Student's Name: ______________________

Student Number: | | | | | | | | |

Teacher's Name: ______________________

ABBOTSLEIGH

2020

HIGHER SCHOOL CERTIFICATE
Assessment 4

Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen.
- **NESA approved** calculators may be used.
- **NESA approved** reference sheet is provided.
- All necessary working should be shown in every question.
- Make sure your HSC candidate Number is on the front cover of each booklet.
- Start a new booklet for Each Question.
- Answer the Multiple Choice questions on the answer sheet provided.
- If you do not attempt a whole question, you must still hand in the Writing Booklet, with the words **'NOT ATTEMPTED'** written clearly on the front cover.

Total marks - 70

- Attempt Sections I and II.

Section I Pages 3 - 7

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II Pages 8- 14

60 marks

- Attempt Questions 11 – 14.
- Allow about 1 hour and 45 minutes for this section.
- All questions are of equal value.

Outcomes to be assessed:

Year 11 Mathematics Extension 1 outcomes

A student:

ME11-1

uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2

manipulates algebraic expressions and graphical functions to solve problems

ME11-3

applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

ME11-4

applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change

ME11-5

uses concepts of permutations and combinations to solve problems involving counting or ordering

ME11-7

communicates making comprehensive use of mathematical language, notation, diagrams and graphs

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1

applies techniques involving proof or calculus to model and solve problems

ME12-2

applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3

applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME12-4

uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

ME12-7

evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

SECTION I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 - 10

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1 In how many different ways can 5 boys and 3 girls stand in a queue if the girls all stand next to each other?

A. 40 320

B. 720

C. 4 320

D. 3 136

2 The curve $y=\sqrt{16-(x+1)^2}$ is rotated about the x-axis to form a solid of revolution. Which of the following integrals would give the volume of the solid?

A. $\pi\int_{-4}^{4}\left(16-(x+1)^2\right)dx$

B. $\pi\int_{-5}^{3}\left(\sqrt{16-(x+1)^2}\right)dx$

C. $\pi\int_{-5}^{3}\left(16-(x+1)^2\right)^2 dx$

D. $\pi\int_{-5}^{3}\left(16-(x+1)^2\right)dx$

3 The polynomial $P(x)=7x^3+9x^2-5cx$ has a factor $(x+c)$, where c is a non-zero real number. What is the value of c?

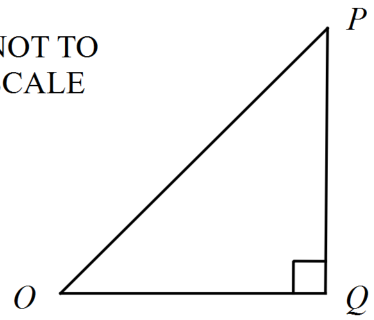
A. 2

B. -2

C. -1

D. 1

4 A right-angled isosceles triangle OPQ is such that $OQ = PQ = 4\,\text{units}$.



If $\underset{\sim}{p} = \overrightarrow{OP}$ and $\underset{\sim}{q} = \overrightarrow{OQ}$, what is the value of the scalar product $\underset{\sim}{p} \bullet \underset{\sim}{q}$

A. 0

B. $8\sqrt{2}$

C. 16

D. $16\sqrt{2}$

5 The polynomial $P(x) = 4x^3 + 8x^2 - 11x + 3$ has a double root at $x = \alpha$.

What is the value of α ?

A. -3

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 3

6 Both A and B are acute angles such that $\tan A = \frac{3}{5}$ and $\tan B = 4$.

What is the size of angle $(A - B)$?

