Student's Name: ______________________

Student Number:

Teacher's Name: ______________________

TEMPUS CELERIUS RADIO FUGIT

ABBOTSLEIGH

2022

HIGHER SCHOOL CERTIFICATE

Assessment 4

Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen.
- **NESA approved** calculators may be used.
- **NESA approved** reference sheet is provided.
- All necessary working should be shown in every question.
- Make sure your HSC Candidate Number is on the front cover of each booklet.
- Start a new booklet for each Question.
- Answer the Multiple Choice questions on the answer sheet provided.
- If you do not attempt a whole question, you must still hand in the Writing Booklet, with the words **'NOT ATTEMPTED'** written clearly on the front cover.

Total marks - 70

- Attempt Sections I and II.

Section I — Pages 3 - 7

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II — Pages 8 - 14

60 marks

- Attempt Questions 11 – 14.
- Allow about 1 hour and 45 minutes for this section.
- All questions are of equal value.

Outcomes to be assessed:

Year 11 Mathematics Extension 1 outcomes

A student:

ME11-1

uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2

manipulates algebraic expressions and graphical functions to solve problems

ME11-3

applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

ME11-4

applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change

ME11-5

uses concepts of permutations and combinations to solve problems involving counting or ordering

ME11-7

communicates making comprehensive use of mathematical language, notation, diagrams and graphs

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1

applies techniques involving proof or calculus to model and solve problems

ME12-2

applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3

applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME12-4

uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

ME12-7

evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Section I (10 marks)

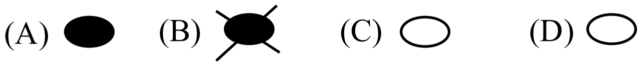
Attempt Questions 1 – 10

Use the multiple-choice answer sheet

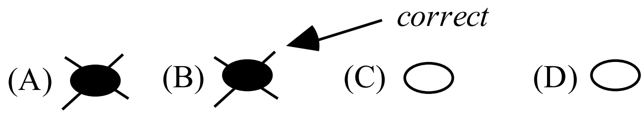
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.



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1 How many distinct arrangements of the letters of the word ARRANGEMENT are possible?

