

SOLUTIONS

HSC Trial Preparation Questions

14. Two cities, X and Y, both lie on the equator and have respective longitudinal positions of 105°W and 155°E .

What is the time in city X when it is 11:15 pm on Monday in city Y?

- (A) 7:55 pm Monday
(B) 5:55 am Monday
(C) 10:43 am Tuesday
(D) 10:35 am Tuesday

Diagram showing cities X and Y on the equator. X is at 105°W and Y is at 155°E . The time difference is calculated as follows:

$$105 + 155 = 260^\circ \times 4 \text{ min} = 1040 \text{ min} \div 60 = 17 \text{ h } 20 \text{ min}$$

$$23:15 - 17:20 = 5:55 \text{ am Monday}$$

17. Hudson is a hospital patient who is given 1.5 L of fluid over 8 hours.

What is the required drip rate?

- (A) 0.2 mL/h
(B) 5.3 mL/h
(C) 12 mL/h
(D) 187.5 mL/h

Diagram showing the calculation of the drip rate:

$$\frac{1500 \text{ mL}}{8 \text{ h}} = 187.5 \text{ mL/h}$$

22. 3000 flathead were caught in Sydney Harbour, tagged and released. Three months later, a sample of 2000 flathead were taken. In this sample, 720 tagged flathead were found.

Which of the following is the best estimate for the number of flathead in Sydney Harbour?

- (A) 1080
(B) 2280
(C) 5000
(D) 8333

Diagram showing the calculation of the population size P:

$$\frac{P}{3000} = \frac{2000}{720}$$

$$P = \frac{2000}{720} \times 3000$$

25. Which of the following correctly expresses r as the subject of $T = ar^2 + c$?

- (A) $r = \pm \sqrt{\frac{T-c}{a}}$
(B) $r = \pm \frac{\sqrt{T-c}}{a}$
(C) $r = \pm \sqrt{\frac{T}{a} - c}$
(D) $r = \pm \sqrt{\frac{T}{a} - c}$

Diagram showing the algebraic steps to solve for r:

$$T = ar^2 + c$$

$$-c \quad -c$$

$$\frac{T-c}{a} = \frac{ar^2}{a}$$

$$r^2 = \frac{T-c}{a}$$

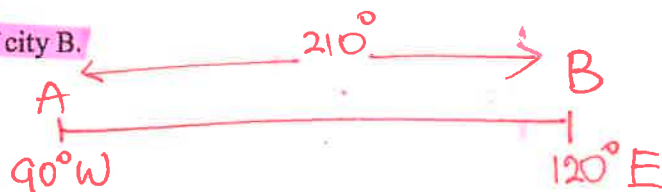
$$r = \pm \sqrt{\frac{T-c}{a}}$$

- 6 The time difference between two world cities A and B, both situated on the 20°N latitude, is 14 hours.

City A lies on the 90°W longitude and is west of city B.

On what longitude does city B lie?

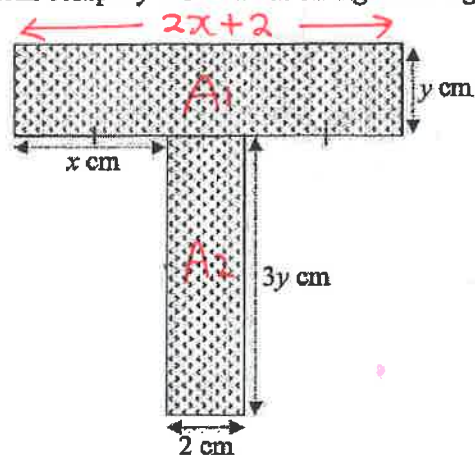
- (A) 30°W
 (B) 30°E
 (C) 120°E
 (D) 210°E



$$14 \text{ h} \times 60 = 840 \text{ min} \div 4 = 210^{\circ}$$

$$210^{\circ} - 90^{\circ} = 120^{\circ}\text{E}$$

- 4 A telecommunications company has a "T" as its signature sign.



