NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

2024 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–39)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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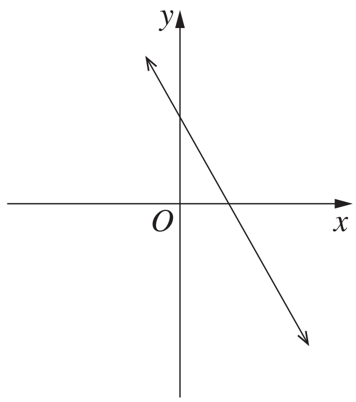
1 Consider the function shown.



NOT TO SCALE

Which of the following could be the equation of this function?

A. $y = 2x + 3$

B. $y = 2x - 3$

C. $y = -2x + 3$

D. $y = -2x - 3$

2 In a group of 60 students, 38 play basketball, 35 play hockey and 5 do not play either basketball or hockey.

How many students play both basketball and hockey?

A. 55

B. 18

C. 13

D. 8

3 Pia's marks in Year 10 assessments are shown. The scores for each subject were normally distributed.

	Pia's mark	*Year 10 mean*	*Year 10 standard deviation*
English	78	66	6
Mathematics	80	71	10
Science	77	70	15
History	85	72	9

In which subject did Pia perform best in comparison with the rest of Year 10?

A. English

B. Mathematics

C. Science

D. History

4 The parabola $y = (x - 3)^2 - 2$ is reflected about the y-axis. This is then reflected about the x-axis.

What is the equation of the resulting parabola?