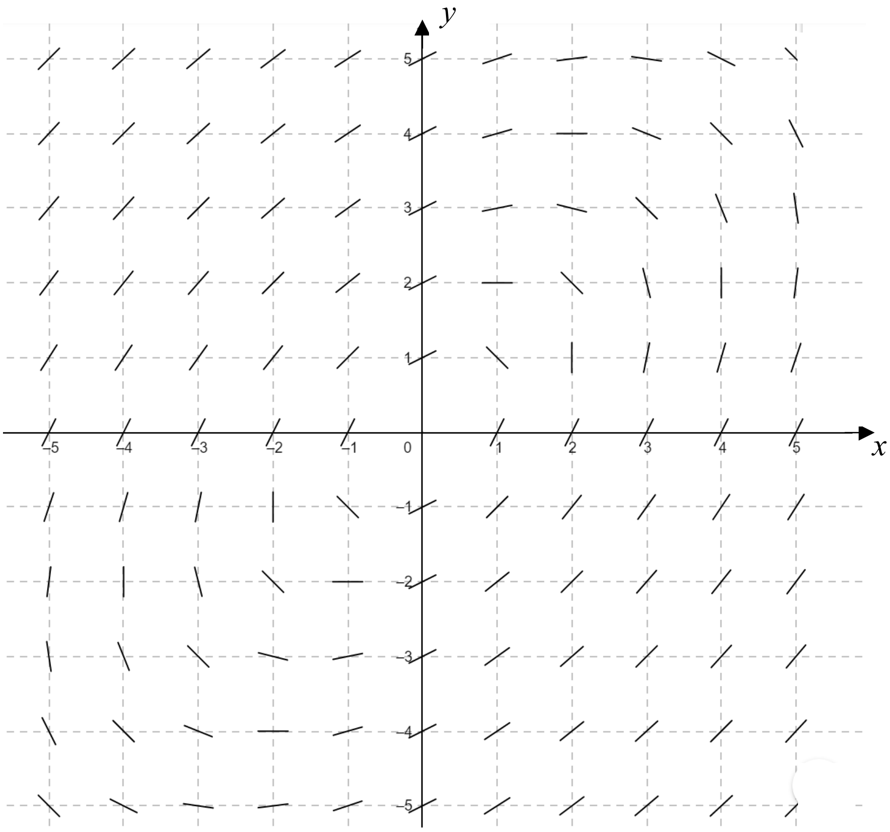
A. -135°

B. 45°

C. 135°

D. -45°

7 The diagram shows a direction (slope) field for a differential equation.



Which differential equation best represents the direction field shown above?

A. $\dfrac{dy}{dx} = \dfrac{y-2x}{2y+x}$

B. $\dfrac{dy}{dx} = \dfrac{y-2x}{2y-x}$

C. $\dfrac{dy}{dx} = \dfrac{2y-x}{y+2x}$

D. $\dfrac{dy}{dx} = \dfrac{x-2y}{2y+x}$

8 Consider the three forces $\underset{\sim}{F}_1 = -\sqrt{3}\underset{\sim}{j}$, $\underset{\sim}{F}_2 = \underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$ and $\underset{\sim}{F}_3 = -\frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}$.

What is the **magnitude** of the sum of the three forces equal to?

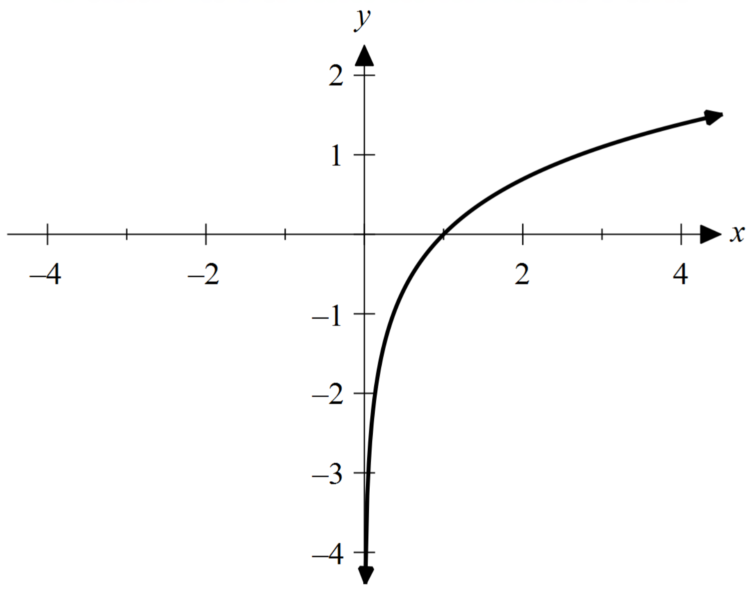
A. The magnitude of $\left(\underset{\sim}{F}_3 - \underset{\sim}{F}_1\right)$.

