

**2022**

**YEAR 12**  
YEARLY  
EXAMINATION

# Mathematics Advanced

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**General  
Instructions**

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In section II, show relevant mathematical reasoning and/or calculations

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**Total marks:  
100**

**Section I – 10 marks**

- Attempt Questions 1-10
- Allow about 15 minutes for this section

**Section II – 90 marks**

- Attempt all questions
- Allow about 2 hours and 45 minutes for this section

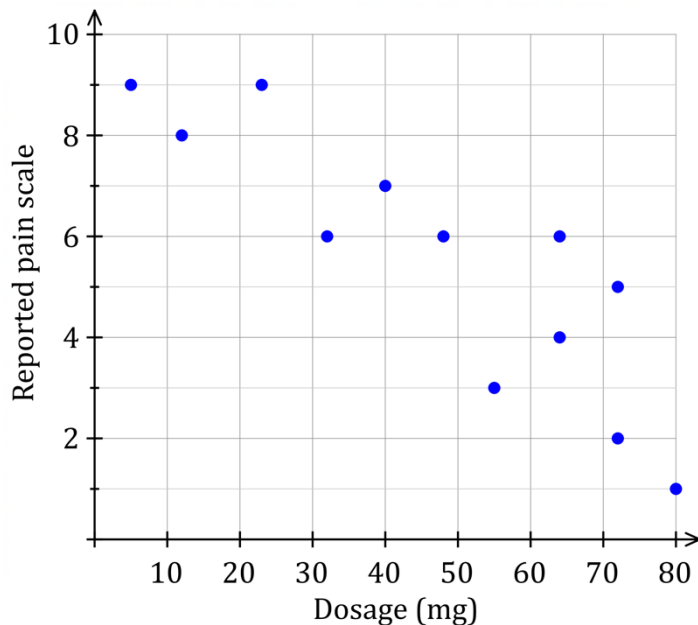
**Section I****10 marks****Attempt questions 1 - 10****Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for questions 1-10

1. Which interval gives the range of the function  $y = 3\cos 2x + 4$  ?

(A)  $[3, 7]$   
 (B)  $[4, 6]$   
 (C)  $[1, 7]$   
 (D)  $[4, 9]$

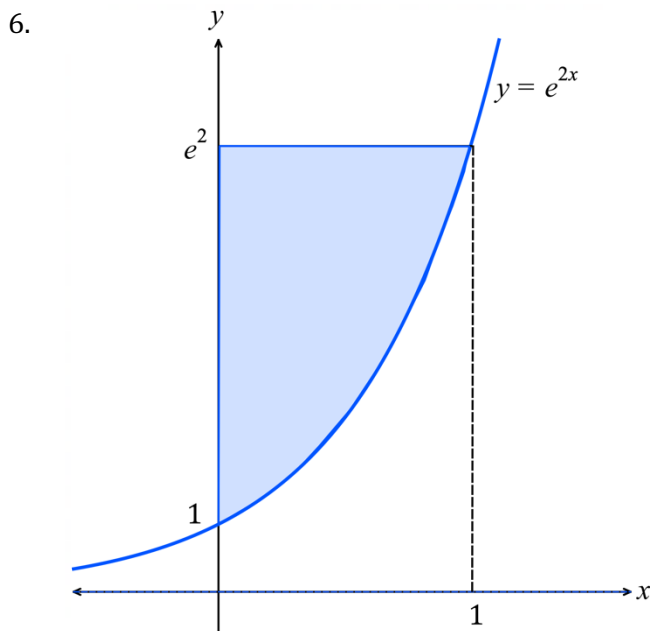
2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage ?