Which of these expressions would give the area (in cm^2) of this front surface of the "T" sign?

- (A) $2xy + 6y$
 (B) $2xy + 8y$
 (C) $4xy + 6y$
 (D) $2x + 8y$

$$A = A_1 + A_2$$

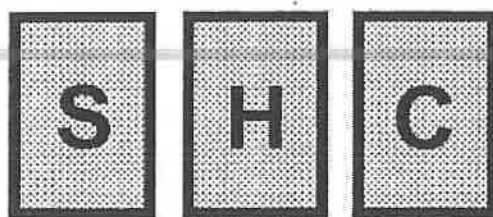
$$= y(2x+2) + 2 \times 3y$$

$$= 2xy + 2y + 6y$$

$$= 2xy + 8y$$

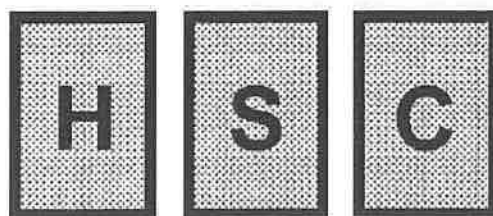
7

Three letters S, H and C are written on separate cards.



The cards are turned over, shuffled and placed face down next to each other on a table.

What is the probability that the cards from left to right appear, as shown below?



(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

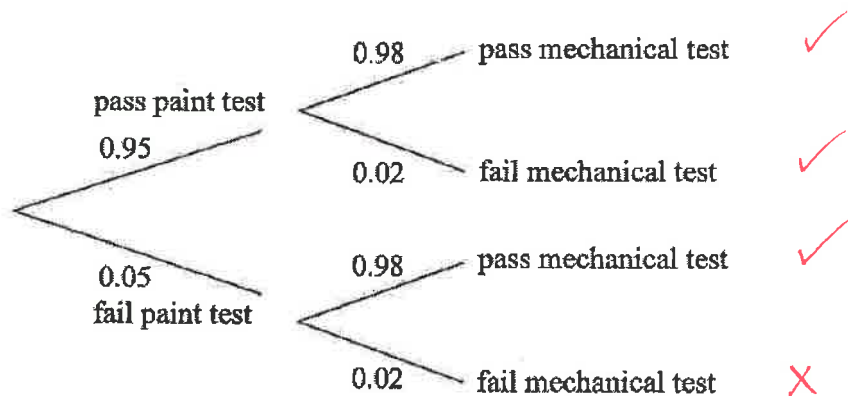
(C) $\frac{2}{3}$

(D) $\frac{1}{6}$

$$\begin{array}{c} \uparrow \\ \frac{1}{3} \end{array} \times \begin{array}{c} \uparrow \\ \frac{1}{2} \end{array} \times \begin{array}{c} \uparrow \\ \frac{1}{1} \end{array} = \frac{1}{6}$$

- 11 New cars off the production line have a probability of 0.95 of passing a paint test and a probability of 0.98 of passing a mechanical test.

A probability tree diagram shows this information for both tests.

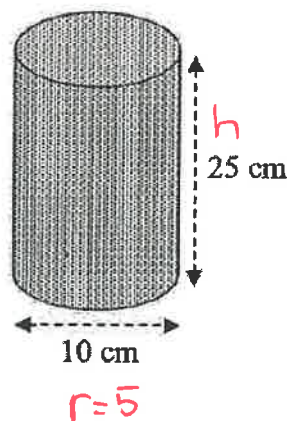


What is the probability that a car randomly selected off the production line will pass at least one of the two tests?

