A C E
EXAM
PAPER

Student name: _____

2022

YEAR 12
YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt all questions
- Allow about 2 hours and 45 minutes for this section

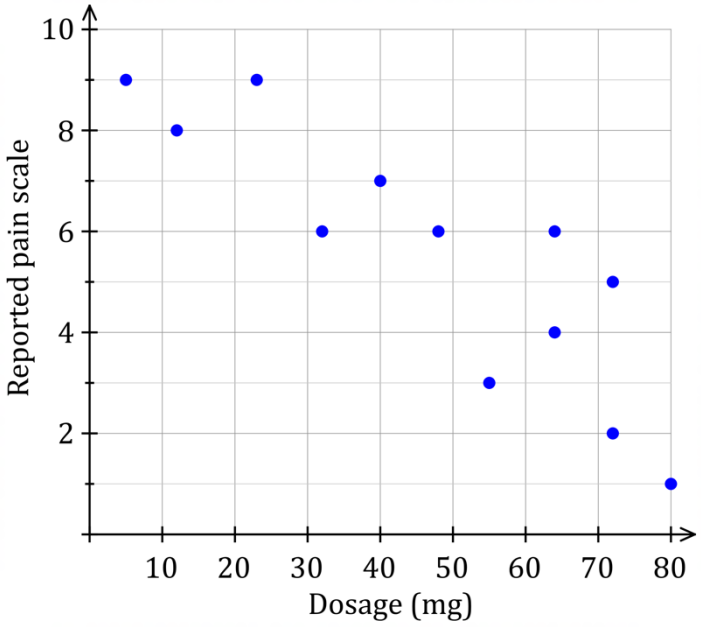
Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. Which interval gives the range of the function $y = 3\cos 2x + 4$?
   (A) [3, 7]
   (B) [4, 6]
   (C) [1, 7]
   (D) [4, 9]

2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



   How could you describe the correlation between the pain and the dosage ?
   (A) A moderate negative correlation
   (B) A moderate positive correlation
   (C) A weak positive correlation.
   (D) No correlation.

3. What is the rule for the image of the graph of $y = f(x)$ after a translation of 3 units in the positive direction of x-axis ?
   (A) $y = f(3x)$
   (B) $y = f(x) + 3$
   (C) $y = f(x - 3)$
   (D) $y = f(x + 3)$

4. What is the value of π^{11} to two significant figures ?

(A) 2.94×10^5

(B) 2.9×10^5

(C) 2.94×10^6

(D) 2.9×10^6

5. The first three terms of an arithmetic series are 2, 7 and 12.
What is the 15th term of this series ?

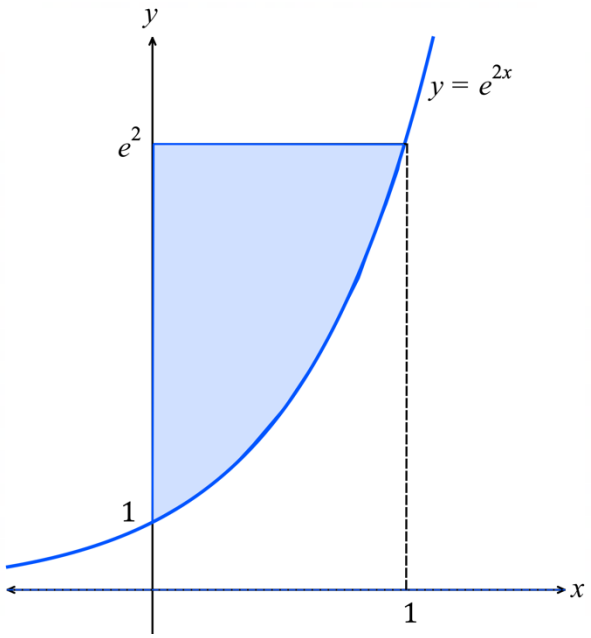
(A) 72

(B) 77

(C) 555

(D) 595

6.