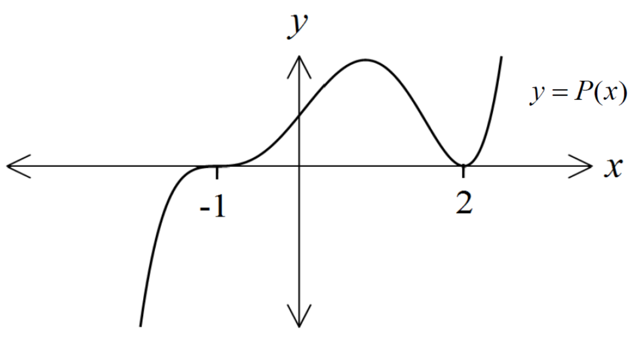
A. 2 494 800

B. 4 989 600

C. 9 979 200

D. 39 916 800

2 Which of the following could be the equation of the graphed polynomial, *P(x)*?



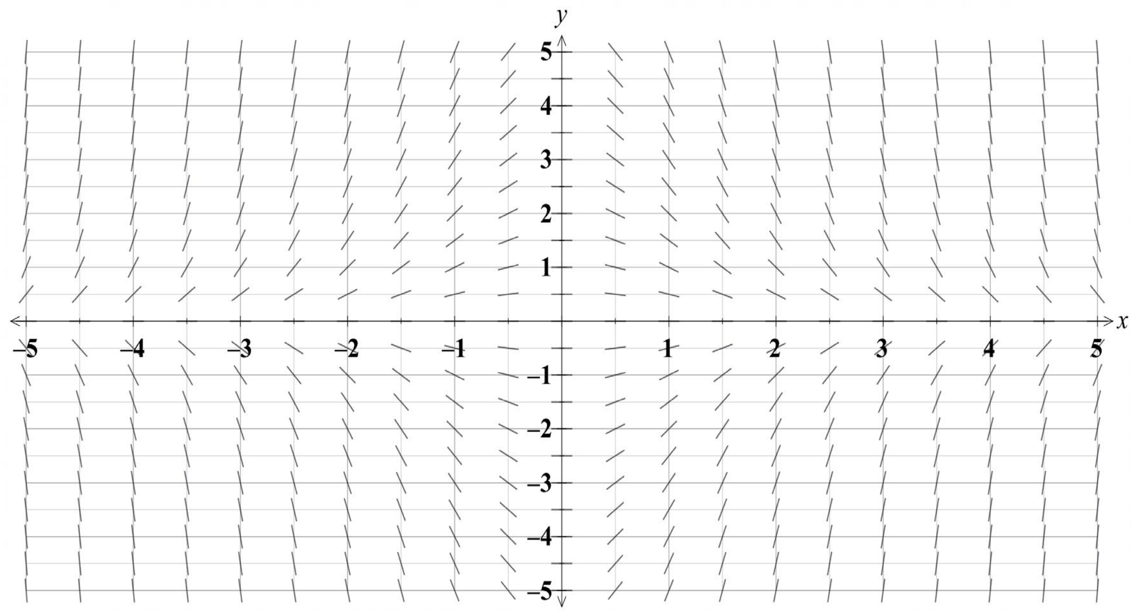
A. $P(x) = -(x+1)^3(x-2)^2$

B. $P(x) = (x-1)^3(x+2)^2$

C. $P(x) = (x+1)^3(x-2)^2$

D. $P(x) = (x-1)^3(2-x)^2$

3 Which of the following differential equations could be represented by the slope field diagram below?



A. $y' = -x\,y$

B. $y' = x\,y$

C. $y' = -x^2\,y$

D. $y' = x^2\,y$

4 If $\tan\theta = t$ then which of the following is equivalent to $\dfrac{2\tan\theta}{1+\tan^2\theta}$?

A. $\cot 2\theta$

B. $\tan 2\theta$

C. $\cos 2\theta$

D. $\sin 2\theta$

5 If $u = t^2 + 5$ and $v = \sqrt{t^2 - 1}$. Find $\dfrac{du}{dv}$.

A. $\dfrac{1}{2v}$

B. $2v$

C. $\dfrac{2}{v}$

D. $\dfrac{1}{2}\sqrt{v}$

6 What is the exact value of $\displaystyle\int_0^4 \frac{dx}{x^2 + 16}$?

A. $-\dfrac{\pi}{4}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{8}$

D. $\dfrac{\pi}{16}$

7 Which of the following is the derivative of $y = x^2 \sin^{-1}(2x)$?

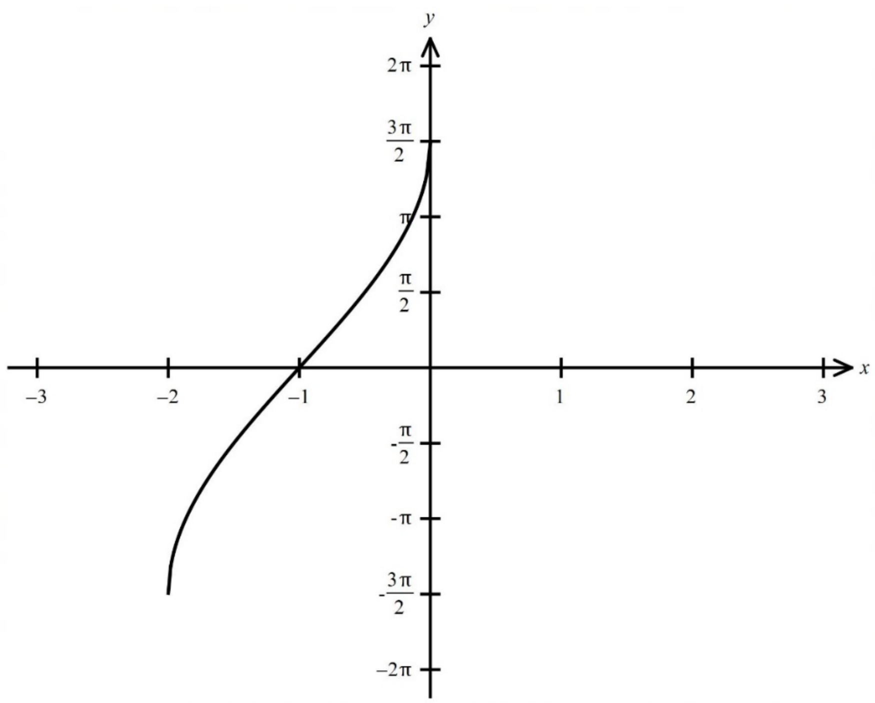
A. $2x\sin^{-1}(2x) + \dfrac{x^2}{\sqrt{1-4x^2}}$