- (A) 93%
(B) 96.5%
(C) 99.9%
(D) 100%

$$\begin{aligned}
 1 - P(\text{FF}) &= 1 - 0.05 \times 0.02 \\
 &= 0.999 \\
 &= 99.9\%
 \end{aligned}$$

- 22 What is the total surface area (in cm^2) of this closed cylinder?



$$\begin{aligned}
 \text{S.A.} &= 2\pi rh + 2\pi r^2 \\
 &= 2\pi \times 5 \times 25 + 2\pi \times 25 \\
 &= 250\pi + 50\pi \\
 &= 300\pi
 \end{aligned}$$

- (A) 175π
(B) 300π
(C) 625π
(D) 700π

- 6 The wingspan of an airplane is 80 metres. Lara builds a model of the plane in which the wingspan measures 16 centimetres.

What scale has Lara used for her model?

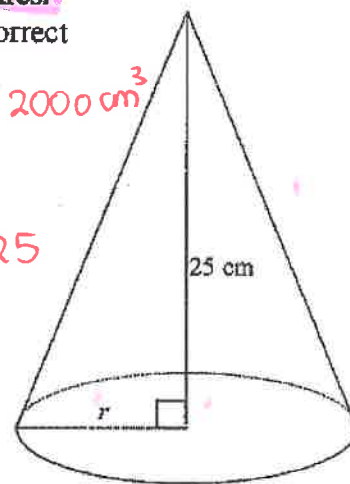
$$\begin{aligned}\text{Scale} &= 16 \text{ cm} : 80 \text{ m} \\ &= 16 \text{ cm} : 8000 \text{ cm} \\ &= 1 : 500\end{aligned}$$

- (A) 1 : 5
(B) 1 : 50
(C) 1 : 500
(D) 1 : 5 000

- 23 The capacity of a cone with a height of 25 cm is 2 litres. Find the radius of the circular base, in centimetres correct to 2 decimal places.

- (A) 2.76 cm
(B) 5.05 cm
(C) 8.74 cm
(D) 76.39 cm

$$\begin{aligned}2 \text{ L} &= 2000 \text{ mL} = 2000 \text{ cm}^3 \\ V &= \frac{1}{3} \pi r^2 h \\ 2000 &= \frac{1}{3} \times \pi \times r^2 \times 25 \\ r^2 &= 76.39 \\ r &\div 8.74\end{aligned}$$



- (b) Make V the subject of the formula

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\begin{aligned}r^3 &= \frac{3V}{4\pi} \\ 4\pi r^3 &= 3V \\ \therefore V &= \frac{4\pi r^3}{3}\end{aligned}$$

- (d) Calculate the percentage error in a measurement of 2.75 metres.

Give your answer correct to 2 decimal places.

$$\text{Limit of reading} = 0.01 \text{ m} \quad \left. \vphantom{\text{Limit of reading}} \right\} \times \frac{1}{2}$$

$$\text{Absolute error} = \pm 0.005$$

$$\% \text{ error} = \pm \frac{0.005}{2.75} \times 100\% \div \pm 0.18\%$$

- (a) An environmental group used the capture-recapture method to estimate the number of sharks in Sydney Harbour.

In one month, 14 sharks were caught, tagged and released back into the harbour. The following month, 18 sharks were caught, including 4 that were already tagged.

Use the data to estimate the total shark population in Sydney Harbour.

1

$$\frac{P}{14} = \frac{18}{4}$$

$$P = 63$$

- (c) An international flight leaves Botswana (22°S, 25°E) at 6 am local time on Friday 2 February and flies to Johannesburg, where it stops for 6 hours and 30 minutes before continuing its journey to Sydney (34°S, 150°E).

3

The flight time from Botswana to Johannesburg is 1 hour and 15 minutes. The flight from Johannesburg to Sydney takes 14 hours.

What is the time and date when the plane arrives in Sydney?