- (A) A moderate negative correlation  
 (B) A moderate positive correlation  
 (C) A weak positive correlation.  
 (D) No correlation.
3. What is the rule for the image of the graph of  $y = f(x)$  after a translation of 3 units in the positive direction of x-axis ?
- (A)  $y = f(3x)$   
 (B)  $y = f(x) + 3$   
 (C)  $y = f(x - 3)$   
 (D)  $y = f(x + 3)$

4. What is the value of  $\pi^{11}$  to two significant figures ?
- (A)  $2.94 \times 10^5$   
 (B)  $2.9 \times 10^5$   
 (C)  $2.94 \times 10^6$   
 (D)  $2.9 \times 10^6$
5. The first three terms of an arithmetic series are 2, 7 and 12.  
 What is the 15th term of this series ?
- (A) 72  
 (B) 77  
 (C) 555  
 (D) 595



To find the area of the shaded region above, which of the following is correct ?

- (A)  $\int_0^1 e^{2x} dx$   
 (B)  $\int_0^1 \frac{1}{2} \ln x dx$   
 (C)  $\int_1^{e^2} e^{2y} dy$   
 (D)  $\int_1^{e^2} \frac{1}{2} \ln y dy$

7. Lachlan did a class test in three topics. The class scores on each test were normally distributed. The table shows the topics and Lachlan's scores as well as the mean and standard deviation of the class scores on each test.

<i>Topic</i>	<i>Lachlan's score</i>	<i>Mean</i>	<i>Standard deviation</i>
Algebra	80	60	10
Measurement	90	81	6
Statistics	88	73	5

Relative to the rest of the class, which row of the table below shows Lachlan's strongest topic and his weakest topic?

	<i>Weakest topic</i>	<i>Strongest topic</i>
(A)	Measurement	Statistics
(B)	Algebra	Statistics
(C)	Algebra	Measurement
(D)	Measurement	Algebra

8. What is the equation of the tangent to the curve  $y = \cos x$  at the point  $\left(\frac{\pi}{2}, 0\right)$  ?

- (A)  $x - y - \frac{\pi}{2} = 0$                       (B)  $x + y - \frac{\pi}{2} = 0$   
 (C)  $y = 0$                                   (D)  $2x + y - \pi = 0$

9. What are the values of  $x$  for which  $|5 - 3x| \geq 11$  ?

- (A)  $x \leq 2$  and  $x \leq \frac{16}{3}$   
 (B)  $x \leq 2$  and  $x \geq \frac{16}{3}$   
 (C)  $x \leq -2$  and  $x \leq \frac{16}{3}$   
 (D)  $x \leq -2$  and  $x \geq \frac{16}{3}$

10. The probability density function for the continuous random variable  $X$  is:

$$f(x) = \begin{cases} x^2 - x + 2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Which of the following is closest to the expected value,  $E(X)$  ?

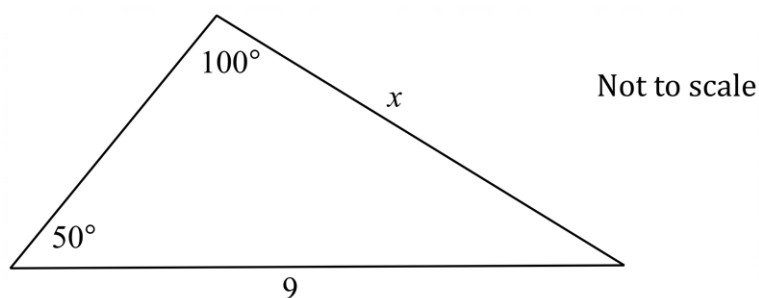
- (A)  $\frac{1}{12}$                                       (B)  $\frac{11}{12}$   
 (C) 1    (D) 2

**Section II****90 marks****Attempt all questions****Allow about 2 hours and 45 minutes for this section**

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

**Question 11 (2 marks)****Marks**Using the sine rule, find the value of  $x$  correct to one decimal place.**2****Question 12 (2 marks)**

Evaluate  $\sum_{r=1}^{10} 3^r$

**2**

**Question 13 (2 marks)**

## Marks

If  $f(x) = x^2 \sin(2x)$ , find  $f' \left( \frac{\pi}{6} \right)$ .

2

1. The first step in the process of creating a business plan is to conduct a thorough market research. This involves identifying the target market, understanding their needs and preferences, and analyzing the competitive landscape. Market research can be conducted through various methods, including surveys, interviews, and focus groups. The data collected from market research is used to develop a clear understanding of the market and to identify opportunities for the business.