To find the area of the shaded region above, which of the following is correct ?

(A) $\int_0^1 e^{2x}\,dx$

(B) $\int_0^1 \frac{1}{2}\ln x dx$

(C) $\int_1^{e^2} e^{2y}\,dy$

(D) $\int_1^{e^2} \frac{1}{2}\ln y dy$

7. Lachlan did a class test in three topics. The class scores on each test were normally distributed. The table shows the topics and Lachlan's scores as well as the mean and standard deviation of the class scores on each test.

Topic	*Lachlan's score*	*Mean*	*Standard deviation*
Algebra	80	60	10
Measurement	90	81	6
Statistics	88	73	5

Relative to the rest of the class, which row of the table below shows Lachlan's strongest topic and his weakest topic?

	Weakest topic	*Strongest topic*
(A)	Measurement	Statistics
(B)	Algebra	Statistics
(C)	Algebra	Measurement
(D)	Measurement	Algebra

8. What is the equation of the tangent to the curve $y = \cos x$ at the point $\left(\frac{\pi}{2}, 0\right)$?

(A) $x - y - \frac{\pi}{2} = 0$ (B) $x + y - \frac{\pi}{2} = 0$

(C) $y = 0$ (D) $2x + y - \pi = 0$

9. What are the values of x for which $|5 - 3x| \geq 11$?

(A) $x \leq 2$ and $x \leq \frac{16}{3}$

(B) $x \leq 2$ and $x \geq \frac{16}{3}$

(C) $x \leq -2$ and $x \leq \frac{16}{3}$

(D) $x \leq -2$ and $x \geq \frac{16}{3}$

10. The probability density function for the continuous random variable X is:

$$f(x) = \begin{cases} x^2 - x + 2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Which of the following is closest to the expected value, E(X) ?

(A) $\frac{1}{12}$ (B) $\frac{11}{12}$

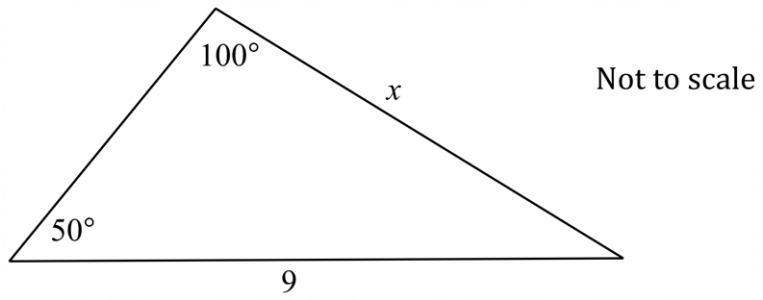
(C) 1 (D) 2

Section II

90 marks
Attempt all questions
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks) **Marks**



Using the sine rule, find the value of x correct to one decimal place. **2**

Question 12 (2 marks)

Evaluate $\sum_{r=1}^{10} 3^r$ **2**

Question 13 (2 marks) **Marks**

If $f(x) = x^2\sin(2x)$, find $f'\left(\frac{\pi}{6}\right)$. **2**

Question 14 (1 mark)

Find $\int (2 + 5x^2)\, dx$ **1**

Question 15 (2 marks)

Solve the equation $(2\sin x + 1)(\sin x + 3) = 0$ over the domain $0 \leq x \leq 2\pi$. **2**

Question 16 (5 marks) **Marks**

The nurses in a hospital were surveyed on the number of hours of sleep per week. The results were normally distributed. The survey indicated that 99.7% of students had between 39 and 57 hours of sleep per week.

(a) Determine the mean number of hours of sleep per week. **2**

(b) What was the standard deviation ? **1**

(c) How many hours of sleep per week would nurses have who recorded a z-score of –2.5 ? **2**

Question 17 (3 marks)

The circle $x^2 - 1x + y^2 + 8y + 8 = 0$ is reflected in the x-axis. **3**
Sketch the reflected circle, showing the coordinates of the centre and the radius.