B. $2x\sin^{-1}(2x) + \dfrac{2x^2}{\sqrt{1-4x^2}}$

C. $2x\sin^{-1}(2x) + \dfrac{2x^2}{\sqrt{1-2x^2}}$

D. $2x\sin^{-1}(2x) + \dfrac{x^2}{\sqrt{1-2x^2}}$

8 Which equation describes the graph shown?



A. $y = 3\cos^{-1}(x+1)$

B. $y = \dfrac{1}{3}\sin^{-1}(x+1)$

C. $y = 3\sin^{-1}(x+1)$

D. $y = 3\cos^{-1}(x-1)$

9 The polynomial $P(x) = 4x^3 - x^2 - 6x + 9$ has roots α, β and γ.

What is the value of $\dfrac{1}{\gamma} + \dfrac{1}{\beta} + \dfrac{1}{\alpha}$?

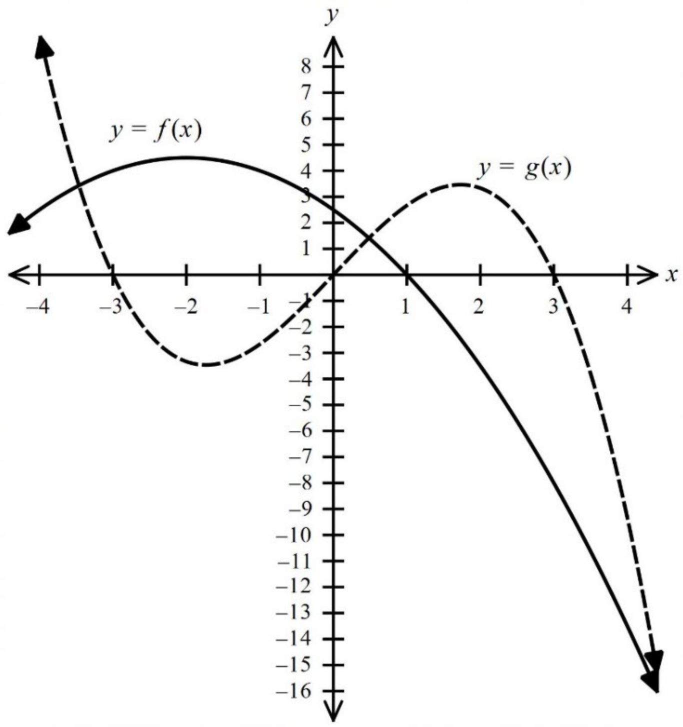
A. $-\dfrac{4}{9}$

B. $\dfrac{27}{8}$

C. $\dfrac{2}{3}$

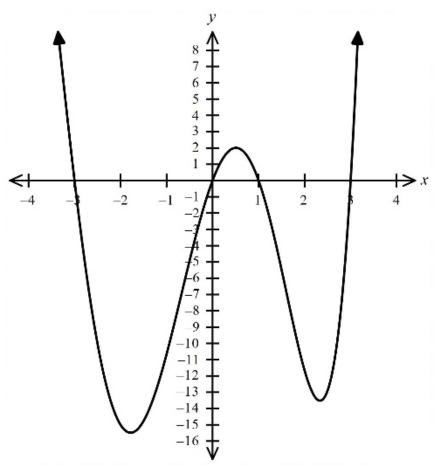
D. $\dfrac{4}{3}$

10 The graphs of $y = f(x)$ and $y = g(x)$ are shown on the axes below.