B. The magnitude of $\left(\underset{\sim}{F}_2 - \underset{\sim}{F}_1\right)$.

C. The magnitude of $\underset{\sim}{F}_2$.

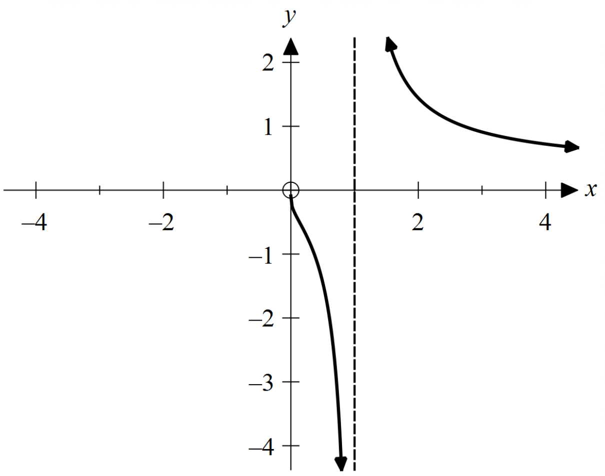
D. The magnitude of $\underset{\sim}{F}_3$.

9 Consider the function $y = f(x)$ graphed below.

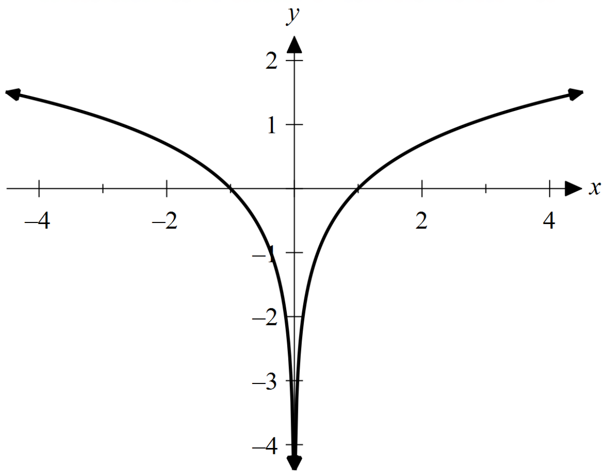


Which of the following graphs best represents the function $y = \dfrac{1}{f(|x|)}$?

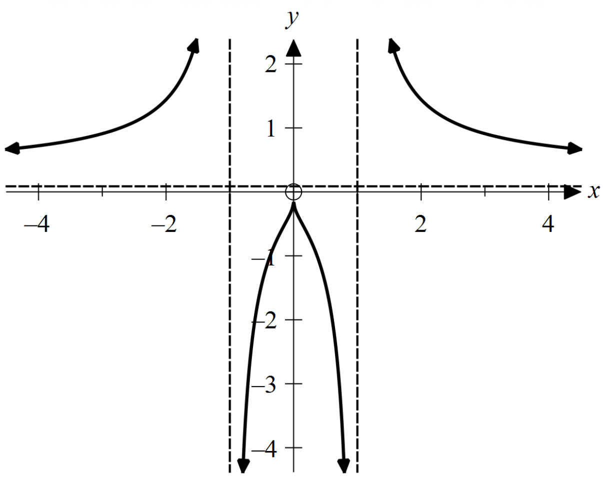
A.



