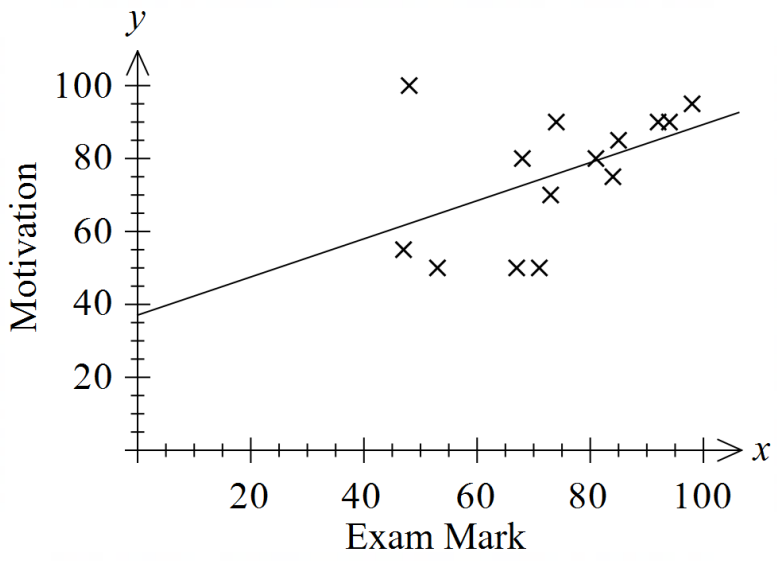
B-C-E-G 22 weeks

Question 41 (3 marks)

The table below shows the results of a small study of students. They were asked to rank their motivation to do well in the subject and later sat an exam in that subject.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*
Mark	47	48	53	67	68	71	73	74	81	84	85	92	94	98
Motivation	55	100	50	50	80	50	70	90	80	75	85	90	90	95

A scatterplot of the data is given below.



a) Calculate Pearson's correlation coefficient, r, for this data set. **1**

From the calculator 0.48

b) Describe the relationship and it's strength between examination mark and motivation. **2**

The relationship is weak positive

Moderate was also accepted.

Question 42 (3 marks)

A photocopier was purchased for $25 000.

During the first three years, the photocopier depreciated at 12% per annum using the declining-balance method.

During the fourth year, the photocopier is then depreciated at 1.4 cents/page printed.

At the end of the fourth year, the photocopier is valued at $14 236.80.

How many pages were printed in the fourth year? **3**

After three years, depreciated value was $25000 \times 0.88^3 = \$17036.80$
In fourth year, the depreciation was $\$17036.80 - \$14236.80 = \$2800$
At 1.4 c/page printed, there were

At 1.4c/page printed, there were $\frac{2800}{0.014} = 200\,000$ pages printed in the fourth year