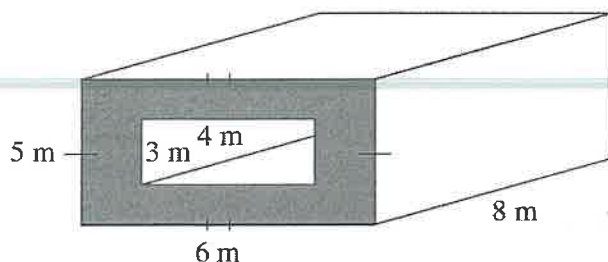


$$\text{Time diff.} = 150^\circ - 25^\circ = 125^\circ \times 4 \text{ min} = 500 \text{ min} \div 60 = 8 \text{ h } 20 \text{ min}$$

$$\text{Total journey time} = 6 \text{ h } 30 \text{ min} + 1 \text{ h } 15 \text{ m} + 14 \text{ h} = 21 \text{ h } 45 \text{ min}$$

$$\begin{aligned} \text{Sydney time} &= B + 21 \text{ h } 45 \text{ m} + 8 \text{ h } 20 \text{ min} \\ &= B + 30 \text{ h } 5 \text{ min} \\ &= 6 \text{ am Fri } 2/2 + 24 \text{ h} + 6 \text{ h } 5 \text{ min} \\ &\quad \quad \quad (1 \text{ day}) \\ &= 6 \text{ am Sat } 3/2 + 6 \text{ h } 5 \text{ min} \\ &= 12:05 \text{ pm Sat } 3/2 \end{aligned}$$

- (c) A composite solid is shown below.



- (i) What is the volume of the inner rectangular prism?

1

$$V = 4 \times 3 \times 8 \\ = 96 \text{ m}^3$$

- (ii) What is the volume of the outer rectangular prism?

1

$$V = 6 \times 5 \times 8 \\ = 240 \text{ m}^3$$

- (iii) What is the volume of the composite solid?

1

$$V = 144 \text{ m}^3$$

- (f) Layla has forgotten her four-digit PIN but knows that it contains the digits 0, 5, 6 and 8.

- (i) How many different PINs are possible using these four digits?

1

Only 4 digits to choose from:

$$\boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 24$$

\uparrow 1st digit \uparrow 2nd \uparrow 3rd \uparrow 4th

- (ii) Layla remembers that the first digit is 5.

1

What is the probability that her PIN is 5860?

Limit the combinations; no longer 24 combinations.

$$\boxed{1} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 6$$

\uparrow Must be a "5" \uparrow 2nd \uparrow 3rd \uparrow 4th

$$P(\text{exactly } 5860) = \frac{1}{6}$$

Question 27 (continued)

Marks

- (c) A discus was thrown at the World Championships a distance of 74.08 m.

(i) What is the absolute error?

1

~~$$\frac{1}{2} \times 0.01 \text{ m} = \pm 0.005 \text{ m}$$~~

$$\frac{1}{2} \times 0.01 \text{ m} = \pm 0.005 \text{ m}$$

- (ii) Find the percentage error, correct to three decimal places. $\pm \frac{0.005}{74.08} \times 100\% = \pm 0.007\%$

- (b) A rain gauge registered 62.5 mm of rain during a storm. The rain fell on a shed with a rectangular roof that measures 20 m by 12 m.

H 0.0625 m

L H

- (i) How many litres of water fell on the shed? Give your answer to the nearest litre.

2

$$V = LBH$$

$$= 20 \times 12 \times 0.0625$$

$$= 15 \text{ m}^3$$

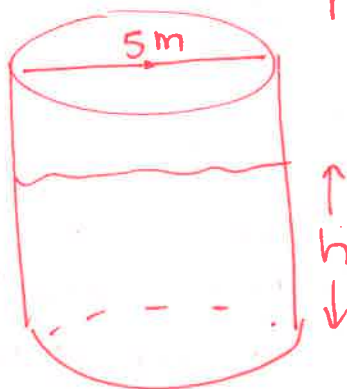
$$= 15 \text{ kL} = 15000 \text{ L}$$

$$1 \text{ m}^3 = 1 \text{ kL}$$

- (ii) The water that fell on the shed was collected in an empty cylindrical tank with a diameter of 5 m.

