HORNSBY GIRLS HIGH SCHOOL

Mathematics Standard 2

Year 12 Higher School Certificate Trial Assessment 4 Term 3 2024

STUDENT NUMBER:_____

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 15 marks (pages 2–7)

- Attempt Questions 1–15
- Allow about 25 minutes for this section

Section II – 85 marks (pages 8–30)

- Attempt Questions 16–36
- Allow about 2 hour and 5 minutes for this section

Outcomes assessed: MS11-1, MS11-2, MS11-3, MS11-4, MS11-5, MS11-6, MS11-7, MS11-8, MS11-10, MS2-12-1, MS2-12-2, MS2-12-3, MS2-12-4, MS2-12-5, MS2-12-6, MS2-12-7, MS2-12-8, MS2-12-10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

15 Marks
Attempt Questions 1-15
Circle the best answer on the Multiple-Choice Answer Sheet provided
Allow about 25 minutes for this section

1. A bank charges 0.05753% simple interest per day on the amount owing on a credit card. The interest charged in four weeks on a balance of $1200 is:

   A) $19.33

   B) $27.61

   C) $69.04

   D) $276.14

2. A survey question asked respondents for their postcode.

   Which of the following best describes the type of data obtained in answering this question?

   A) Nominal categorical

   B) Ordinal categorical

   C) Continuous numerical

   D) Discrete numerical

3. A number is increased by 5. Then this amount is halved to give a result of 29. The equation that correctly represents this is:

   A) $\frac{x}{2}+5=29$

   B) $2x+5=29$

   C) $x+\frac{5}{2}=29$

   D) $\frac{x+5}{2}=29$

4. The probability that it rains in Sydney on any particular day is 20%.

If three consecutive days of the year are chosen, what is the probability that it rains on all three of those days?

A) 0.6 %

B) 0.8 %

C) 8 %

D) 60 %

5. In a normally distributed set of scores, the mean is 72 and the standard deviation is 6. Approximately what percentage of the scores will lie between 60 and 84?

A) 34%

B) 68%

C) 95%

D) 99.7%

6. $5000 is invested at a simple interest rate of 9% per annum. $300 interest is earned.

For how many years was the amount invested?

A) 6.7 months

B) 8 months

C) 1.5 years

D) 8 years

7. Two standard dice are thrown together 100 times. After each throw the sum of the numbers showing on the upper face of each dice is recorded.

Which calculation below illustrates the expected number of times you would expect to get a total of 12?

A) $\frac{1}{6} \times 100$

B) $\left(\frac{1}{6} + \frac{1}{6}\right) \times 100$

C) $\left(\frac{1}{6} \times \frac{1}{6}\right) \times 100$

D) $\frac{12}{36} \times 100$

8. A map has a scale of 1:5000. If Oak Street is 800 m, what is the map distance of Oak Street?

A) 0.16 mm

B) 16 mm

C) 160 mm

D) 1600 mm

9. A small business invests a total of $10 000 towards advertising and new technology in the ratio 3 : 5.

The owner of the business adds $1000 to the advertising budget.

What is the new ratio of advertising budget to new technology?

A) 2 : 3

B) 4 : 5

C) 19 : 21

D) 19 : 25

10. Sixty tickets are sold in a raffle. There are two prizes. Harry buys 5 tickets. Which expression gives the probability that Harry wins both prizes?

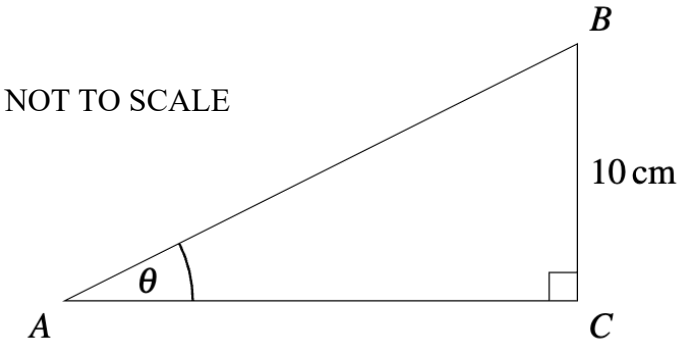
A) $\frac{5}{60}+\frac{4}{60}$

B) $\frac{5}{60}+\frac{4}{59}$

C) $\frac{5}{60}\times\frac{4}{60}$

D) $\frac{5}{60}\times\frac{4}{59}$

11. A right-angled triangle, *ABC*, has a perpendicular height, *BC,* of 10 cm.
The area of triangle *ABC* is 90 cm^2.



What is the size of the angle marked θ, to the nearest degree?

A) 15°

B) 29°

C) 45°

D) 61°

12. Max is driving on a racetrack at a speed of 210 kilometres per hour and has to brake suddenly. He has a reaction time of 0.82 seconds and a braking distance of 79.2 metres.
Stopping distance can be calculated using the following formula:

Stopping Distance = {Reaction Distance} + {Braking Distance}

What is Max's stopping distance, correct to the nearest metre?

A) 31 metres

B) 93 metres

C) 127 metres

D) 251 metres

13. The weights of 348 male players in a football competition are normally distributed with a mean of 76.6 kg and a standard deviation of 5.4 kg.

How many players are expected to weigh more than 65.8 kg?

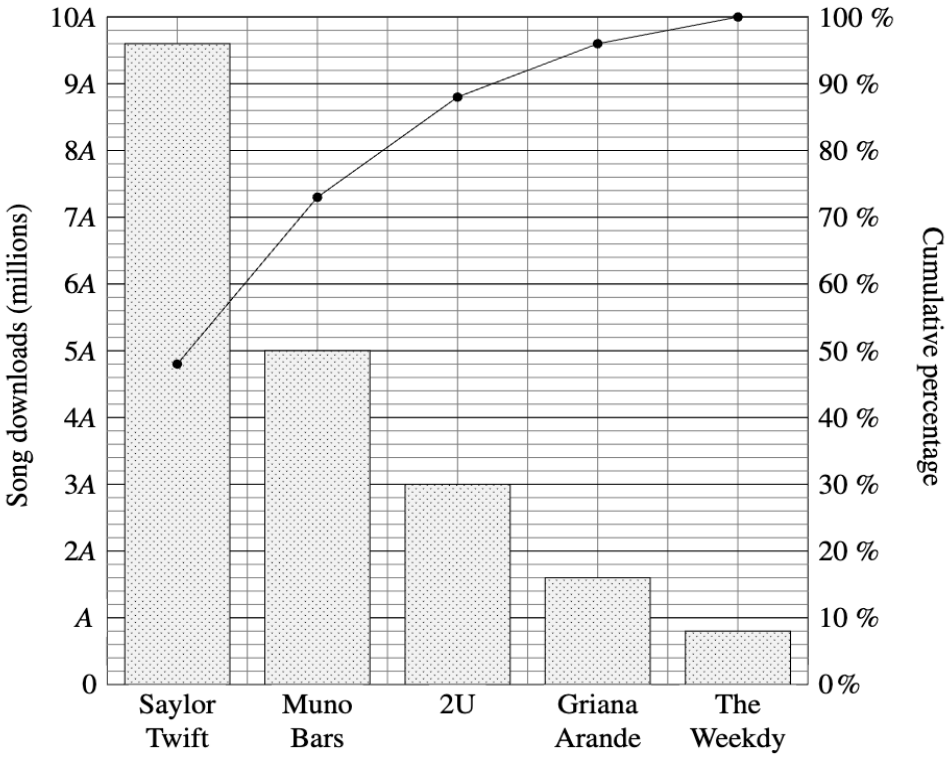
A) 8

B) 16

C) 332

D) 340

14. A streaming service collected data on the top five artists by songs downloaded in the last calendar month. There were 800 million downloads in total among the top five artists. The pareto chart shows the data collected, with the scale on the left-hand vertical axis increasing by an unknown quantity A.



What is the value of A?

A) 20

B) 40

C) 60

D) 80

15. Henry is concerned about the lizard population in the local community. He collects 170 lizards and tags them. A couple of months later he collects 32 lizards and found that 10 of them were tagged.

What is his estimate of the lizard population using the capture-recapture method, before tagging?

A) 53

B) 524

C) 536

D) 544

Section II

85 Marks
Attempt Questions 16-36
Write answers in the spaces provided
Allow about 2 hours and 5 minutes for this section

Question 16 (4 marks)

Kai measures his heart rate at the end of ten sets of push-ups. The results listed are measured in beats per minute (bpm).

108, 104, 112, 99, 102, 110, 108, 102, 102, 109

(a) Find the mean and standard deviation in Kai's heart rate, correct to two decimal places. **2**

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(b) Kai performs another set of push-ups and measures his heart rate at 108 bpm. **2**

Describe the effect of this score on the mean and standard deviation found in part (a), justifying your answer with an appropriate explanation.

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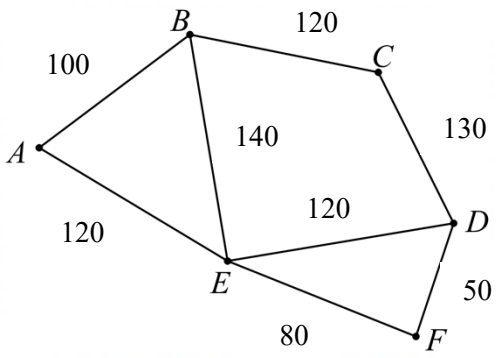
Question 17 (3 marks)

There are six towns labelled A, B, C, D, E and F. The network below shows the road connections and distances between these towns in kilometres.