2. The second step in the process of creating a business plan is to develop a clear and concise executive summary. This summary provides a high-level overview of the business, including its mission, vision, and key financial projections. The executive summary is typically the first section of the business plan and is used to attract the attention of potential investors and lenders.

3. The third step in the process of creating a business plan is to develop a detailed financial plan. This plan includes a variety of financial statements, such as the income statement, balance sheet, and cash flow statement. It also includes a variety of financial ratios and metrics that are used to evaluate the financial health of the business. The financial plan is a critical component of the business plan and is used to demonstrate the financial viability of the business.

4. The fourth step in the process of creating a business plan is to develop a marketing and sales plan. This plan outlines the strategies and tactics that will be used to promote the business and to generate sales. It includes a variety of marketing and sales activities, such as advertising, public relations, and direct sales. The marketing and sales plan is a key component of the business plan and is used to demonstrate the business's ability to generate revenue.

5. The fifth step in the process of creating a business plan is to develop a management and organizational plan. This plan outlines the structure of the business, including the roles and responsibilities of the management team and the organizational chart. It also includes a variety of information about the business's operations, such as its location, its equipment, and its personnel. The management and organizational plan is a key component of the business plan and is used to demonstrate the business's ability to manage its operations effectively.

6. The sixth step in the process of creating a business plan is to develop a risk management plan. This plan outlines the strategies and tactics that will be used to identify, assess, and mitigate the risks that the business faces. It includes a variety of risk management activities, such as risk assessment, risk mitigation, and risk monitoring. The risk management plan is a key component of the business plan and is used to demonstrate the business's ability to manage its risks effectively.

7. The seventh step in the process of creating a business plan is to develop a conclusion. This section provides a summary of the key findings of the business plan and a final statement of the business's viability. It is typically the last section of the business plan and is used to leave a lasting impression on the reader.

8. The eighth step in the process of creating a business plan is to develop a final review. This review involves a thorough review of the entire business plan, from the executive summary to the conclusion. It is used to ensure that the business plan is clear, concise, and accurate. The final review is a critical step in the process of creating a business plan and is used to ensure that the business plan is of the highest quality.

**Question 14** (1 mark)

Find  $\int (2 + 5x^2) dx$

**1**

[illegible]

**Question 15 (2 marks)**

Solve the equation  $(2\sin x + 1)(\sin x + 3) = 0$  over the domain  $0 \leq x \leq 2\pi$ .

2

[illegible]







**Question 20 (2 marks)****Marks**

The equation of least-squares line of best fit is given by  $y = mx + c$  where

**2**

$$m = r \frac{S_y}{S_x} \text{ and } c = \bar{y} - m\bar{x}$$

What is the gradient of the least-squares line of best fit given  $r = 0.617$ ,  $S_x = 2.185$  and  $S_y = 5.036$ ? Answer correct to two decimal places.

**Question 21 (4 marks)**

Differentiate with respect to  $x$  :