A. $y = (x + 3)^2 + 2$

B. $y = (x - 3)^2 + 2$

C. $y = -(x + 3)^2 + 2$

D. $y = -(x - 3)^2 + 2$

5 What is $\int (6x+1)^3\,dx$?

A. $\frac{1}{24}(6x+1)^4 + C$

B. $\frac{1}{4}(6x+1)^4 + C$

C. $\frac{2}{3}(6x+1)^4 + C$

D. $\frac{3}{2}(6x+1)^4 + C$

6 What is the domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 1}}$?

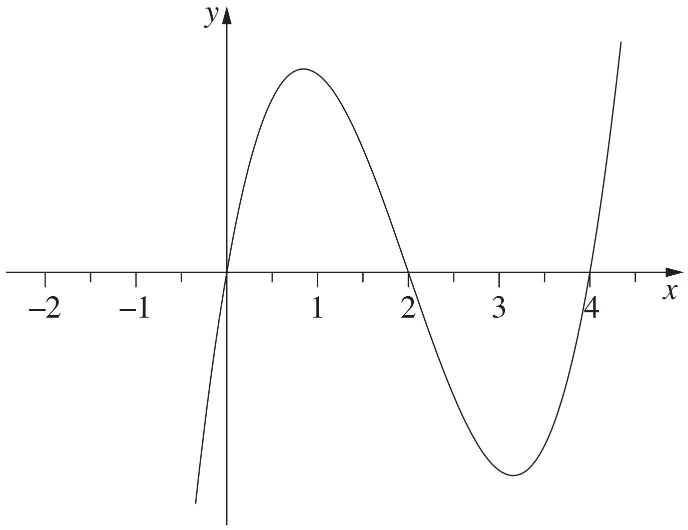
A. $[-1, 1]$

B. $(-\infty, -1] \cup [1, \infty)$

C. $(-1, 1)$

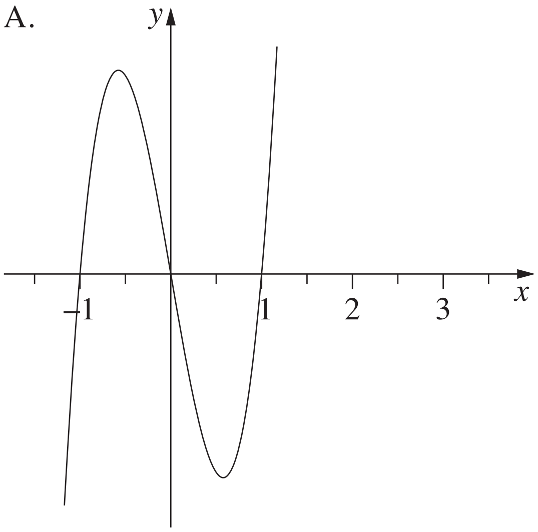
D. $(-\infty, -1) \cup (1, \infty)$

7 The diagram shows the graph $y = f(x)$.

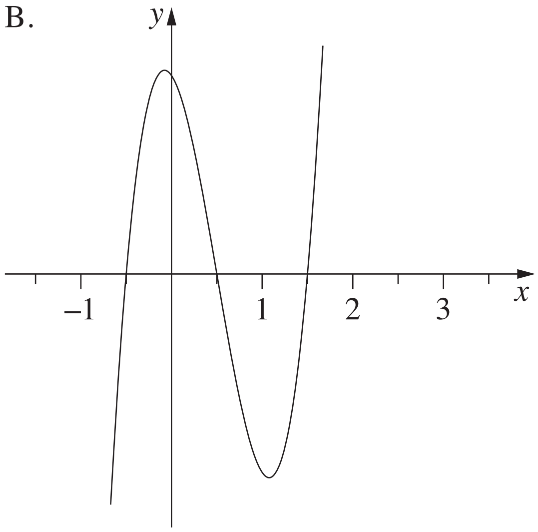


Which of the following best represents the graph $y = f(2x - 1)$?

A.



B.



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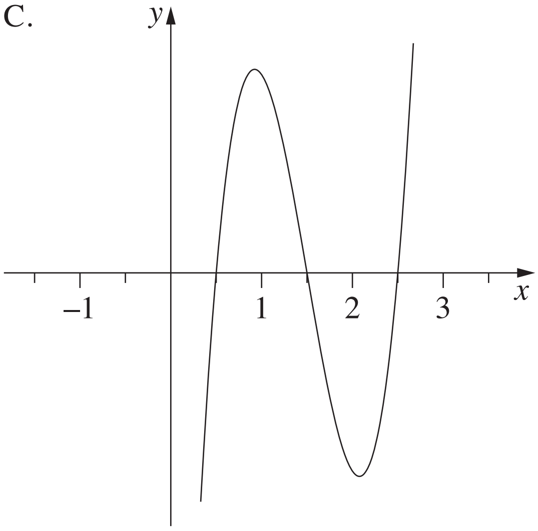
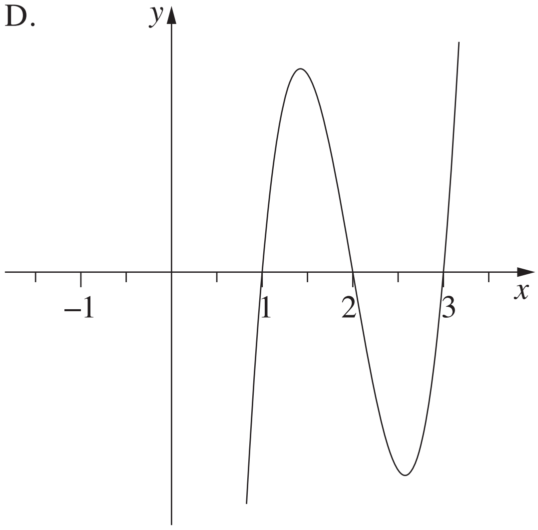


D.

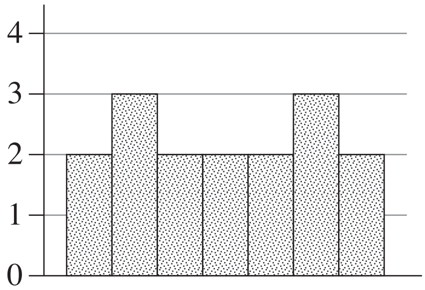


8 Some data are used to create a box plot shown.

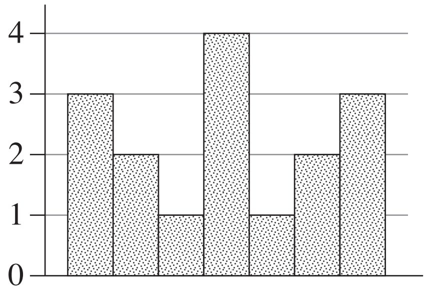
A histogram is created from the same set of data.

Which of these histograms is NOT possible for the given box plot?

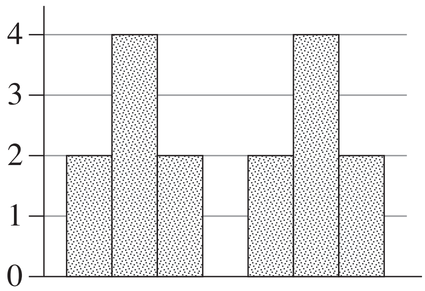
A.