Question 18 (5 marks) **Marks**

Hannah is training for a marathon. Her training includes a run every Sunday, starting with a run of 5 km on the first Sunday. Each Sunday she increases the length of her run from the previous Sunday by 2 km.

(a) Show that on the 4th Sunday of training she runs 11 km. **1**

(b) Find an expression for the length of her training run on the nth Sunday. **1**

(c) Find the total distance she runs on Sundays, in n weeks of training. **2**

(d) On the nth Sunday Hannah runs 43 km. Find the value of n. **1**

Question 19 (3 marks)

Find $f'(4)$ given $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$ **3**

Question 20 (2 marks) **Marks**

The equation of least-squares line of best fit is given by $y = mx + c$ where **2**

$m = r\frac{S_y}{S_x}$ and $c = \bar{y} - m\bar{x}$

What is the gradient of the least-squares line of best fit given $r = 0.617$, $S_x = 2.185$ and $S_y = 5.036$? Answer correct to two decimal places.

Question 21 (4 marks)

Differentiate with respect to x :

(a) $e^{2x}\sin x$ **2**

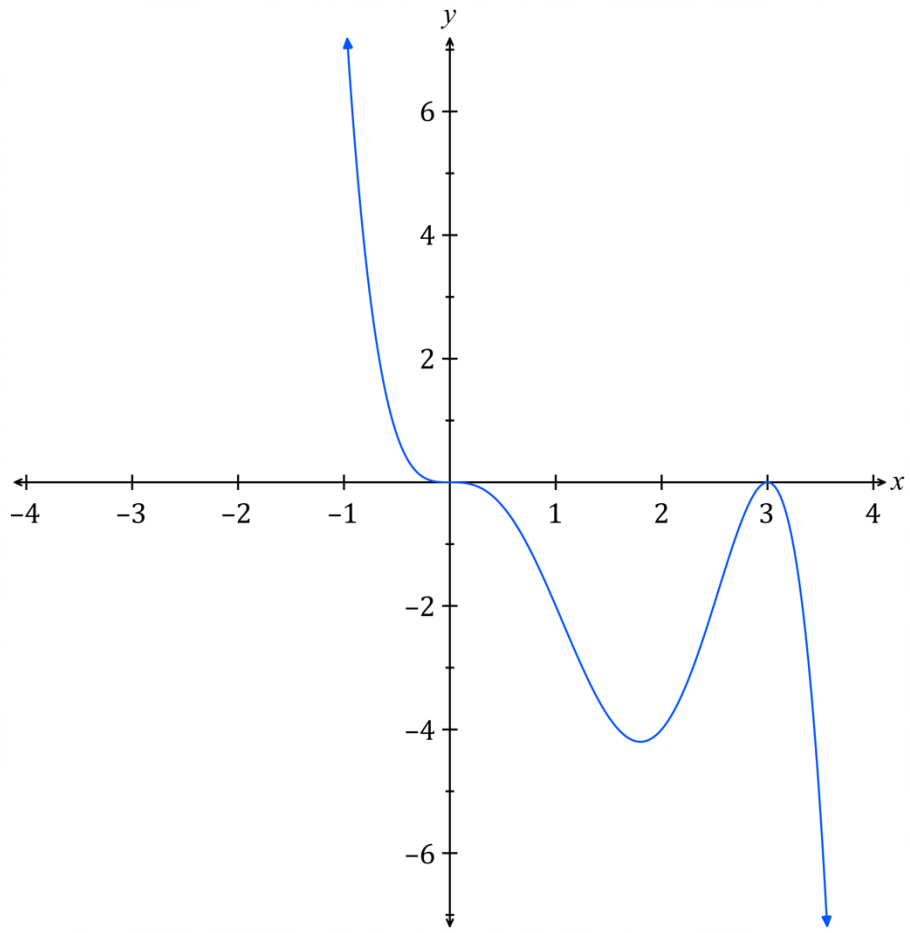
(b) $\dfrac{\cos x}{6 - x}$ **2**

Question 22 (2 marks)

A class compared their shoe size to their height. The Pearson's correlation coefficient for these quantities was 0.8. What is the meaning of this correlation ? **2**

Question 23 (4 marks) **Marks**

The graph of $y = f(x)$ is shown below.