(a)  $e^{2x} \sin x$

**2**

(b)  $\frac{\cos x}{6 - x}$

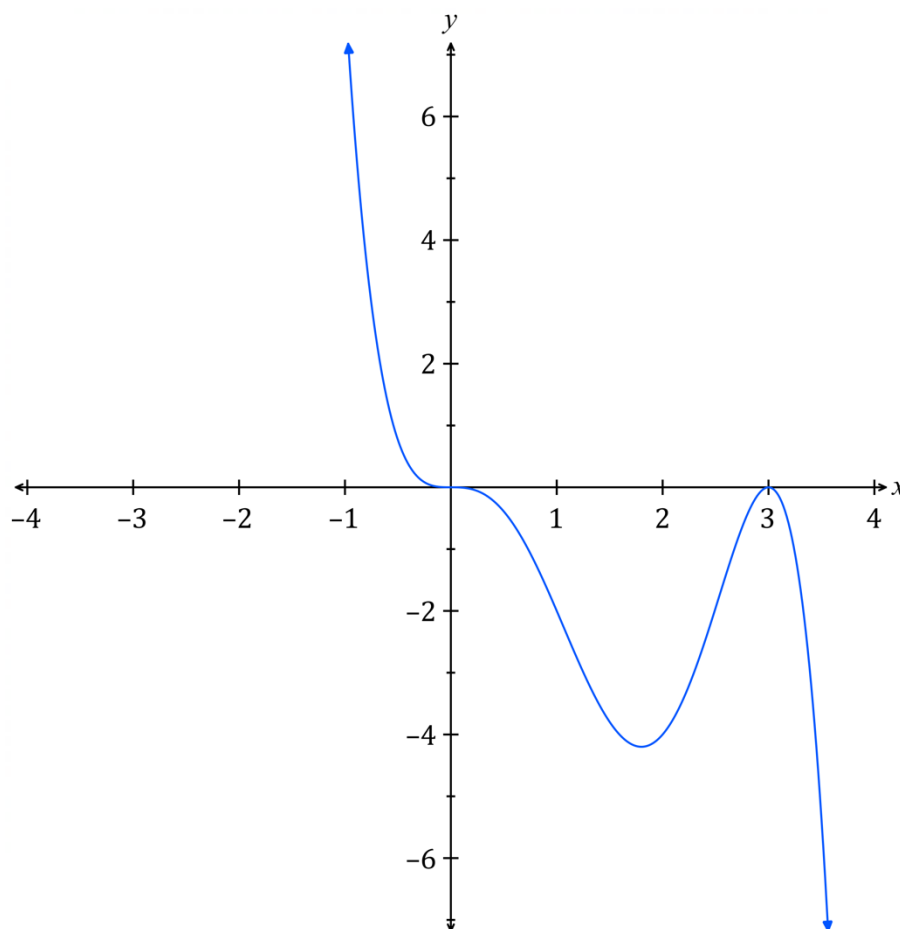
**2****Question 22 (2 marks)**

A class compared their shoe size to their height. The Pearson's correlation coefficient for these quantities was 0.8. What is the meaning of this correlation?

**2**

**Question 23** (4 marks)**Marks**

The graph of  $y = f(x)$  is shown below.



Draw sketches of the following functions on the above number plane. Clearly label each sketch. Indicate any intercepts with the axes.

(a)  $y = f(x + 2)$

**2**

(b)  $y = f(x) + 4$

**2****Question 24** (2 marks)

Find the period and amplitude for the graph  $y = 4\cos\left(3x - \frac{\pi}{2}\right)$ .

**2**


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**Question 25 (2 marks)**

### Marks

A student was asked to differentiate  $f(x) = \frac{1}{x}$  from first principles.

2

The student began the solution as shown below.

Complete the solution.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

1. The first step in the process of creating a new product is to identify a market need. This involves conducting market research to understand the preferences and behaviors of potential customers.

2. Once a market need is identified, the next step is to develop a concept. This involves brainstorming ideas and creating a rough sketch of the product.

3. The third step is to create a prototype. This involves building a small-scale model of the product to test its functionality and appearance.

4. After the prototype is created, the next step is to conduct a feasibility study. This involves evaluating the technical, financial, and market viability of the product.

5. Once the feasibility study is complete, the next step is to develop a business plan. This involves outlining the marketing, sales, and financial strategies for the product.

6. The final step in the process is to launch the product. This involves manufacturing the product and distributing it to the market.

**Question 26 (2 marks)**

Find the antiderivative of  $\frac{1}{3x-4}$  with respect to  $x$ .

2

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Question 27 (4 marks)

Marks

Sketch the graph of the curve  $y = x^3 - 2x^2 + 1$ , labelling the stationary points and any points of inflection. Do NOT determine the  $x$  intercepts of the curve.

4

Blank area for sketching the graph of the curve  $y = x^3 - 2x^2 + 1$ .