(a) Find the shortest distance between town F and town B. **1**

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(b) Samantha is an engineer who plans to inspect all the roads in this network. **2**

She starts at D and intends to inspect each road only once. Find a possible path that Samantha could take.

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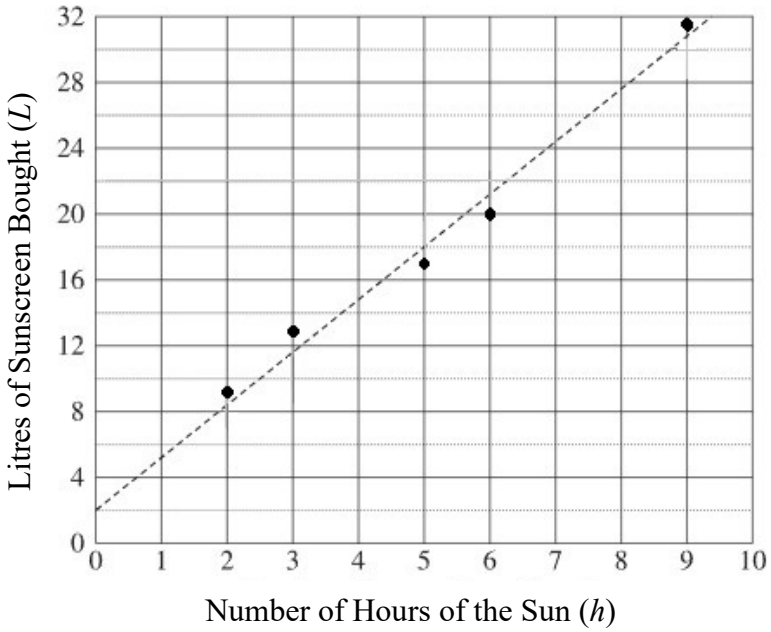
Question 18 (2 marks)

A salesperson is interested in the relationship between the litres (L) of sunscreen bought per day and the number of hours (h) of sunshine on the day. **2**

The diagram below shows the dataset used in the investigation and the least-squares regression line.



Find the equation of the least-squares regression line relating to the dataset.

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Question 19 (2 marks)

A painting was purchased at the start of 2006 for $15 000. **2**

Assume that the value of the painting increases by 5% every year since 2006.

What is the value of the painting at the end of 2024, to the nearest $1000?

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Question 20 (4 marks)

Sally lives in Helsinki, which is at latitude $60°N$ and longitude $25°E$. She plans to travel to Bangkok, which is $44°$ south of Helsinki and $75°$ east of Helsinki.

(a) What is the latitude and longitude of Bangkok? **1**

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(b) Sally travels non-stop from Helsinki to Bangkok. Her plane leaves Helsinki at 5.20 pm on Thursday local time. The flight takes 11 hours and 50 minutes. **3**

What time and day is it in Bangkok when the plane lands?

It is given that $15°$ = 1 hour time difference. Ignore daylight saving.

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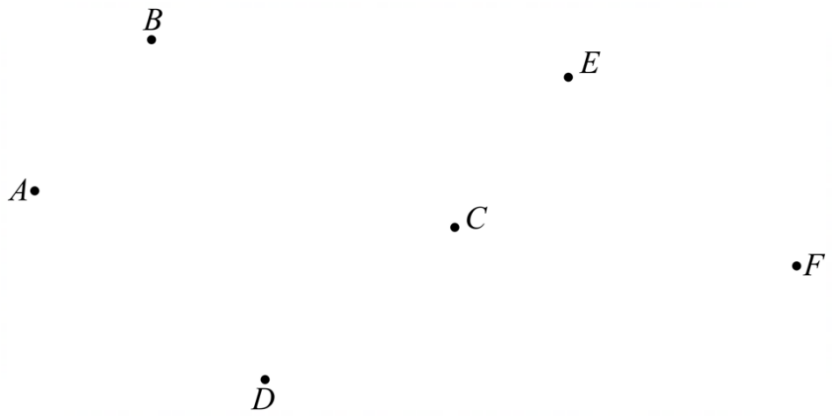
Question 21 (5 marks)

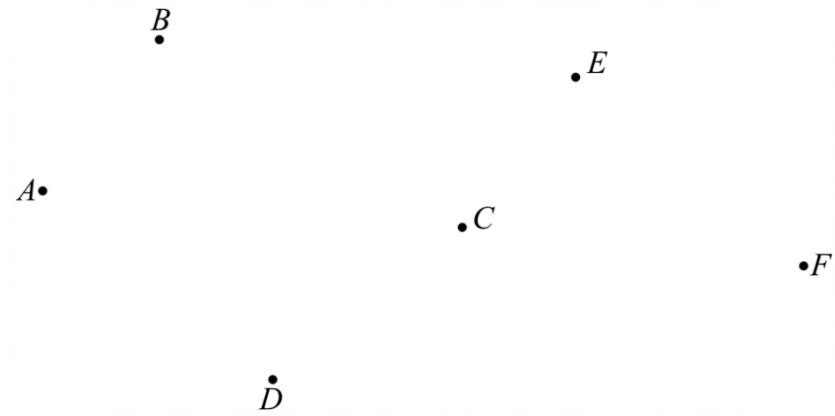
The table below shows the distance, in kilometres, between a number of towns.

Towns	A	B	C	D	E	F
A		15		8		
B	15		5		6	
C		5		5		15
D	8		5			16
E		6				7
F			15	16	7	

(a) Using the vertices given, draw a weighted network diagram to represent the information shown in the table. **2**



(b) Patrick wishes to visit each town. Draw the minimum spanning tree which will allow for this and determine its length. **3**



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Question 22 (3 marks)

The number of people (N) who attend a show varies inversely with the amount of floor space (in cm^2) allowed per person (A).

A venue can hold 3200 people if each person is allowed 300 cm^2.

(a) How many people can a venue hold if a person is allowed 250 cm^2? **2**

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(b) What is the space allowed per person if 4000 people attend the show? **1**

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Question 23 (3 marks)

The share price of a company is \$16.25.

(a) The predicted dividend yield is 2.2%. What would be the dividend? Answer correct to the nearest cent. **1**

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(b) The company decides to pay a dividend of \$0.52. What is the dividend yield? **2**

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Question 24 (3 marks)

A professional cyclist wishes to continue his training with producing electricity for his home. He calculates that he can burn 600 kilocalories per hour while training for an upcoming competition. He connects his stationary bike to a generator that stores electricity for use in his home. **3**

The following table contains the conversions between kilocalories, kilojoules, and kilowatt-hours (kWh).

1 kilocalorie = 4.184 kilojoules 3600 kilojoules = 1 kilowatt-hour (kWh)

The cyclist finds that his daily electricity usage is 5.5 kWh per day.

How long does the cyclist need to train per day on his stationary bake in order to generate enough electricity for his home? Give your answer in hours and minutes, to the nearest minute.

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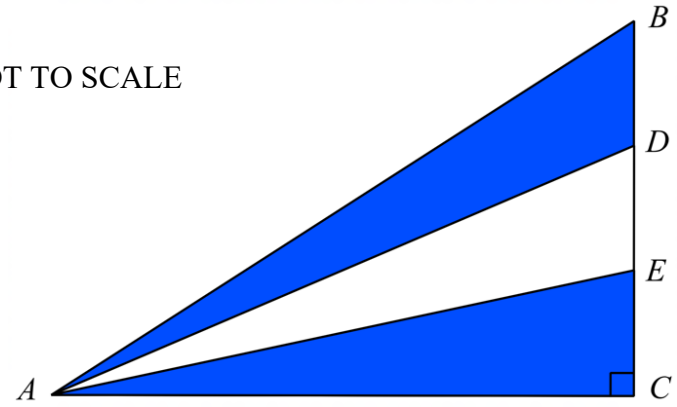
Question 25 (5 marks)

The flag shown below has three sections. ΔABC is right-angled where $AC = 120$ cm and $BC = 45$ cm. The distance BD, DE and EC are equal and $\angle DEA = 105°$.

NOT TO SCALE



(a) Find the distance AE. Answer to the nearest centimetre. **2**

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(b) Find the area of ΔDEA. Answer correct to the nearest square centimetre. **2**

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(c) What percentage of the area of the banner is ΔDEA? Answer correct to one decimal place. **1**

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Question 26 (5 marks)

The cost, $\$C$, of land in the suburb *Cartesia* is \$455 for every square metre.

The local council also charges a fee of \$12 500 for every purchase.

(a) Express the cost of purchasing land, $\$C$, in terms of the area, A square metres, in *Cartesia.* **1**

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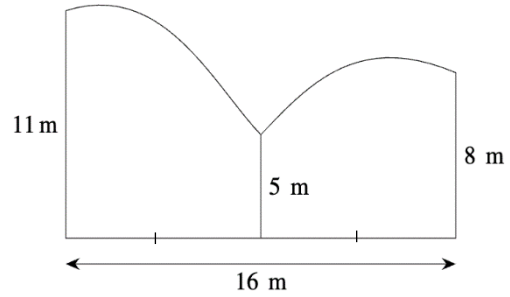
(b) John wishes to purchase a piece of land in *Cartesia*. The shape of the land is pictured below. Since the shape is irregular, he will estimate the cost in weighing up his decision. **3**



NOT TO SCALE

Using two applications of the trapezoidal rule, estimate the cost of purchasing this piece of land.