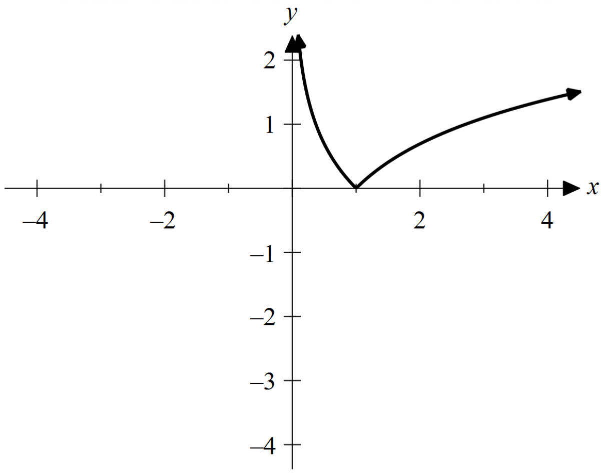
B.



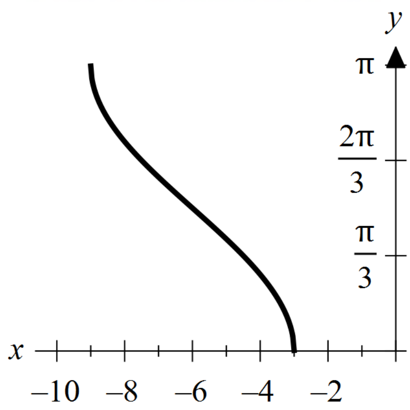
C.



D.



10 The graph of the function $y = \cos^{-1} x$ is transformed to give the graph shown in the diagram.



What is the equation of the transformed graph?

A. $y = \cos^{-1}\left(\dfrac{x+6}{3}\right)$

B. $y = \cos^{-1}\left(\dfrac{x-2}{3}\right)$

C. $y = \cos^{-1}(3x+18)$

D. $y = \cos^{-1}(3x-6)$

SECTION II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a **SEPARATE** writing booklet. Extra writing booklets are available.

In Questions 11-14 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Use the *t*-formulae to solve the equation $\sin 4\theta + \sqrt{3}\cos 4\theta = 0$ for $0 \le \theta \le \pi$. **4**

(b) A school has 2733 classes scheduled over 40 periods a fortnight.

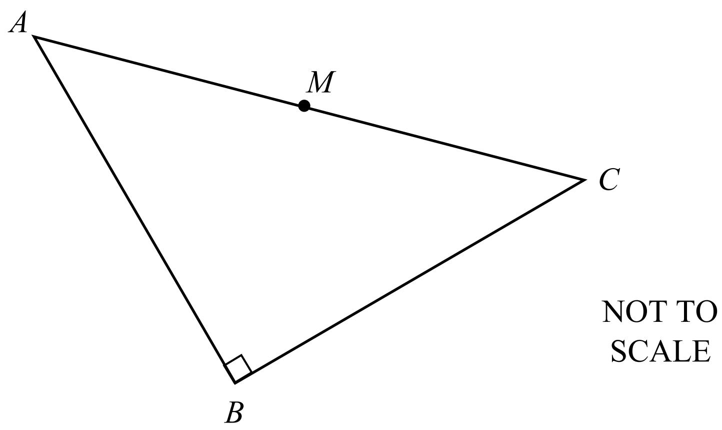
(i) Use the pigeonhole principle to find the minimum number of classrooms the school needs so that each class is able to have a separate classroom. **1**

(ii) Explain why the minimum number of classrooms may not be enough for every class to have a separate classroom in every period. **1**

(c) Differentiate $y = x\cos^{-1} 4x$. **2**

Question 11 continues on the next page

(d) Triangle ABC is right-angled at B. The point M is the midpoint of AC.



(i) Given that $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$, write expressions for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(ii) Use vectors to prove that M is equidistant from the three vertices A, B and C. **2**

(e) A function is defined as $f(x) = \sqrt{2 - \sin^2 x}$, in the domain $\left[0, \frac{\pi}{2}\right]$. **3**

Find the inverse function $f^{-1}(x)$, stating its domain and range.

End of Question 11

Question 12 (15 marks) **Start a new writing booklet**

(a) By expressing $\sin 6x \sin 2x$ as the sum of trigonometric functions, evaluate **3**

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\sin 6x \sin 2x \, dx.$$

(b) Find the particular solution for the differential equation $\frac{dy}{dx} = y^2 \cos x$, given **3**

that $y\left(\frac{\pi}{6}\right) = -\frac{1}{2}$.