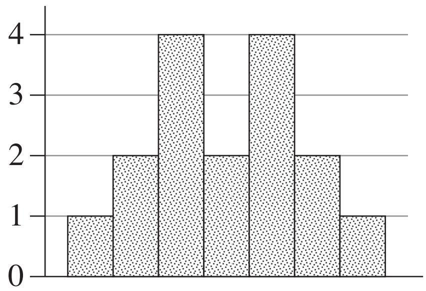
B.



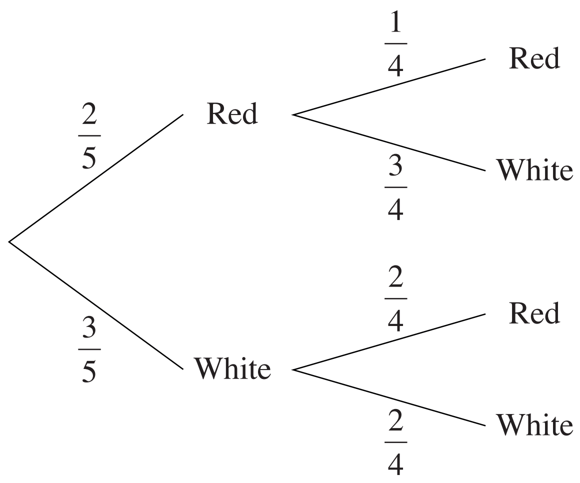
C.



D.



9 A bag contains 2 red and 3 white marbles. Jovan randomly selects two marbles at the same time from this bag. The probability tree diagram shows the probabilities for each of the outcomes.



Given that one of the marbles that Jovan has selected is red, what is the probability that the other marble that he has selected is also red?

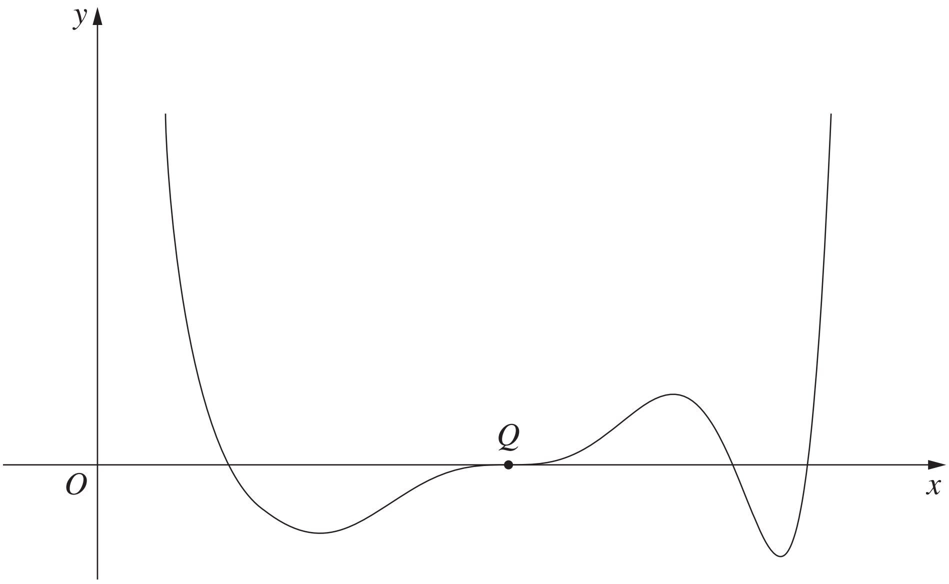
A. $\frac{1}{10}$

B. $\frac{1}{7}$

C. $\frac{1}{4}$

D. $\frac{7}{10}$

10 The diagram shows the graph $y = f(x)$.



The point Q is a horizontal point of inflection.

Let $A(x) = \int_0^x f(t)\,dt$.

How many points of inflection does the graph $y = A(x)$ have?