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(c) John decides to go ahead with the purchase. The price will be based on the exact area of the land. **1**

Will he pay more or less than his estimate? Briefly explain your answer.

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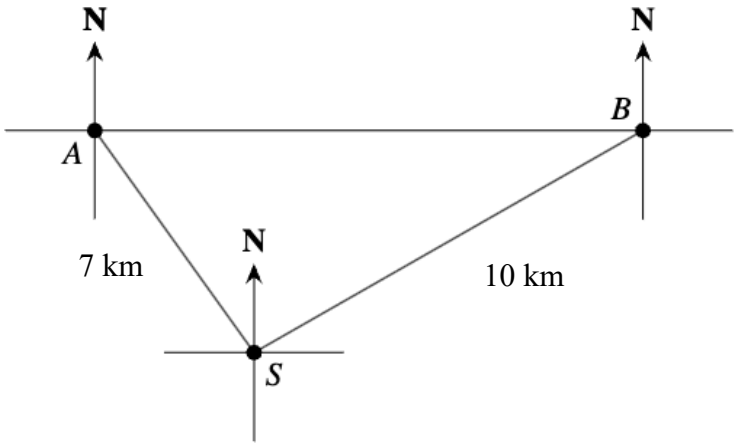
Question 27 (4 marks)

Two friends, Ali and Brian, are calculating the distance between their houses. Brian's house is due east of Ali's house.

Ali knows that he travels to school for 10 km at a bearing of $150°$ while Brian travels to the same school for 7 km at a bearing of $240°$.



NOT TO SCALE

(a) Explain why $\angle ASB = 90°$, labelling your diagram to aid your response. **1**

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(b) Hence, find the distance between Ali and Brian's houses, correct to one decimal place. **2**

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(c) What is the bearing of Ali's house from the school? **1**

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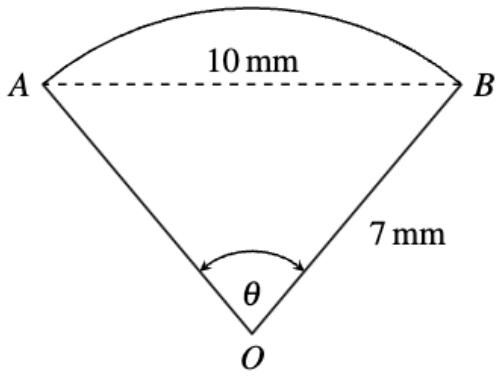
Question 28 (3 marks)

A sector AOB has a radius 7 mm and a central angle θ is shown. The length of the chord AB is 10 mm. **3**



NOT TO SCALE

Find the area of the flat surface of the sector, correct to the nearest square millimetre.

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Question 29 (5 marks)

Elliot is an engineer who earns $130 000 per year through his business. He purchases a $55 000 Ute and uses it for work purposes. **5**

He has two options for calculating the depreciation on the Ute, which will be used as a work-related tax deduction.

Method of Depreciation	*Rate of Depreciation*
Straight-line Method	$5000 per annum
Declining-balance method	10% per annum

Elliot's accountant recommends that Elliot claim 5 years' worth of depreciation when he calculates his tax, using the method that will produce the greater amount of depreciation.

The table shows the income tax rates for the 2023-24 financial year. The Medicare Levy is calculated at 2% of a person's taxable income.

Taxable Income	*Tax on this Income*
0 - $18 200	Nil
$18 200 - $45 000	19c for each $1 over $18 200
$45 001 - $120 000	$5092 plus 32.5c for each $1 over $45 000
$120 001 - $180 000	$29 467 plus 37c for each $1 over $120 000
$180 001 and over	$51 667 plus 45c for each $1 over $180 000

By using the higher depreciation amount as a tax deduction, calculate Elliot's total tax, including the Medicare Levy.

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Question 30 (4 marks)

A plane flies from Sydney to Los Angeles.

Correct to the nearest 100 km, the flight distance is 12 100 km. The plane travels at an average speed of 880 km/h, correct to the nearest 10 km/h.

(a) What is the lower bound for the flight distance? **2**

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(b) Calculate the shortest possible flight time for the trip. **2**
Give your answer in hours and minutes, correct to the nearest minute.

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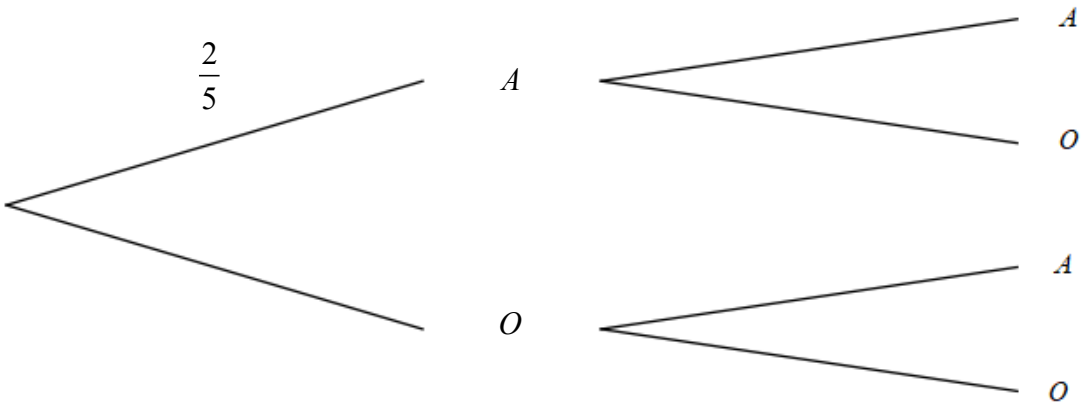
Question 31 (3 marks)

In a population of 200 people, $\frac{2}{5}$ are from blood group A and the rest are from blood group O.

Two different people are chosen at random from this population.

(a) Complete the tree diagram representing this situation. **1**



(b) Find the probability that both people are of blood group A. **1**

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(c) Find the probability at least one person is of blood group O. **1**

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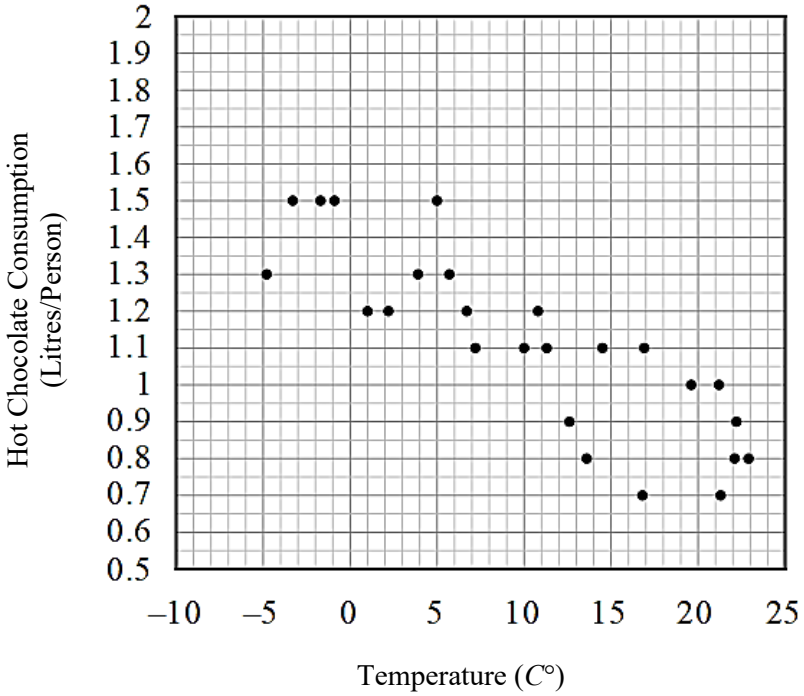
Question 32 (6 marks)

A researcher assumes that there is a linear relationship between the temperature and hot chocolate consumption.

Data was collected randomly in cities across Europe and the result were recorded in the scatterplot below.



The scatterplot shows the average monthly hot chocolate consumption, in litres/person against average monthly temperature, in $C°$.

The correlation (r) is found to be -0.8492.

When a least squares regression line is fitted to the scatterplot, the equation is found to be:

$$\text{Consumption} = 1.3599 - 0.0241 \times \text{temperature}$$

(a) Draw the least squares regression line on the scatterplot graph above. **1**

(b) Describe the association between average hot chocolate consumption and average monthly temperature in terms of strength, direction and form. **2**

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(c) Interpret the slope of the least squares regression line in terms of the variables hot chocolate consumption and temperature. **1**

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(d) Use the equation of the least squares regression line to predict the average monthly hot chocolate consumption, in litres per person, when the average monthly temperature is $-6°C$. **1**

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(e) State whether your prediction in part (d) is interpolation or extrapolation. **1**

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Question 33 (7 marks)

Noah purchases a home and takes out a reducing-balance loan of $350 000 to be repaid over 25 years. The interest rate charged is 6% per annum, compounding monthly.