**Question 28** (4 marks)**Marks**

The probability density function for the continuous random variable  $X$  is given by:

$$f(x) = \begin{cases} \frac{x}{16} & 0 \leq x \leq 4 \\ 0.25e^{-0.5(x-4)} & x > 4 \end{cases}$$

Find correct to three decimal places.

(a)  $P(0 \leq X \leq 3)$

**2**


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(b)  $P(3 \leq X \leq 5)$

**2**


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**Question 29** (2 marks)

Evaluate  $\int_2^7 \frac{1}{\sqrt{x}-1} dx$

**2**


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**Question 30 (5 marks)****Marks**

Ryan opens a bank account. At the start of each month he deposits  $\$M$  into the bank account. At the end of each month, after interest is added into the bank account, the bank withdraws  $\$1950$  from the bank account as a loan repayment. Let  $A_n$  be the amount in the bank account after the  $n$ th withdrawal. The bank account pays interest of 3.6% per annum compounded monthly.

- (a) Show that after the second withdrawal the amount in the bank account is given by:

**2**

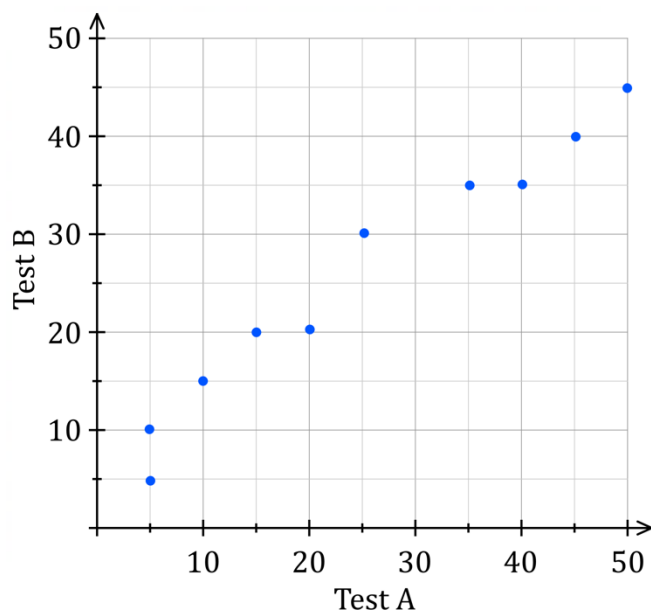
$$A_2 = M[(1.003)^2 + (1.003)] - 1950(1.003 + 1)$$

- (b) What is the amount Ryan needs to deposit into the bank account each month, or  $\$M$ , so that  $\$70\,000$  remains after the last withdrawal of the fifth year? Answer correct to nearest dollar.

**3**

**Question 31** (6 marks)**Marks**

The scatterplot shows results of Test A and Test B for ten students..



(a) Draw a line of best fit on the scatterplot. **1**

(b) What is the gradient of the line of best fit? **1**

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(c) Find Pearson's correlation coefficient. Answer correct to two decimal places. **1**

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(d) Use a calculator to determine the equation of the least-squares line of best fit. Answer correct to two decimal places. **2**

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(e) Noah was absent for the Test B. Use algebra to predict his Test B result if he scored 40 on Test A. **1**