(c) Amelia has a remote-controlled toy boat which can go at 6 m/s in still water. Amelia is on the west bank of a section of Lake Burley Griffin which is 80 metres wide. There is a current flowing north to south at 5 m/s and a wind blowing at 2 m/s from the south west. Both the current and the wind affect the boat's velocity. Amelia steers her boat perpendicular to the west bank, towards the east bank where her brother is standing.

(i) Show that the velocity of Amelia's boat, taking into account the wind and the current, is given by the vector $\left(6+\sqrt{2}\right)\underset{\sim}{i} + \left(\sqrt{2}-5\right)\underset{\sim}{j}$, where $\underset{\sim}{i}$ represents 1 m/s east and $\underset{\sim}{j}$ represents 1 m/s north. **2**

(ii) Find the speed of Amelia's boat as it travels across the lake, correct to three significant figures. **1**

(iii) Find how long Amelia's boat takes to reach the eastern bank. **3**

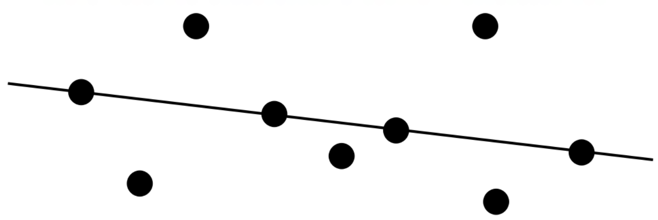
(d) A curve has gradient function $\frac{dy}{dx} = \frac{6e^{2x}}{1+e^{4x}}$. Using the substitution $u = e^{2x}$, or otherwise, find the equation of the curve with this gradient function which passes through $(0, \pi)$. **3**

End of Question 12

Question 13 (15 marks) **Start a new writing booklet**

(a) Nine points are drawn on a page as shown, with four of them lying on one line.

Other than these four points, no other set of three points is collinear.



How many different triangles can be drawn using the nine points as vertices? **2**

(b) Miriam takes a saucepan of soup from the stove and measures its temperature as $96\,°\mathrm{C}$.

The air temperature in Miriam's kitchen is $21°\mathrm{C}$. Miriam knows that the temperature T of her soup at time t minutes after she has removed it from the stove can be modelled using the differential equation

$$\frac{dT}{dt} = -k\left(T - R\right),$$

where R is the air temperature of the kitchen and k is a positive constant.

(i) Show that $T = 21 + 75e^{-kt}$ is a solution of the differential equation. **1**

(ii) Miriam finds the temperature of the soup 5 minutes after she removes it from the stove is $87\,°\mathrm{C}$. If she likes to start eating it when it reaches $70\,°\mathrm{C}$, find how much longer Miriam will need to wait until the soup is cool enough. **3**