(a) The table below shows the present value interest factors for a loan of $1 for the various interest rates (r) and number of periods (N). **2**

Table of present value interest factors					
N \ r	Interest rate per period (as a decimal)				
	0.002	0.003	0.004	0.005	0.006
100	90.5529	86.2830	82.2862	78.5426	75.0339
150	129.4799	120.6474	112.6328	105.3500	98.7226
200	164.7061	150.2317	137.4884	126.2406	116.2873
250	196.5832	175.7009	157.8465	142.5203	129.3113
300	225.4297	197.6274	174.5210	155.2069	138.9683
350	251.5337	216.5039	188.1784	165.0933	146.1288

How much should the monthly payment be in order to repay the loan in equal monthly repayments? Give your answer to the nearest cent.

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(b) The interest charged and the balance owing for the first three months of the loan are shown in the table below. **1**

Month	Opening Balance	Interest Charged	Repayment	End Balance
1	350 000	**A**	2255.05	349 494.95
2	349 494.95	1747.47	2255.05	348 987.36
3	348 987.36	1744.94	2255.05	348 477.25

What is the value of **A**?

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(c) How much interest did Noah pay over the course of the 25 years? **2**

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(d) After 15 years, Noah decides to make a lump sum payment of $50 000 and continues to make the same monthly repayments. The rest of the loan was then repaid over a further 86 months. **2**

How much interest did Noah save by making the lump sum payment?

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Question 34 (6 marks)

The income of a travel kit manufacturer is given by $I = 7x$ (dollars) and their costs is given by $C = 5x + 1000$ (dollars), where x represents the number of the travel kits.

(a) Draw a sketch to represent the costs and income for producing a travel kit. **3**

(b) Using the graph above, estimate how many travel kits are needed to break-even? **1**

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(c) Determine the number of travel kits that need to be sold to break-even, if the total cost increases by 5%. **2**

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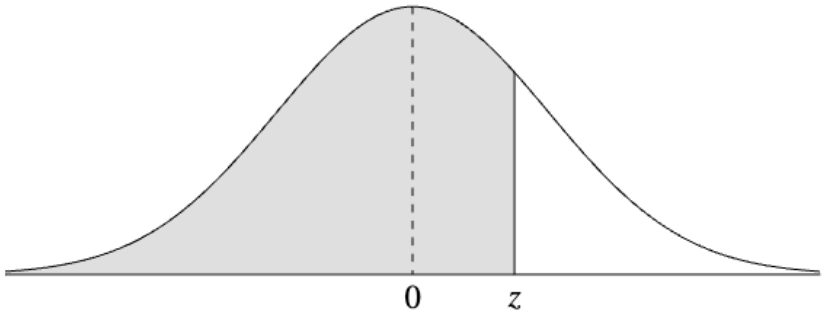
Question 35 (3 marks) **3**

The table below gives the probability that a random variable lies below some z-score, where the random variable is normally distributed.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549

The probability values given in the table below represent the shaded area in the diagram.



The results of a university entrance test sat by Year 12 students is normally distributed with a mean of 35 and a standard deviation of 5. Students who score in the 67th percentile and above are offered a place at the university.

What is the minimum mark students need to achieve in the entrance test in order to be offered a place at the university?

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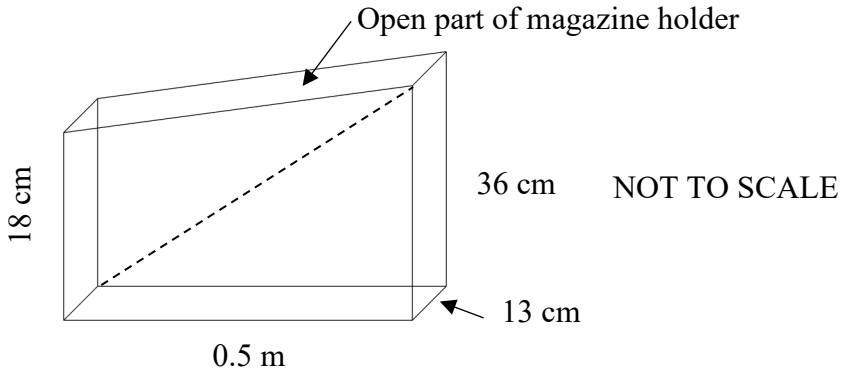
Question 36 (5 marks)

A magazine holder has only one slot to insert magazines as shown below. **5**

The width of the holder is 13 cm.



The cost of making such a holder is based on the external surface area of the holder.
If the cost of heavy duty coloured cardboard is $6.75 per square metre, calculate the cost of making 2550 of these magazine holders, correct to the nearest dollar.

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END OF EXAMINATION

HORNSBY GIRLS HIGH SCHOOL

Mathematics Standard 2

Year 12 Higher School Certificate Trial Assessment 4 Term 3 2024

STUDENT NUMBER: Solutions

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Section I

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B) $2x+5=29$

C) $x+\frac{5}{2}=29$

(D) $\frac{x+5}{2}=29$

4. The probability that it rains in Sydney on any particular day is 20%.

If three consecutive days of the year are chosen, what is the probability that it rains on all three of those days?

A) 0.6 %

B) 0.8 %

C) 8 %

D) 60 %

$0.2^3 \times 100$

5. In a normally distributed set of scores, the mean is 72 and the standard deviation is 6. Approximately what percentage of the scores will lie between 60 and 84?

A) 34%

B) 68%

C) 95%

D) 99.7%

$z = \frac{x-\mu}{\sigma}$

$= \frac{60-72}{6}$

$= 2$

$z = \frac{x-\mu}{\sigma}$

$= \frac{84-72}{6}$

$= -2$

6. $5000 is invested at a simple interest rate of 9% per annum. $300 interest is earned.

For how many years was the amount invested?

A) 6.7 months

B) 8 months

C) 1.5 years

D) 8 years

$I = Prn$

$300 = 5000 \times 0.09 \times n$

$\therefore n = \frac{300}{450}$

7. Two standard dice are thrown together 100 times. After each throw the sum of the numbers showing on the upper face of each dice is recorded.

Which calculation below illustrates the expected number of times you would expect to get a total of 12?

A) $\frac{1}{6} \times 100$

B) $\left(\frac{1}{6} + \frac{1}{6}\right) \times 100$

C) $\left(\frac{1}{6} \times \frac{1}{6}\right) \times 100$

D) $\frac{12}{36} \times 100$

8. A map has a scale of 1:5000. If Oak Street is 800 m, what is the map distance of Oak Street?

$800 \div 5000$
$= 0.16\text{m}$
$= 160\text{mm}$

A) 0.16 mm

B) 16 mm

C) 160 mm

D) 1600 mm

9. A small business invests a total of $10 000 towards advertising and new technology in the ratio 3 : 5.

The owner of the business adds $1000 to the advertising budget.

What is the new ratio of advertising budget to new technology?

$\frac{3}{8} \times 10000 + 1000$
$= 4750$
$4750 : 6250$

A) 2 : 3

B) 4 : 5

C) 19 : 21

D) 19 : 25

10. Sixty tickets are sold in a raffle. There are two prizes. Harry buys 5 tickets. Which expression gives the probability that Harry wins both prizes?

A) $\frac{5}{60}+\frac{4}{60}$

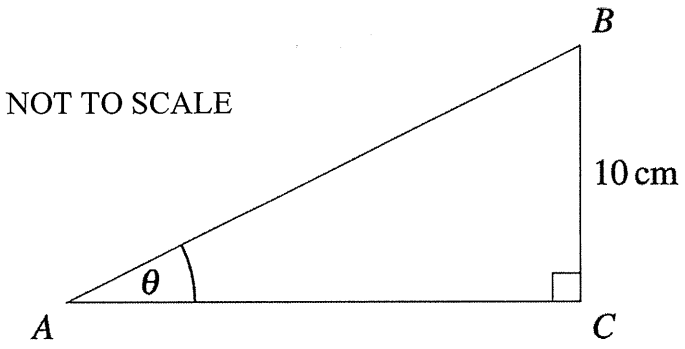
B) $\frac{5}{60}+\frac{4}{59}$

$P(WW) = \frac{5}{60} \times \frac{4}{59}$

C) $\frac{5}{60}\times\frac{4}{60}$

D) $\frac{5}{60}\times\frac{4}{59}$

11. A right-angled triangle, *ABC*, has a perpendicular height, *BC*, of 10 cm. The area of triangle *ABC* is 90 cm^2.



What is the size of the angle marked θ, to the nearest degree?

A) 15°

$90 = \frac{1}{2} \times AC \times 10$

$AC = 18$

B) 29°

$\theta = \tan^{-1}\left(\frac{10}{18}\right)$

C) 45°

D) 61°

12. Max is driving on a racetrack at a speed of 210 kilometres per hour and has to brake suddenly. He has a reaction time of 0.82 seconds and a braking distance of 79.2 metres.
Stopping distance can be calculated using the following formula:

Stopping Distance = {Reaction Distance} + {Braking Distance}

What is Max's stopping distance, correct to the nearest metre?

A) 31 metres

B) 93 metres

C) 127 metres

D) 251 metres

Speed in m/s $= 210 \div 3.6$
$= 58.\dot{3}$

reaction distance $= 0.8 \times 58.\dot{3}$
$= 47.8\dot{3}$

$\therefore$ stopping distance $= 47.8\dot{3} + 79.2$

13. The weights of 348 male players in a football competition are normally distributed with a mean of 76.6 kg and a standard deviation of 5.4 kg.