2

What depth of water will be in the tank? Give your answer correct to two decimal places.



$$r = 2.5 \text{ m}$$

$$V = \pi r^2 h$$

$$15 = \pi \times 2.5^2 \times h$$

$$h = 0.7639... \text{ m}$$

$$h \approx 0.76 \text{ m}$$

- (c) Ships A and B leave port C at 10:00 am in different directions.

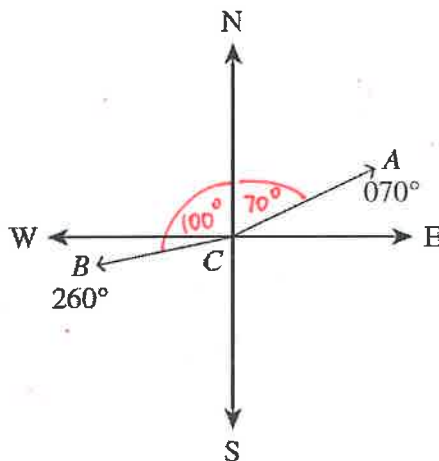
Ship	Bearing from C	Speed
A	70°	25 km/h
B	260°	40 km/h

Distance

$$\times 5 = 125 \text{ km}$$

$$\times 5 = 200 \text{ km}$$

The diagram below shows the courses of each of the ships from port C.



$$3 \text{ pm} - 10 \text{ am}$$

$$15 - 10 = 5 \text{ h}$$

$$A = 125 \text{ km}$$

$$B = 200 \text{ km}$$

2

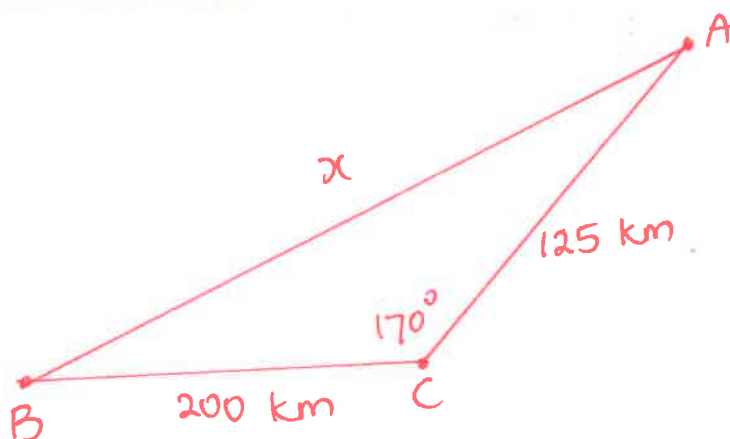
- (ii) What is the size of $\angle ACB$?

$$\angle ACB = 70^\circ + 100^\circ = 170^\circ$$

1

- (iii) What is the distance AB between the ships at 3.00 pm? Answer correct to the nearest kilometre.

2



$$x^2 = 125^2 + 200^2 - 2 \times 125 \times 200 \times \cos 170^\circ$$

$$x^2 = 104,865.387 \dots$$

$$x \div 324 \text{ km}$$

(a) A boat leaves Port Vila (17°S, 168°E) and travels to Kingston (29°S, 168°E).

- (i) Find the distance the boat has travelled between Port Vila and Kingston, correct to the nearest kilometre. 1

$$\begin{array}{l}
 \begin{array}{c}
 +0^\circ \\
 \text{P.V.} \times 17^\circ\text{S} \\
 \vdots \\
 \text{K} \times 29^\circ\text{S}
 \end{array}
 \quad \left. \vphantom{\begin{array}{c} \text{P.V.} \times 17^\circ\text{S} \\ \vdots \\ \text{K} \times 29^\circ\text{S} \end{array}} \right\} \begin{array}{l} 29^\circ - 17^\circ = 12^\circ \\ d = \frac{12}{360} \times 2\pi \times 6400 \\ \div 1340 \text{ km} \end{array}
 \end{array}$$

radius of Earth

- (ii) The boat travels at an average speed of 16 km/h. 1

How many hours and minutes will it take to travel from Port Vila to Kingston?