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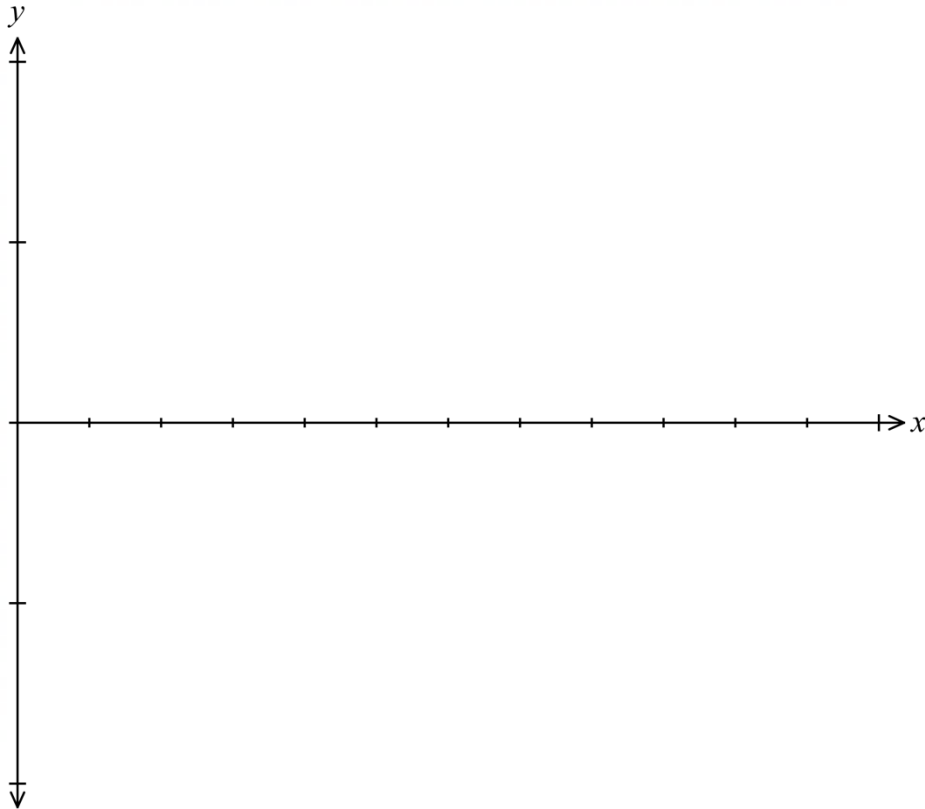
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**Question 32** (6 marks)**Marks**

- (a) Sketch the graphs of  $y = \sin x$  and  $y = \sqrt{3}\cos x$  over the domain  $0 \leq x \leq 2\pi$ .

**2**

- (b) The graphs intersect at points  $A$  and  $B$ . What are the coordinates of  $A$  and  $B$ ?

**2**


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- (c) Find the area enclosed by  $y = \sin x$  and  $y = \sqrt{3}\cos x$  between  $A$  and  $B$ .

**2**


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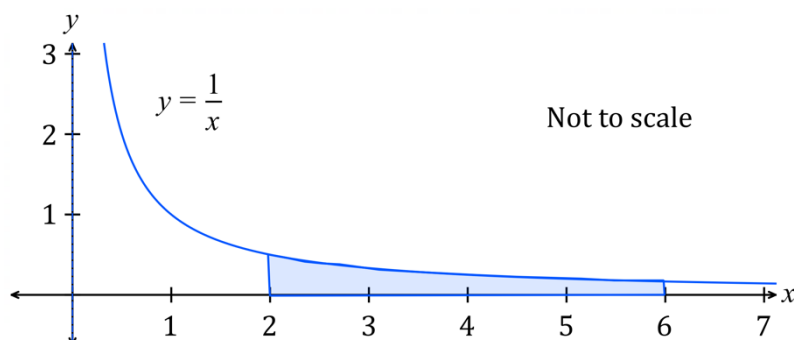
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**Question 33** (4 marks)**Marks**

Consider the curve  $y = \frac{1}{x}$



- (a) Find the area bounded by the curve,  $x$ -axis and the lines  $x = 2$  and  $x = 6$  by using the trapezoidal rule with five function values. Answer correct to three decimal places.

**2**

- (b) Calculate the same area by evaluating  $y = \int_2^6 \frac{1}{x} dx$ .

**1**

- (c) Explain the difference between your answers in parts (a) and (b).

**1**

### Question 34 (7 marks)

## Marks

A closed cylinder has a height  $h$  cm and a radius  $r$  cm. The volume of the cylinder is  $450 \text{ cm}^3$ .

- 1**

[illegible]

- 2

$$SA = 2\pi r^2 + \frac{900}{r}$$

[illegible]

- 4

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**Question 35 (4 marks)****Marks**

A scientist grows the number of bacteria according to the equation

$$N = N_0 e^{0.15t}$$

where  $t$  is measured in days and  $N_0$  is a constant.