How many players are expected to weigh more than 65.8 kg?

A) 8

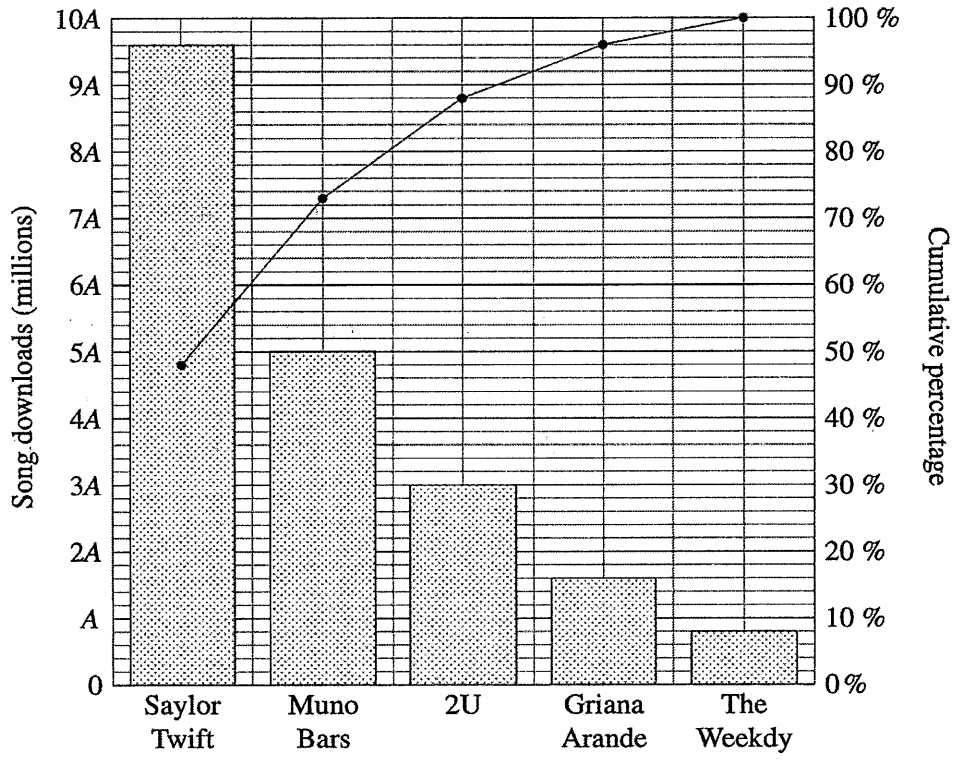
B) 16

C) 332

D) 340

97.5% of 348

14. A streaming service collected data on the top five artists by songs downloaded in the last calendar month. There were 800 million downloads in total among the top five artists. The pareto chart shows the data collected, with the scale on the left-hand vertical axis increasing by an unknown quantity A.



What is the value of A?

A) 20

B) 40

C) 60

D) 80

8% × 800
= 64 million
∴ 1.6A = 64
A = 40

15. Henry is concerned about the lizard population in the local community. He collects 170 lizards and tags them. A couple of months later he collects 32 lizards and found that 10 of them were tagged.

What is his estimate of the lizard population using the capture-recapture method, before tagging?

A) 53

B) 524

C) 536

D) 544

$\frac{170}{P} = \frac{10}{32}$

$10P = 5440$

$P =$

Section II

85 Marks
Attempt Questions 16-36
Write answers in the spaces provided
Allow about 2 hours and 5 minutes for this section

Question 16 (4 marks)

Kai measures his heart rate at the end of ten sets of push-ups. The results listed are measured in beats per minute (bpm).

108, 104, 112, 99, 102, 110, 108, 102, 102, 109

(a) Find the mean and standard deviation in Kai's heart rate, correct to two decimal places. **2**

$\bar{x} = 105.60$ (2.d.p)

$\sigma = 4.10$ (2.d.p)

(b) Kai performs another set of push-ups and measures his heart rate at 108 bpm. **2**

Describe the effect of this score on the mean and standard deviation found in part (a), justifying your answer with an appropriate explanation.

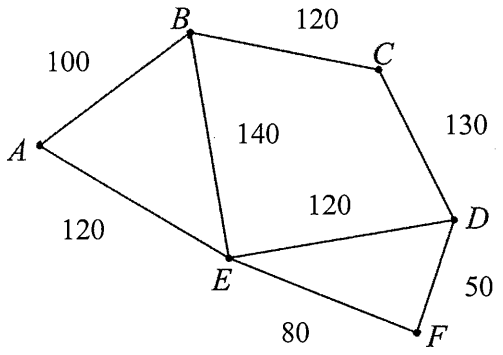
The new score is above the current mean therefore the mean will increase.
New score is 108 - 105.60 = 2.4 bpm from the mean, thus standard deviation should be lower.

$\bar{x}_{new} = 105.82$ (2.d.p)

$\sigma_{new} = 3.97$ (2.d.p)

Question 17 (3 marks)

There are six towns labelled A, B, C, D, E and F. The network below shows the road connections and distances between these towns in kilometres.



(a) Find the shortest distance between town F and town B. **1**

FEB → 220km

(b) Samantha is an engineer who plans to inspect all the roads in this network. **2**

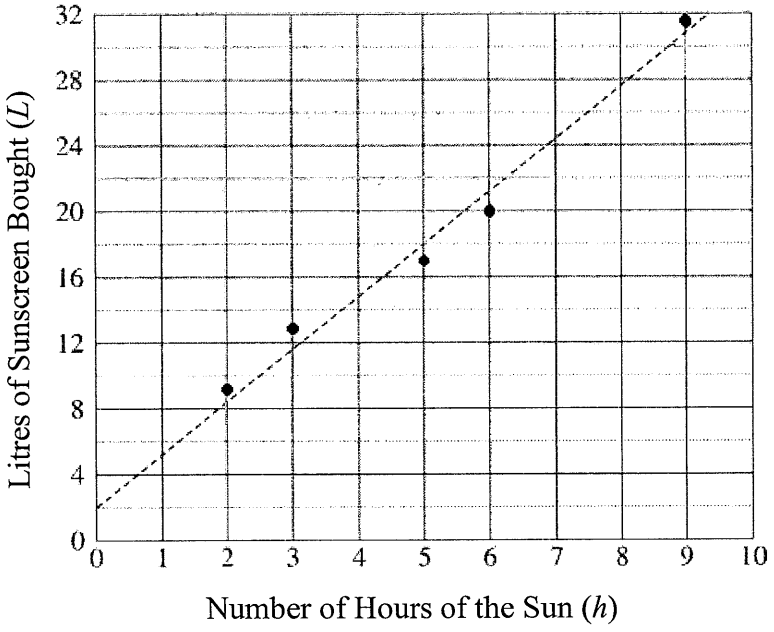
She starts at D and intends to inspect each road only once. Find a possible path that Samantha could take.

DFEABEDCB

Question 18 (2 marks)

A salesperson is interested in the relationship between the litres (L) of sunscreen bought per day and the number of hours (h) of sunshine on the day. **2**

The diagram below shows the dataset used in the investigation and the least-squares regression line.



Find the equation of the least-squares regression line relating to the dataset.

$y = mx + c$

$c = 2, \quad m = \frac{18-2}{5-0} = \frac{16}{5}$

$y = \frac{16}{5}x + 2$

$L = \frac{16}{5}h + 2$

Question 19 (2 marks)

A painting was purchased at the start of 2006 for \$15 000. **2**

Assume that the value of the painting increases by 5% every year since 2006.

What is the value of the painting at the end of 2024, to the nearest \$1000?

Value = 15000×1.05^{19}

$= 37904.25$

$= \$38000$ (nearest \$1000)

Question 20 (4 marks)

Sally lives in Helsinki, which is at latitude $60°N$ and longitude $25°E$. She plans to travel to Bangkok, which is $44°$ south of Helsinki and $75°$ east of Helsinki.

(a) What is the latitude and longitude of Bangkok? **1**

latitude = $60° - 44° = 16°$ North

longitude = $25° + 75° = 100°$ East

$(16°N, 100°E)$

(b) Sally travels non-stop from Helsinki to Bangkok. Her plane leaves Helsinki at 5.20 pm on Thursday local time. The flight takes 11 hours and 50 minutes. **3**

What time and day is it in Bangkok when the plane lands?

It is given that $15° = 1$ hour time difference. Ignore daylight saving.