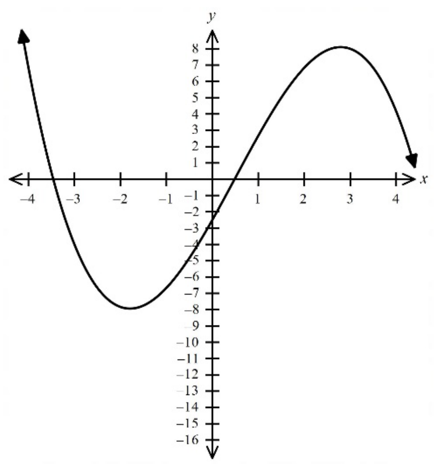


Which graph could represent $y = f(x) \times g(x)$ for the domain shown above?

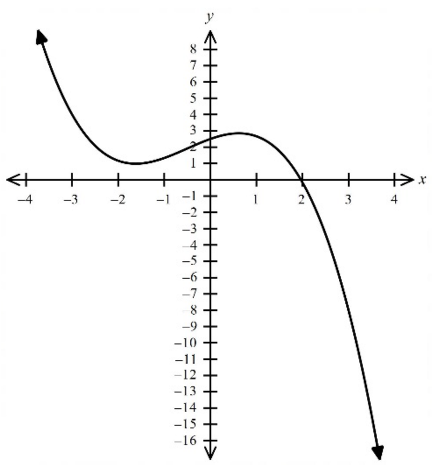
A.



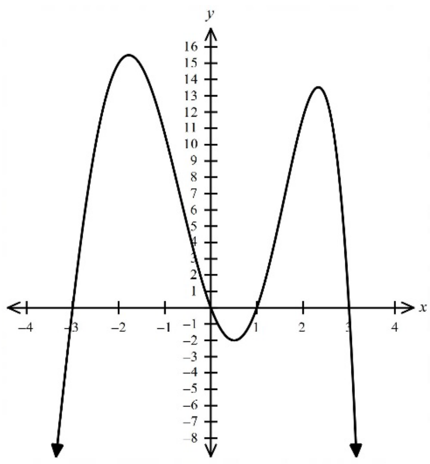
B.



C.



D.



End of Section I

SECTION II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a **SEPARATE** writing booklet. Extra writing booklets are available.

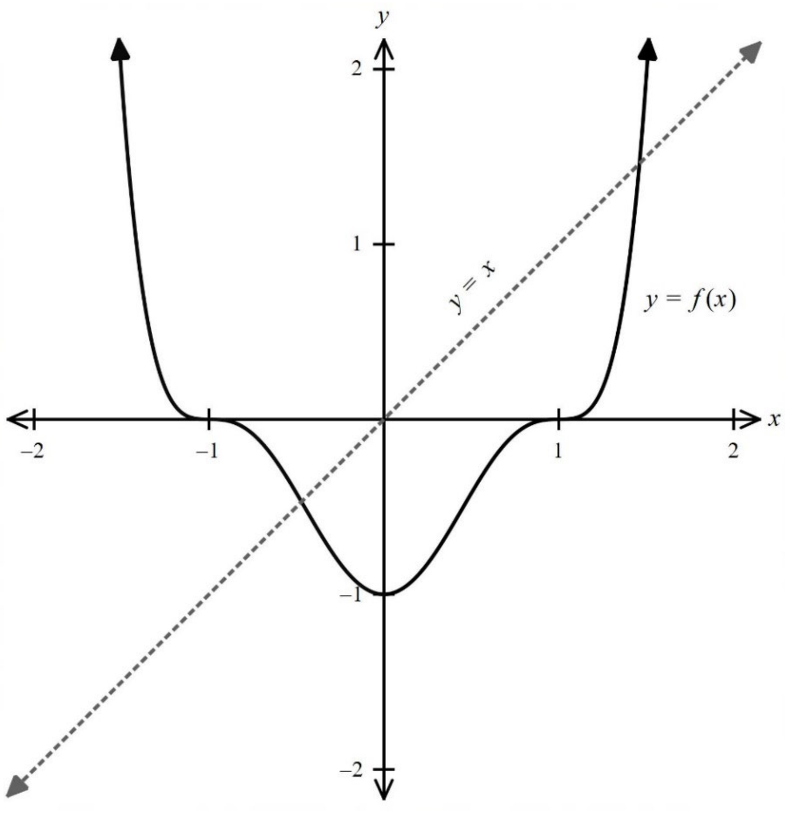
In Questions 11-14 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The point A has a position vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$ and the point B has position vector $\begin{pmatrix} 8 \\ 3 \end{pmatrix}$. **2**