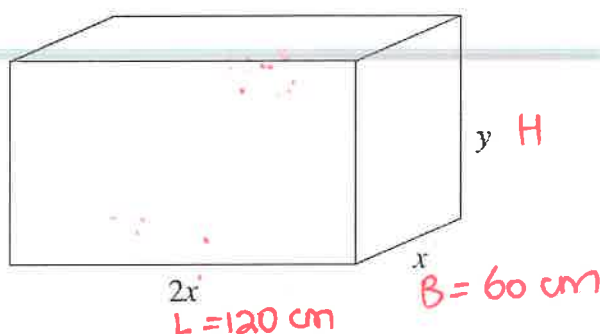
$$T = \frac{D}{S} = \frac{1340 \text{ km}}{16 \text{ km/h}} \div 83 \text{ h } 45 \text{ min}$$

- (iii) The boat leaves Port Vila at 7:00 am on Wednesday 10th July. 2

What is the date and time of arrival of the boat in Kingston?

$$\begin{array}{l}
 \begin{array}{c}
 +0^\circ \\
 \text{P.V.} \times 17^\circ\text{S} \\
 \vdots \\
 \text{K} \times 29^\circ\text{S}
 \end{array}
 \quad \begin{array}{l}
 7:00 \text{ am Wed } 10^{\text{th}} \text{ July} \\
 + 3 \text{ days } + 11 \text{ h } 45 \text{ min} \\
 7:00 \text{ am Sat } 13^{\text{th}} \text{ July} \\
 + 11 \text{ h } 45 \text{ min} \\
 18:45 \text{ Sat } 13^{\text{th}} \text{ July}
 \end{array}
 \end{array}$$

- (d) A concrete block has dimensions $2x$ cm, x cm and y cm as shown below.



- (i) Find an expression for the surface area of the block.

1

$$\begin{aligned}
 \text{S.A} &= 2 \times (\text{Front} + \text{Side} + \text{Top}) \\
 &= 2 \times (2xy + xy + 2x^2) \\
 &= 2 \times (3xy + 2x^2) \\
 &= 6xy + 4x^2
 \end{aligned}$$

- (ii) The block has a surface area $43\,200 \text{ cm}^2$.

1

Show that the height is given by $y = \frac{43\,200 - 4x^2}{6x}$.

$$\begin{aligned}
 43200 &= 6xy + 4x^2 \\
 -4x^2 &\quad -4x^2 \\
 \hline
 43200 - 4x^2 &= \frac{6xy}{6x} \quad \therefore y = \frac{43200 - 4x^2}{6x}
 \end{aligned}$$

- (iii) Find an expression for the volume (V) of the block.

1

$$\begin{aligned}
 V &= LBH \\
 &= 2x \times x \times y \\
 \therefore V &= 2x^2y
 \end{aligned}$$

- (iv) What is the volume of the block if the block has base dimensions of 120 cm by 60 cm?

1

$$\begin{aligned}
 y &= \frac{43200 - 4(60)^2}{6 \times 60} \\
 &= 80 \\
 V &= 2x^2y = 2 \times 60^2 \times 80 \\
 &= 576,000 \text{ cm}^3
 \end{aligned}$$

- (b) Ally is putting together the table of data below from information collected on her sports watch. It shows the z-scores corresponding to the number of active minutes done each day. The data is assumed to be normally distributed.

Number of active minutes/day		16	30				
z-score	-3	-2	-1	0	1	2	3

- (i) What is the standard deviation for this data?

$$30 - 16 = 14$$

- (ii) What is the median? = mean

$$30 + 14 = 44$$

- (iii) Health experts recommend a minimum of 30 minutes of active minutes each day. Calculate the number of days that Ally could expect to achieve the recommended number of active minutes in the month of August.

↓
31 days

$$34\% + 50\% = 84\%$$

$$84\% \times 31 = 26.04$$

$$\approx 26 \text{ days}$$