Draw sketches of the following functions on the above number plane.
Clearly label each sketch. Indicate any intercepts with the axes.

(a) $y = f(x + 2)$ **2**

(b) $y = f(x) + 4$ **2**

Question 24 (2 marks)

Find the period and amplitude for the graph $y = 4\cos\left(3x - \frac{\pi}{2}\right)$. **2**

Question 25 (2 marks)

Marks

A student was asked to differentiate $f(x) = \frac{1}{x}$ from first principles. **2**

The student began the solution as shown below.

Complete the solution.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 26 (2 marks)

Find the antiderivative of $\frac{1}{3x-4}$ with respect to x. **2**

Question 27 (4 marks)

Marks

Sketch the graph of the curve $y = x^3 - 2x^2 + 1$, labelling the stationary points and any points of inflection. Do NOT determine the x intercepts of the curve.

4

Question 28 (4 marks) **Marks**

The probability density function for the continuous random variable X is given by:

$$f(x) = \begin{cases} \frac{x}{16} & 0 \leq x \leq 4 \\ 0.25e^{-0.5(x-4)} & x > 4 \end{cases}$$

Find correct to three decimal places.

(a) $P(0 \leq X \leq 3)$ **2**

(b) $P(3 \leq X \leq 5)$ **2**

Question 29 (2 marks)

Evaluate $\int_2^7 \frac{1}{\sqrt{x-1}} dx$ **2**

Question 30 (5 marks) **Marks**

Ryan opens a bank account. At the start of each month he deposits $\$M$ into the bank account. At the end of each month, after interest is added into the bank account, the bank withdraws \$1950 from the bank account as a loan repayment. Let A_n be the amount in the bank account after the nth withdrawal. The bank account pays interest of 3.6% per annum compounded monthly.

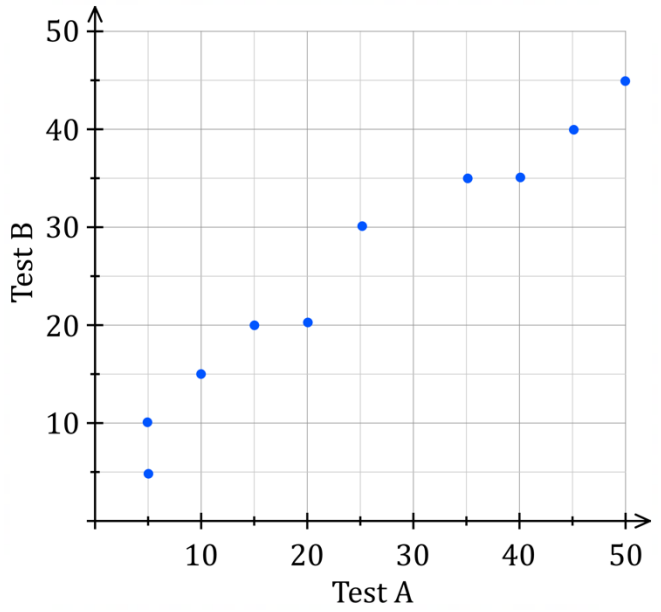
(a) Show that after the second withdrawal the amount in the bank account is given by: **2**

$$A_2 = M[(1.003)^2 + (1.003)] - 1950(1.003 + 1)$$

(b) What is the amount Ryan needs to deposit into the bank account each month, or $\$M$, so that \$70 000 remains after the last withdrawal of the fifth year ? Answer correct to nearest dollar. **3**

Question 31 (6 marks) **Marks**

The scatterplot shows results of Test A and Test B for ten students..