Bangkok is 75° east of Helsinki → 5hrs ahead

Arrival time = 5.20pm + 11hr 50min

= 5:10am (Helsinki time)

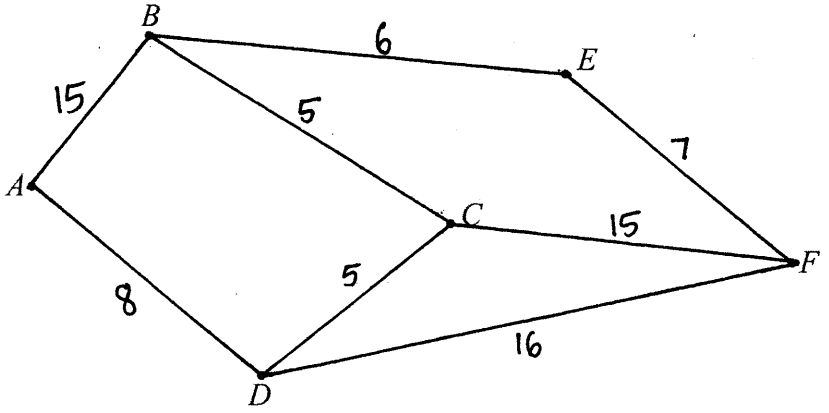
= 10:10am (Bangkok time)

Question 21 (5 marks)

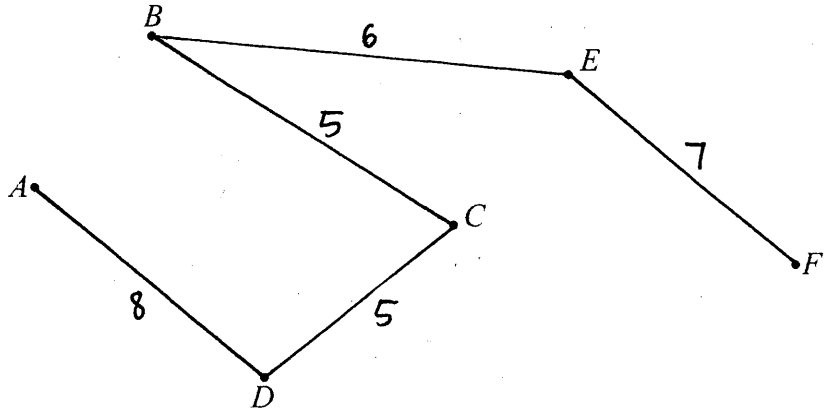
The table below shows the distance, in kilometres, between a number of towns.

Towns	A	B	C	D	E	F
A		15		8		
B	15		5		6	
C		5		5		15
D	8		5			16
E		6				7
F			15	16	7	

(a) Using the vertices given, draw a weighted network diagram to represent the information shown in the table. **2**



(b) Patrick wishes to visit each town. Draw the minimum spanning tree which will allow for this and determine its length. **3**



Length = 8+5+5+6+7

= 31 km

Question 22 (3 marks)

The number of people (N) who attend a show varies inversely with the amount of floor space (in cm^2) allowed per person (A).

A venue can hold 3200 people if each person is allowed 300 cm^2.

(a) How many people can a venue hold if a person is allowed 250 cm^2? **2**

$N = \frac{k}{A}$ $N = \frac{960000}{A}$

$3200 = \frac{k}{300}$ $= \frac{960000}{250}$

$k = 960000$ $= 3840$

$\therefore$ The venue could hold 3840 people.

(b) What is the space allowed per person if 4000 people attend the show? **1**

$N = \frac{960000}{A}$

$4000 = \frac{960000}{A}$ $\therefore$ The space allowed

$A = 240$ is 240 cm^2

Question 23 (3 marks)

The share price of a company is \$16.25.

(a) The predicted dividend yield is 2.2%. What would be the dividend? Answer correct to the nearest cent. **1**

Dividend $= 0.022 \times 16.25$

$= 0.3575$

$= \$0.36$

(b) The company decides to pay a dividend of \$0.52. What is the dividend yield? **2**

Dividend yield $= \frac{\text{Annual dividend}}{\text{Market Price}} \times 100$

$= \frac{0.52}{16.25} \times 100$

$= 3.2\%$

Question 24 (3 marks)

A professional cyclist wishes to continue his training with producing electricity for his home. He calculates that he can burn 600 kilocalories per hour while training for an upcoming competition. He connects his stationary bike to a generator that stores electricity for use in his home. **3**

The following table contains the conversions between kilocalories, kilojoules, and kilowatt-hours (kWh).

1 kilocalorie = 4.184 kilojoules 3600 kilojoules = 1 kilowatt-hour (kWh)

The cyclist finds that his daily electricity usage is 5.5 kWh per day.

How long does the cyclist need to train per day on his stationary bake in order to generate enough electricity for his home? Give your answer in hours and minutes, to the nearest minute.

Energy produced per hour = 600×4.184

$= 2510.4\text{ kJ}$

$= 2510.4 \div 3600$

$= 0.697\dot{3}\text{ kWh}$

Time to produce 5.5 kWh = $5.5 \div 0.697\dot{3}$

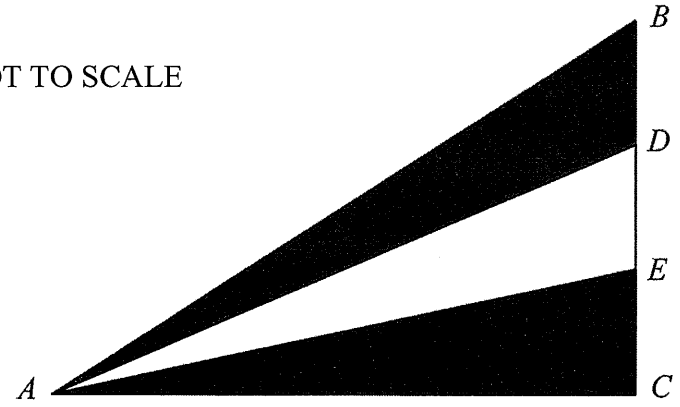
$= 7\text{hr } 53\text{min}$

∴ The cyclist must train for 7 hours and 53 minutes to produce 5.5 kWh to power his home.

Question 25 (5 marks)

The flag shown below has three sections. ΔABC is right-angled where $AC = 120$ cm and $BC = 45$ cm. The distance BD, DE and EC are equal and $\angle DEA = 105°$.

NOT TO SCALE



(a) Find the distance AE. Answer to the nearest centimetre. **2**

$AE^2 = 15^2 + 120^2$

$AE = \sqrt{15^2 + 120^2}$

$AE = 120.9338...$

$= 121$ cm (nearest cm)

(b) Find the area of ΔDEA. Answer correct to the nearest square centimetre. **2**

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 15 \times 120.93... \times \sin 105$

$= 876.0985...$

$\approx 876 \text{cm}^2$ (nearest square centimetre)

(c) What percentage of the area of the banner is ΔDEA? Answer correct to one decimal place. **1**

$\text{Percentage} = \frac{876.0985...}{\frac{1}{2} \times 120 \times 45} \times 100$

$= 32.4\%$ (1 d.p.)

Question 26 (5 marks)

The cost, $\$C$, of land in the suburb *Cartesia* is \$455 for every square metre.

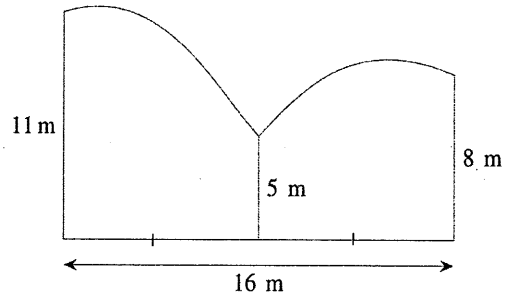
The local council also charges a fee of \$12 500 for every purchase.

(a) Express the cost of purchasing land, $\$C$, in terms of the area, A square metres, in *Cartesia.* **1**

$C = mA + b$

$= 455A + 12500$

(b) John wishes to purchase a piece of land in *Cartesia*. The shape of the land is pictured below. Since the shape is irregular, he will estimate the cost in weighing up his decision. **3**



NOT TO SCALE

Using two applications of the trapezoidal rule, estimate the cost of purchasing this piece of land.

$A \approx \frac{8}{2}(11+5) + \frac{8}{2}(5+8)$

$\approx 116 m^2$

$\therefore$ cost $\approx 455 \times 116 + 12500$

$\approx \$65280$

(c) John decides to go ahead with the purchase. The price will be based on the exact area of the land. **1**

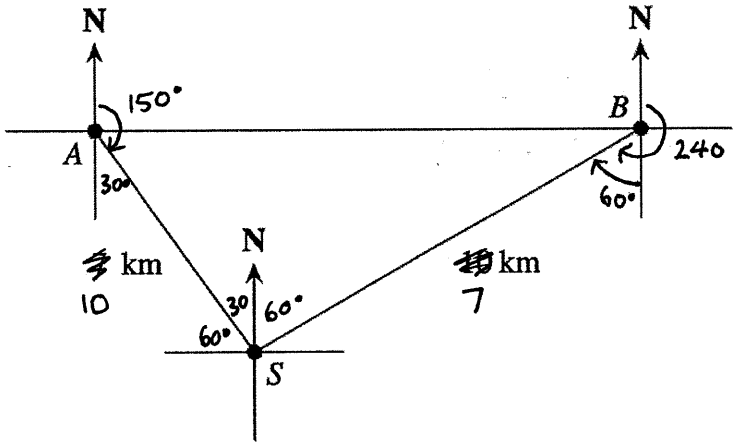
Will he pay more or less than his estimate? Briefly explain your answer.

The shape of his land is concave down, so the trapezoidal rule underestimates the area so John will pay more than his estimate.

Question 27 (4 marks)

Two friends, Ali and Brian, are calculating the distance between their houses. Brian's house is due east of Ali's house.

Ali knows that he travels to school for 10 km at a bearing of 150° while Brian travels to the same school for 7 km at a bearing of 240° .