Question 13 continues on the next page

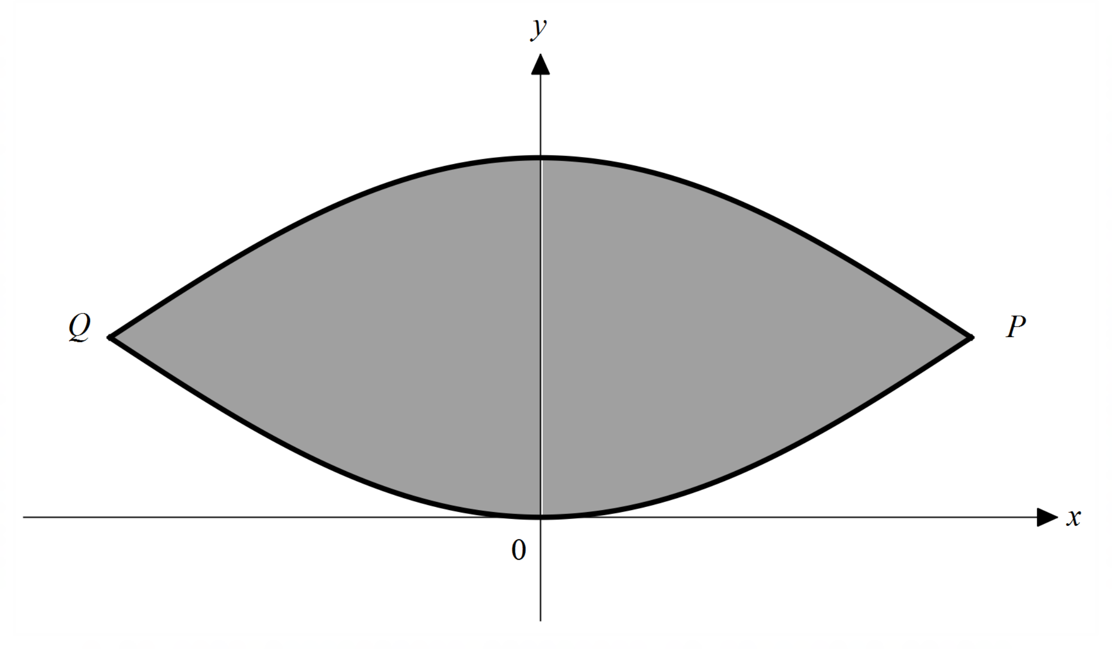
(c) A machine produces a sound which can be modelled using the equation $y = \cos t - \sqrt{3}\sin t$, where y gives a measure of the sound at time t seconds after the machine is switched on. The sound is audible to humans when $y \geq 1$.

(i) Express $\cos t - \sqrt{3}\sin t$ in the form $R\cos(t+\alpha)$, where $R > 0$ and $0 \leq \alpha < \dfrac{\pi}{2}$. **2**

(ii) Hence find the time intervals during which the noise is audible to humans for $0 < t \leq 4\pi$. **3**

(d) An artist uses mathematical functions to create shapes for decorative works made out of etched metal sheet.

She creates a design which uses the shaded area enclosed between $y = \cos^2\left(\dfrac{x}{3}\right)$ and $y = \sin^2\left(\dfrac{x}{3}\right)$ as shown in the diagram.



(i) Find the x-coordinates of the points P and Q where the functions $y = \cos^2\left(\dfrac{x}{3}\right)$ and $y = \sin^2\left(\dfrac{x}{3}\right)$ meet. **1**

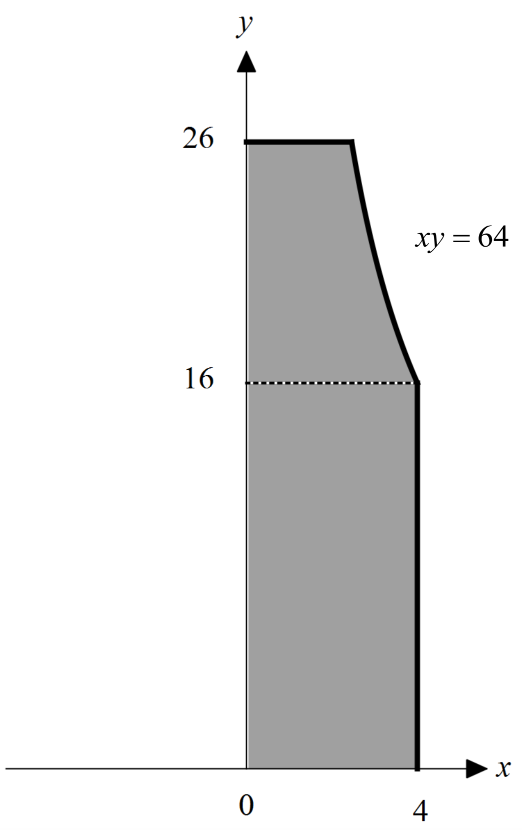
(ii) If the measurements are in metres, find the area of metal sheeting required for the design. **3**

End of Question 13

Question 14 (15 marks)

(a) Using the substitution $x = (u-4)^2 + 1$, or otherwise, evaluate $\displaystyle\int_2^5 \frac{1}{4+\sqrt{x-1}}\,dx$ **5**