(a) Draw a line of best fit on the scatterplot. **1**

(b) What is the gradient of the line of best fit ? **1**

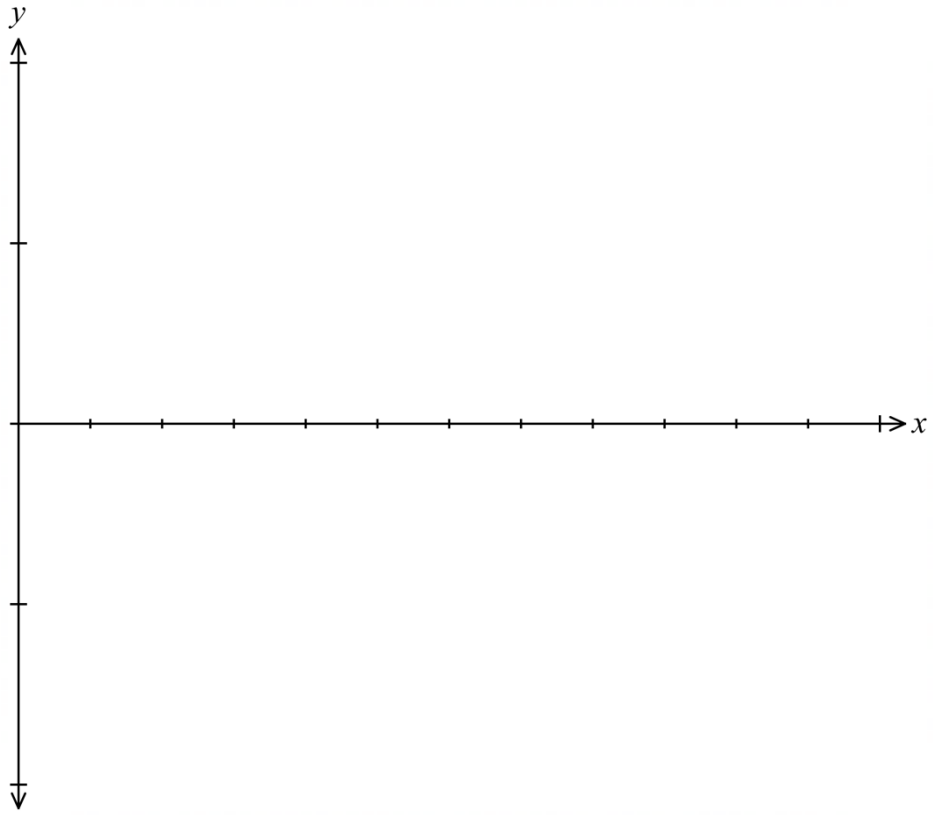
(c) Find Pearson's correlation coefficient. Answer correct to two decimal places. **1**

(d) Use a calculator to determine the equation of the least-squares line of best fit. Answer correct to two decimal places. **2**

(e) Noah was absent for the Test B. Use algebra to predict his Test B result if he scored 40 on Test A. **1**

Question 32 (6 marks) **Marks**

(a) Sketch the graphs of $y = \sin x$ and $y = \sqrt{3}\cos x$ over the domain $0 \leq x \leq 2\pi$. **2**

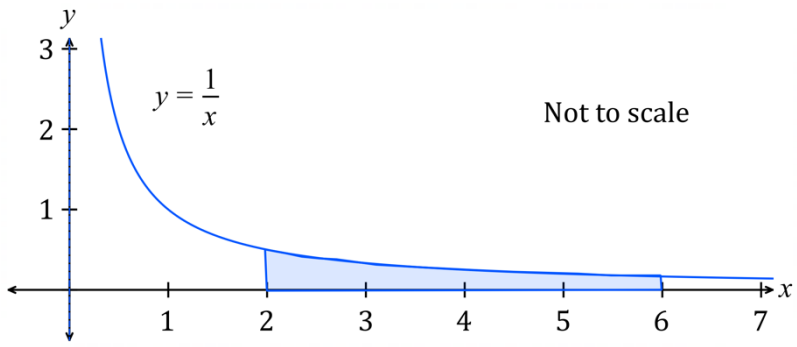


(b) The graphs intersect at points A and B. What are the coordinates of A and B? **2**