NOT TO SCALE

(a) Explain why $\angle ASB = 90°$, labelling your diagram to aid your response. **1**

$\angle ASB = (180° - 150°) + (240° - 180°)$

$= 60° + 30° = 90°$

(b) Hence, find the distance between Ali and Brian's houses, correct to one decimal place. **2**

$AB^2 = 7^2 + 10^2$

$AB = \sqrt{149}$

$AB = 12.2\text{km} \quad (AB > 0) \quad (1.d.p)$

(c) What is the bearing of Ali's house from the school? **1**

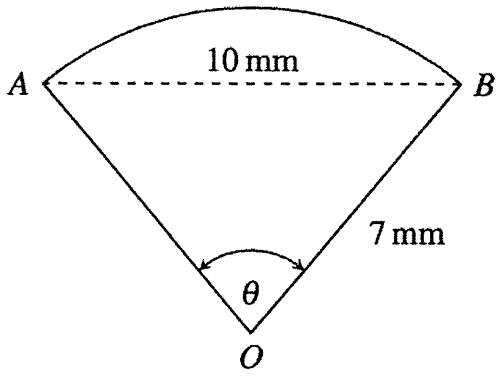
$\angle BAS = 60°$

$\therefore$ Bearing of Ali's house from the school is $270° + 60°$

$= 330°$

Question 28 (3 marks)

A sector AOB has a radius 7 mm and a central angle θ is shown. The length of the chord AB is 10 mm. **3**



NOT TO SCALE

Find the area of the flat surface of the sector, correct to the nearest square millimetre.

$\cos\theta = \frac{7^2+7^2-10^2}{2\times7\times7}$

$\theta = \cos^{-1}\left(\frac{7^2+7^2-10^2}{2\times7\times7}\right)$

$= 91.1694\ldots$

$A_{sector} = \frac{91.1694\ldots}{360} \times \pi \times 7^2$

$= 38.98$

$= 39\text{mm}^2$ (nearest mm^2)

Question 29 (5 marks)

Elliot is an engineer who earns $130 000 per year through his business. He purchases a $55 000 Ute and uses it for work purposes. **5**

He has two options for calculating the depreciation on the Ute, which will be used as a work-related tax deduction.

Method of Depreciation	*Rate of Depreciation*
Straight-line Method	$5000 per annum
Declining-balance method	10% per annum

Elliot's accountant recommends that Elliot claim 5 years' worth of depreciation when he calculates his tax, using the method that will produce the greater amount of depreciation.

The table shows the income tax rates for the 2023-24 financial year. The Medicare Levy is calculated at 2% of a person's taxable income.

Taxable Income	*Tax on this Income*
0 - $18 200	Nil
$18 200 - $45 000	19c for each $1 over $18 200
$45 001 - $120 000	$5092 plus 32.5c for each $1 over $45 000
$120 001 - $180 000	$29 467 plus 37c for each $1 over $120 000
$180 001 and over	$51 667 plus 45c for each $1 over $180 000

By using the higher depreciation amount as a tax deduction, calculate Elliot's total tax, including the Medicare Levy.

Straight-line: $S = 55000 - 5000 \times 5 = \$30000$

↳ depreciated more

Declining-bal: $S = 55000(1-0.1)^5 = \$32476.95$

taxable income = 130000 - 25000

= $105000

tax payable = 5092 + 0·325(105000 - 45000)

= $24592

medicare levy = 0·02 x 105000 = $2100

∴ total tax owed by Elliot is

$24592 + $2100

= $26692

Question 30 (4 marks)

A plane flies from Sydney to Los Angeles.

Correct to the nearest 100 km, the flight distance is 12 100 km. The plane travels at an average speed of 880 km/h, correct to the nearest 10 km/h.

(a) What is the lower bound for the flight distance? **2**

Absolute error $= \frac{1}{2} \times 100 = \pm 50$ km

$\therefore$ lower bound $= 12100 - 50$

$= 12050$ km

(b) Calculate the shortest possible flight time for the trip. **2**
Give your answer in hours and minutes, correct to the nearest minute.

For flight speed, absolute error $= \frac{1}{2} \times 10 = \pm 5$ km

upper bound $= 880 + 5 = 885$ km/h

$\therefore$ Shortest flight time $= \frac{12050}{885}$

$= 13$ hr 37 min

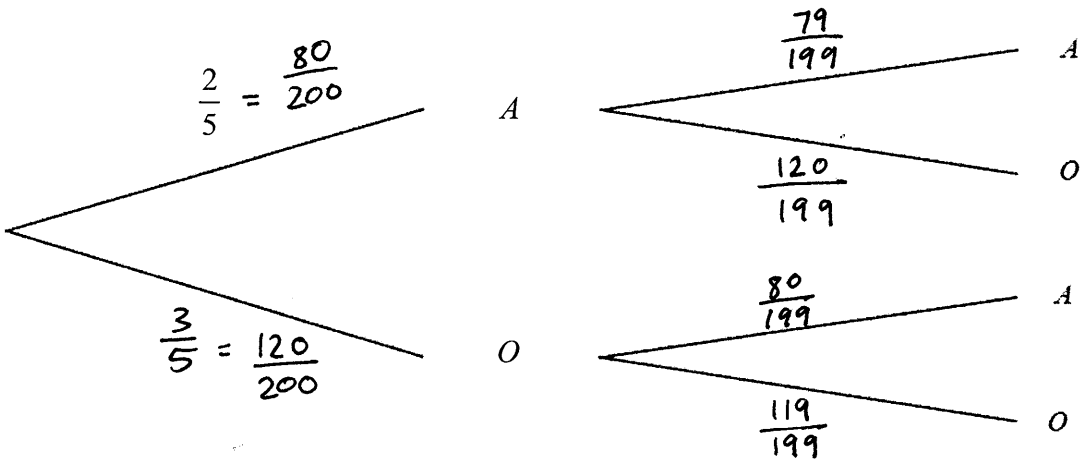
(nearest min)

Question 31 (3 marks)

In a population of 200 people, $\frac{2}{5}$ are from blood group A and the rest are from blood group O.

Two different people are chosen at random from this population.

(a) Complete the tree diagram representing this situation. **1**



(b) Find the probability that both people are of blood group A. **1**

$P(AA) = \frac{80}{200} \times \frac{79}{199} = \frac{158}{995}$

(c) Find the probability at least one person is of blood group O. **1**

$P(\text{at least 1 O}) = 1 - P(AA)$

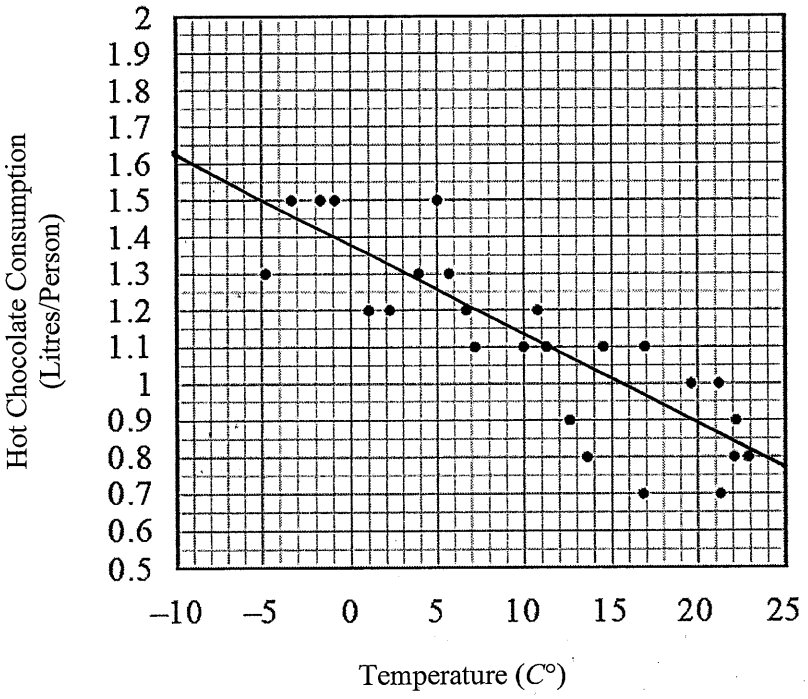
$= 1 - \frac{158}{995}$

$= \frac{837}{995}$

Question 32 (6 marks)

A researcher assumes that there is a linear relationship between the temperature and hot chocolate consumption.

Data was collected randomly in cities across Europe and the result were recorded in the scatterplot below.



The scatterplot shows the average monthly hot chocolate consumption, in litres/person against average monthly temperature, in $C°$.

The correlation (r) is found to be -0.8492.

When a least squares regression line is fitted to the scatterplot, the equation is found to be:

$$\text{Consumption} = 1.3599 - 0.0241 \times \text{temperature}$$

(a) Draw the least squares regression line on the scatterplot graph above. **1**

(b) Describe the association between average hot chocolate consumption and average monthly temperature in terms of strength, direction and form. **2**

Strong, negative, linear, associations.

(c) Interpret the slope of the least squares regression line in terms of the variables hot chocolate consumption and temperature. **1**

For every 1° increase in temperature, the average hot chocolate consumption per person decreases.