- (a) When  $t = 3$  the number of bacteria was estimated at  $1.5 \times 10^8$ .  
Evaluate  $N_0$ . Answer correct to two significant figures.

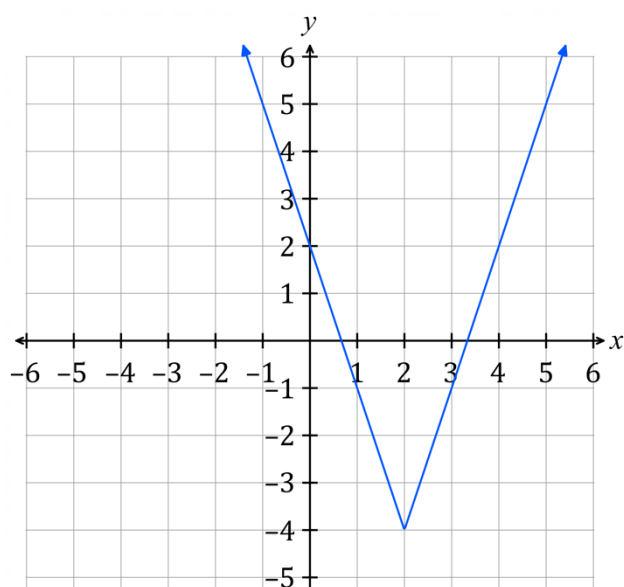
**1**

- (b) The number of bacteria doubles every  $x$  days. Find  $x$ . Answer correct to one decimal place.

**3**

**Question 36** (3 marks)**Marks**

The function  $y = |x|$  is transformed and the equation of the new function is of the form  $y = kf(x + b) + c$ , where  $k$ ,  $b$  and  $c$  are constants. The graph of the new function is shown below.

**3**

What are the values of  $k$ ,  $b$  and  $c$ ?

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**Question 37** (2 marks)

The velocity,  $v$  metres per second, of a body  $t$  seconds after starting from rest, is given by  $v = 3t - t^2$ . How far has the body travelled when it next comes to rest?

**2**


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**End of paper**



NSW Education Standards Authority

**2020** HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Advanced

## Mathematics Extension 1

## Mathematics Extension 2

### REFERENCE SHEET

#### Measurement

##### Length

$$l = \frac{\theta}{360} \times 2\pi r$$

##### Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

##### Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

##### Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

#### Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For  $ax^3 + bx^2 + cx + d = 0$ :

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

##### Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

#### Financial Mathematics

$$A = P(1 + r)^n$$

##### Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

#### Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

## Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

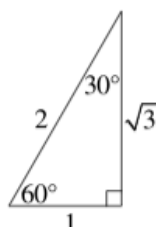
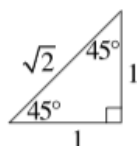
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



## Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

## Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

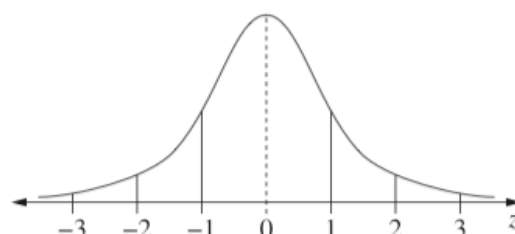
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

## Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score  
less than  $Q_1 - 1.5 \times IQR$   
or  
more than  $Q_3 + 1.5 \times IQR$

## Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

## Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

## Continuous random variables

$$P(X \leq x) = \int_a^x f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

## Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

**Differential Calculus****Function****Derivative**

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

**Integral Calculus**

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where  $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \}$$

where  $a = x_0$  and  $b = x_n$

**Combinatorics**

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

**Vectors**

$$|\underline{u}| = |x_1\underline{i} + y_1\underline{j}| = \sqrt{x_1^2 + y_1^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

**Complex Numbers**

$$\begin{aligned} z &= a + ib = r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

**Mechanics**

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$