Find the vector $\overrightarrow{AB}$.

(b) The graph shows the function $y = f(x) = \left(x^2 - 1\right)^3$ for all real x, and the line $y = x$.



So that it will have an inverse, the function $g(x)$ is defined as $g(x) = \left(x^2 - 1\right)^3$ for $x \geq 0$.

(i) Draw a sketch showing $y = g(x)$ and $y = g^{-1}(x)$ on the same axes. **1**

(ii) Write an expression for $g^{-1}(x)$. **2**

Question 11 continues on the next page

(c) Find the coefficient of y^4 in $(1+2y)^6$. **1**

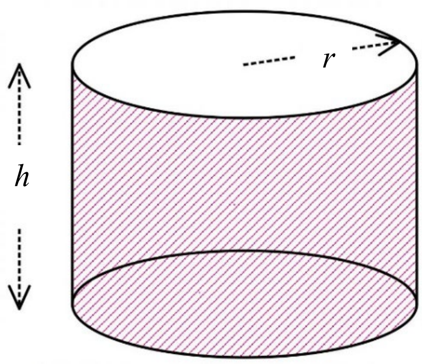
(d) (i) There are 16 adults and 9 children living in a building and six of these people are to be chosen at random to complete a survey. **1**

How many ways can the six people be chosen?

(ii) The six people chosen to complete the survey are to comprise four adults and two children. **1**

How many different groups of people are possible with this restriction?

(e) The solid shown below is a cylinder which has its height equal to its diameter.



A virtual 3-D model is created which maintains the ratio of height and diameter.

In an animation, the model is being scaled up so that its volume is increasing at a steady rate of 120 cm^3 per second.

(i) Show that the equation of the volume in terms of the radius r is $V = 2\pi r^3$. **1**

(ii) Find the rate at which the radius is increasing when it reaches 2 cm. **3**

(f) Use the substitution $x = \cos 2\theta$ to show that $\int \sqrt{\dfrac{1-x}{1+x}}\, dx = \int -4\sin^2\theta \, d\theta$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\dfrac{4x}{x+2} \le 3$ **3**

(b) Show that $y = Ae^{3x} + Be^{-2x}$ satisfies the differential equation **2**

$y'' - y' - 6y = 0$ for any real values of A and B.

(c) Use the substitution $u = 1 + x$ to find $\displaystyle\int_{-1}^{0} \frac{x}{\sqrt{1+x}}\, dx$. **3**

(d) (i) Express $\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $0 < \alpha < \dfrac{\pi}{2}$. **2**

(ii) Hence solve the equation $\cos\theta - \sin\theta = 1$ for $0 \le \theta \le 2\pi$. **2**

(e) Prove by mathematical induction that, for all integers $n \ge 1$

$$2 + 2^3 + 2^5 + \dots + 2^{2n-1} = \frac{2(2^{2n} - 1)}{3}.$$ **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the binomial theorem to find the term independent of x in the expansion of **3**

$$\left(\frac{2}{x}-x^2\right)^{12}.$$

(b) Consider the vectors $\underset{\sim}{a}=2\underset{\sim}{i}+3\underset{\sim}{j}$ and $\underset{\sim}{b}=9\underset{\sim}{i}+\underset{\sim}{j}$.

Calculate the angle, in radians correct to 3 decimal places, between vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(c) Points A, B have position vectors $\overrightarrow{OA}=3\underset{\sim}{i}-2\underset{\sim}{j}$ and $\overrightarrow{OB}=-\underset{\sim}{i}+\underset{\sim}{j}$.