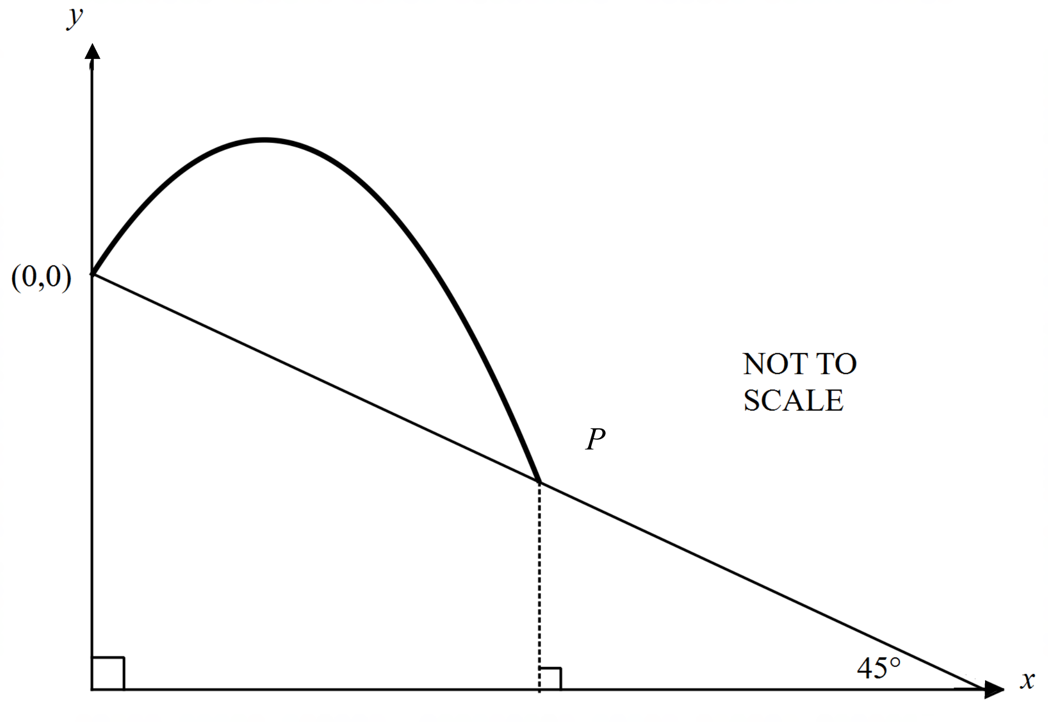
(b) The shaded area in the diagram is bounded by the curve $xy = 64$, the lines $x = 4$ and $y = 26$, and the x and y axes. A water flask is created by rotating the shaded area about the y-axis.



Find the capacity of the flask correct to the nearest millilitre given that all measurements in the diagram are in centimetres. **4**

Question 14 continues on the next page

(c) A stone is projected from the top of a slope, as shown in the diagram.



The stone is projected with speed 15 ms^{-1} at an angle of 60° to the horizontal, and the slope is at 45° to the horizontal.

Taking $g = 10\,\text{ms}^{-2}$ and the point of projection of the stone as the origin, the equations of motion for the stone are:

$$x = \frac{15t}{2} \qquad \text{and} \qquad y = \frac{15\sqrt{3}}{2}t - 5t^2 \qquad \text{OR}$$

$$\underset{\sim}{r}(t) = \left(\frac{15t}{2}\right)\underset{\sim}{i} + \left(\frac{15\sqrt{3}}{2}t - 5t^2\right)\underset{\sim}{j} \qquad \text{[in vector notation]}$$

You do **NOT** need to prove that these are the equations of motion.

(i) Show that the cartesian equation for the motion of the stone is $y = \sqrt{3}x - \dfrac{4x^2}{45}$. **1**

(ii) What is the greatest height above the point of projection reached by the stone? **2**

(iii) How far down the slope from the point of projection is the point P where the stone lands? **3**

End of Paper