(d) Use the equation of the least squares regression line to predict the average monthly hot chocolate consumption, in litres per person, when the average monthly temperature is $-6°C$. **1**

Consumption = 1·3599 − 0·0241 x −6

= 1·5045L

(e) State whether your prediction in part (d) is interpolation or extrapolation. **1**

Extrapolation

Question 33 (7 marks)

Noah purchases a home and takes out a reducing-balance loan of $350 000 to be repaid over 25 years. The interest rate charged is 6% per annum, compounding monthly.

(a) The table below shows the present value interest factors for a loan of $1 for the various interest rates (r) and number of periods (N). **2**

Table of present value interest factors					
r / N	Interest rate per period (as a decimal)				
	0.002	0.003	0.004	0.005	0.006
100	90.5529	86.2830	82.2862	78.5426	75.0339
150	129.4799	120.6474	112.6328	105.3500	98.7226
200	164.7061	150.2317	137.4884	126.2406	116.2873
250	196.5832	175.7009	157.8465	142.5203	129.3113
300	225.4297	197.6274	174.5210	155.2069	138.9683
350	251.5337	216.5039	188.1784	165.0933	146.1288

How much should the monthly payment be in order to repay the loan in equal monthly repayments? Give your answer to the nearest cent.

$PV = a \times x$

$a = \frac{PV}{x}$

$= \frac{350000}{155.2069}$

$= \$2255.05$ (nearest cent)

(b) The interest charged and the balance owing for the first three months of the loan are shown in the table below. **1**

Month	Opening Balance	Interest Charged	Repayment	End Balance
1	350 000	**A**	2255.05	349 494.95
2	349 494.95	1747.47	2255.05	348 987.36
3	348 987.36	1744.94	2255.05	348 477.25

What is the value of **A**?

$A = 350000 \times \frac{0.06}{12}$

$= \$1750$

(c) How much interest did Noah pay over the course of the 25 years? **2**

Interest = total repayments − amount borrowed

$I = 2255.05 \times (25 \times 12) - 350000$

$= 676515 - 350000$

$= \$326515$

(d) After 15 years, Noah decides to make a lump sum payment of $50 000 and continues to make the same monthly repayments. The rest of the loan was then repaid over a further 86 months. **2**

How much interest did Noah save by making the lump sum payment?

total regular repayments $= (180+86) \times 2255.05$

$= \$599843.30$

$\therefore$ total repaid $= 599843.30 + 50000$

$= \$649843.30$

$\therefore$ interest saved $= 676515 - 649843.30$

$= \$26671.70$

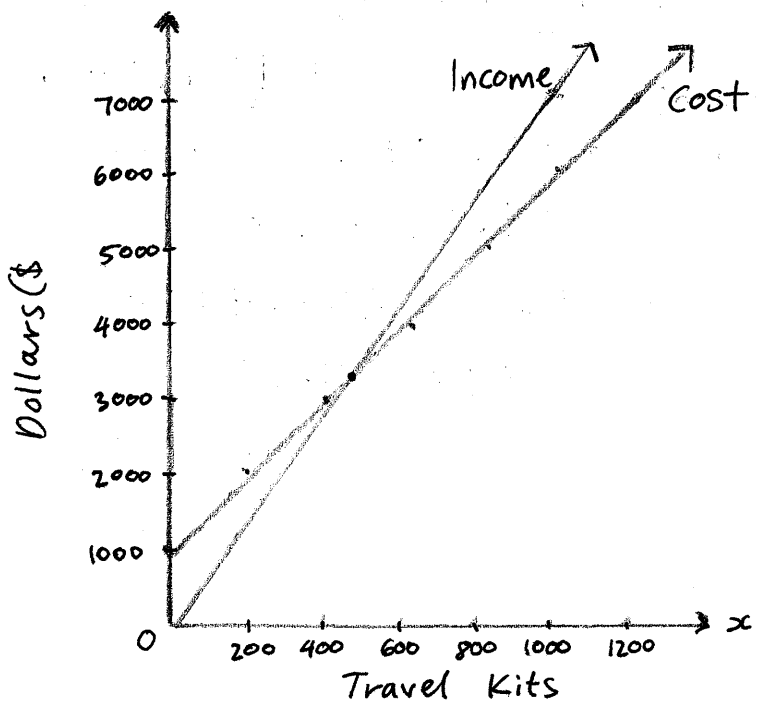
Question 34 (6 marks)

The income of a travel kit manufacturer is given by $I = 7x$ (dollars) and their costs is given by $C = 5x + 1000$ (dollars), where x represents the number of the travel kits.

(a) Draw a sketch to represent the costs and income for producing a travel kit. **3**

x	200	400	600	800	1000	1200
I	1400	2800	4200	5600	7000	8400

x	200	400	600	800	1000	1200
C	2000	3000	4000	5000	6000	7000



(b) Using the graph above, estimate how many travel kits are needed to break-even? **1**

≈ 500 travel kits are needed to break-even.

(c) Determine the number of travel kits that need to be sold to break-even, if the total cost increases by 5%. **2**

Cost increase → $C_{new} = 1{\cdot}05(5x + 1000)$

$= 5.25x + 1050$

Break-even when $I = C \rightarrow 7x = 5.25x + 1050$

$1.75x = 1050$

$x = 600$

∴ 600 travel kits need to be made to break-even.

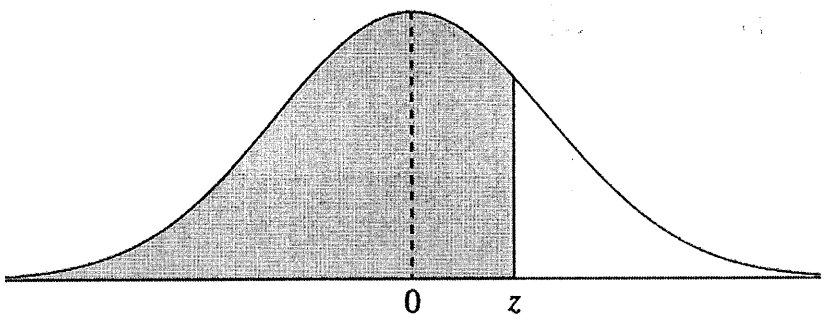
Question 35 (3 marks)

3

The table below gives the probability that a random variable lies below some z-score, where the random variable is normally distributed.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549

The probability values given in the table below represent the shaded area in the diagram.



The results of a university entrance test sat by Year 12 students is normally distributed with a mean of 35 and a standard deviation of 5. Students who score in the 67th percentile and above are offered a place at the university.

What is the minimum mark students need to achieve in the entrance test in order to be offered a place at the university?

$P(Z \leq z) = 0.6700 \rightarrow z = 0.44$

$0.44 = \frac{x - 35}{5}$

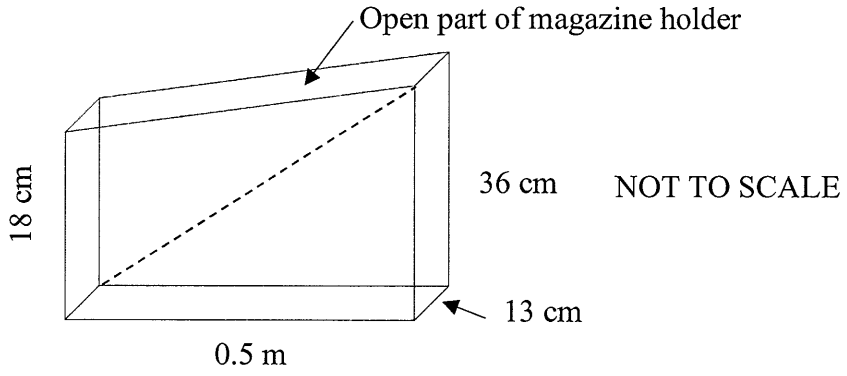
$x = 0.44 \times 5 + 35$

$= 37.2$

Question 36 (5 marks)

A magazine holder has only one slot to insert magazines as shown below. **5**

The width of the holder is 13 cm.



The cost of making such a holder is based on the external surface area of the holder.

If the cost of heavy duty coloured cardboard is $6.75 per square metre, calculate the cost of making 2550 of these magazine holders, correct to the nearest dollar.

$SA = 2 \times \left(\frac{50}{2}(36+18)\right) + (36 \times 13) + (50 \times 13) + (18 \times 13)$

$= 4052 cm^2$

$= 4052 \div 100^2$

$= 0.4052 m^2$

Cost of 1 $= 0.4052 \times 6.75$

$= 2.74$

Cost of 2550 $= 2.74 \times 2550$

$= 6974.51$

$= \$6975$ (nearest dollar)

END OF EXAMINATION

HORNSBY GIRLS HIGH SCHOOL

Mathematics Standard 2

Year 12 Higher School Certificate Trial
Assessment 4 Term 3 2024

STUDENT NUMBER: _____

Section I – Multiple Choice Answer Sheet

Allow about 25 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ● (correct) C ○ D ○

1.	A ●	B ○	C ○	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ●	D ○
6.	A ○	B ●	C ○	D ○
7.	A ○	B ○	C ●	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ○	D ●
10.	A ○	B ○	C ○	D ●
11.	A ○	B ●	C ○	D ○
12.	A ○	B ○	C ●	D ○
13.	A ○	B ○	C ○	D ●
14.	A ○	B ●	C ○	D ○
15.	A ○	B ○	C ○	D ●