(i) Find the unit vector along $\overrightarrow{AB}$. **2**

(ii) Suppose P is a point on AB such that $\overrightarrow{OP}\perp\overrightarrow{AB}$. Find $\overrightarrow{OP}$. **3**

(d) Find the volume generated when $y=\sin x$ between $x=0$ and $x=\dfrac{\pi}{3}$ is rotated around the x-axis. **3**

(e) Find $\int 2\cos 5x\sin 6x\ dx$. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Show that $\tan 2x + \tan x = \dfrac{\sin 3x}{\cos 2x \cos x}$. **2**

(ii) Hence solve the equation $\tan 2x + \tan x = 0$ for $0 < x < \dfrac{\pi}{2}$. **2**

(b) The population, P, of animals in an environment in which there are scarce resources is increasing such that $\dfrac{dP}{dt} = P(100 - P)$, where t is time and $0 < P < 100$. When $t = 0$, $P = 10$.

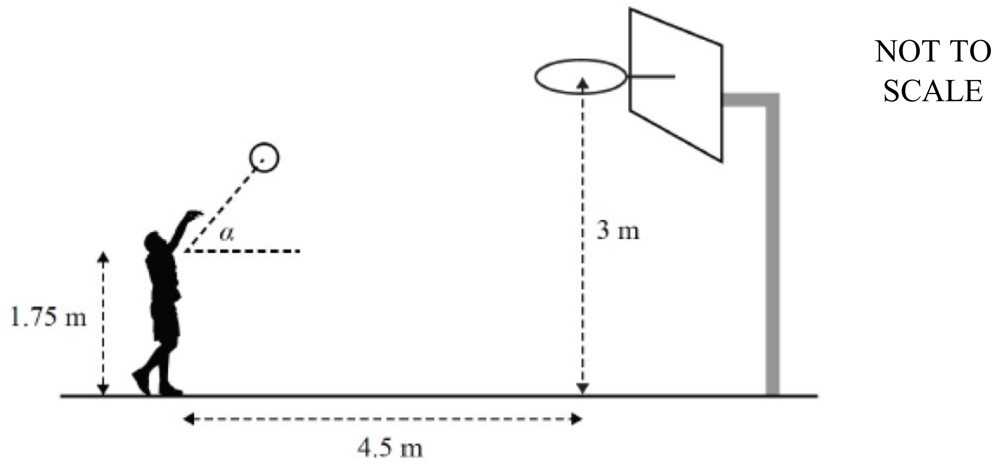
(i) Show that $\dfrac{1}{100}\left(\dfrac{1}{P} + \dfrac{1}{100 - P}\right) = \dfrac{1}{P(100 - P)}$. **1**

(ii) Find an expression for P in terms of t. **3**

Question 14 continues on next page

Question 14 (continued)

(c) A basketball player aims to throw a basketball through a ring, the centre of which is at a horizontal distance of 4.5 m from the point of release of the ball and 3 m above floor level. The ball is released at a height of 1.75 m above floor level, at an angle of projection α to the horizontal and at a speed of V ms^{-1}.
Air resistance is assumed to be negligible.



The position vector of the centre of the ball at any time t seconds, for $t \geq 0$, relative to the point of release is given by

$$\underset{\sim}{r} = \begin{pmatrix} Vt\cos\alpha \\ Vt\sin\alpha - 5t^2 \end{pmatrix}$$

Displacement components are measured in metres.
For the player's first shot at goal, $V = 7\text{ms}^{-1}$ and $\alpha = 45°$.

(i) Find the time, in seconds, taken for the ball to reach its maximum height. **3**
Give your answer in the form $\dfrac{a\sqrt{b}}{c}$, where a, b and c are positive integers.

(ii) Find the maximum height, in metres above floor level, reached by the centre of the ball. **2**

(iii) Find the distance between the centre of the ball and the centre of the ring when the ball reaches its maximum height. Give your answer in metres, correct to two decimal places. **2**

End of Paper

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