(c) Find the area enclosed by $y = \sin x$ and $y = \sqrt{3}\cos x$ between A and B. **2**

Question 33 (4 marks) **Marks**

Consider the curve $y = \frac{1}{x}$



(a) Find the area bounded by the curve, x-axis and the lines $x = 2$ and $x = 6$ by using the trapezoidal rule with five function values. Answer correct to three decimal places. **2**

(b) Calculate the same area by evaluating $y = \int_2^6 \frac{1}{x}\,dx$. **1**

(c) Explain the difference between your answers in parts (a) and (b). **1**

Question 34 (7 marks) **Marks**

A closed cylinder has a height h cm and a radius r cm. The volume of the cylinder is 450 cm^3.

(a) Find an expression for h in terms of r. **1**

(b) Show that the surface area SA cm^2 of the cylinder is given by: **2**

$$SA = 2\pi r^2 + \frac{900}{r}$$

(c) If the surface area of cylinder is to be minimised, find the radius of the cylinder. Answer correct to one decimal place. **4**

Question 35 (4 marks) **Marks**

A scientist grows the number of bacteria according to the equation

$N = N_0 e^{0.15t}$

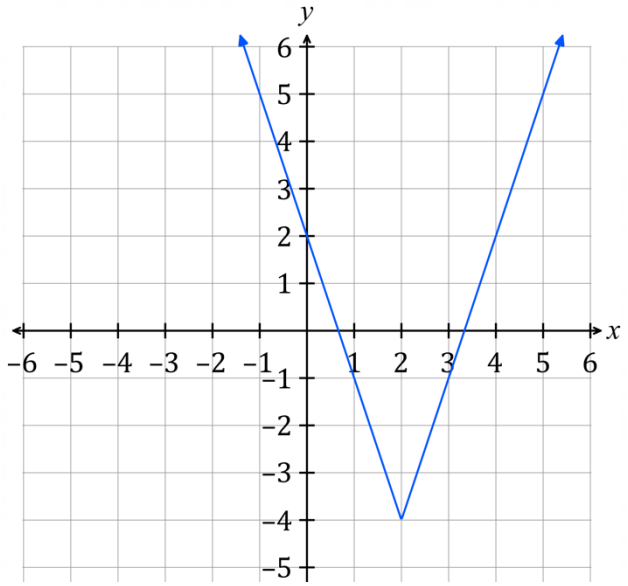
where t is measured in days and N_0 is a constant.

(a) When $t = 3$ the number of bacteria was estimated at 1.5×10^8. Evaluate N_0. Answer correct to two significant figures. **1**

(b) The number of bacteria doubles every x days. Find x. Answer correct to one decimal place. **3**

Question 36 (3 marks) **Marks**

The function $y = |x|$ is transformed and the equation of the new function is of the form $y = kf(x + b) + c$, where k, b and c are constants. The graph of the new function is shown below. **3**



What are the values of k, b and c ?

Question 37 (2 marks)

The velocity, v metres per second, of a body t seconds after starting from rest, is given by $v = 3t - t^2$. How far has the body travelled when it next comes to rest ? **2**

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $$\alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

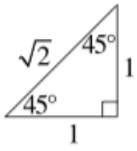
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

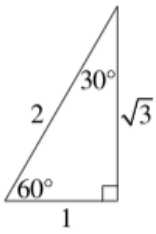
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

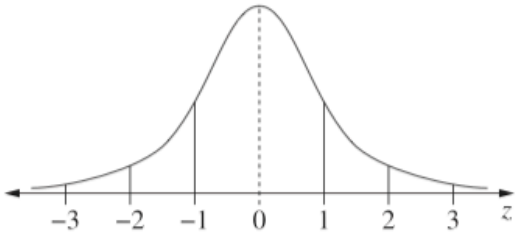
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$