A. 2

B. 3

C. 4

D. 5

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2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

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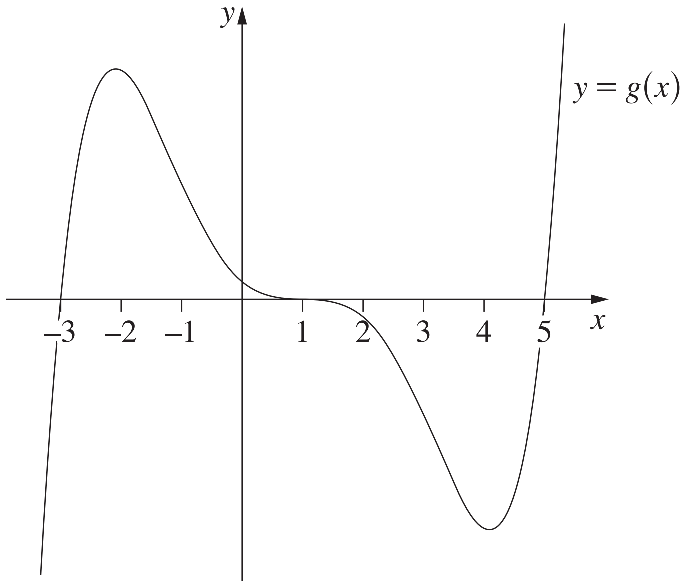
Question 11 (3 marks)

The graph of the function $g(x)$ is shown. **3**



Using the graph, complete the table with the words *positive*, *zero* or *negative* as appropriate.

x-value	*First derivative of* $g(x)$ *at* x	*Second derivative of* $g(x)$ *at* x
$x = -3$		
$x = 1$		
$x = 5$		

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Question 12 (3 marks)

Find the sum of the terms in the arithmetic series **3**

$$50 + 57 + 64 + \cdots + 2024.$$

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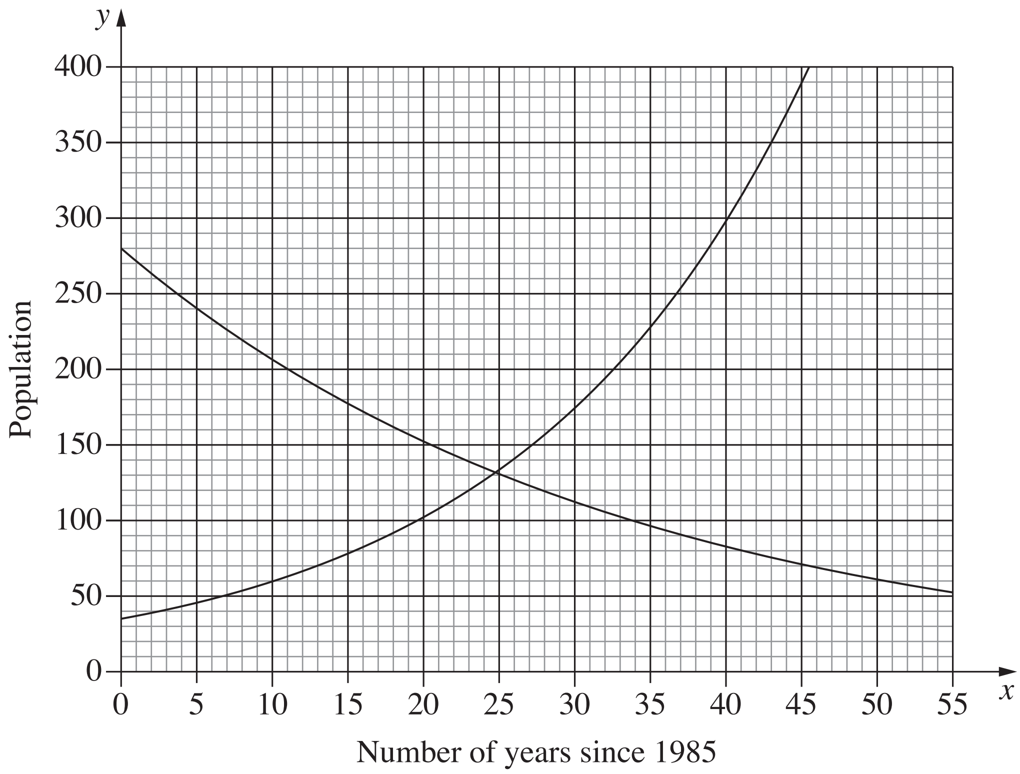
Question 13 (3 marks) **3**

The graph shows the populations of two different animals, W and K, in a conservation park over time. The y-axis is the size of the population and the x-axis is the number of years since 1985.

Population W is modelled by the equation $y = A \times (1.055)^x$.

Population K is modelled by the equation $y = B \times (0.97)^x$.

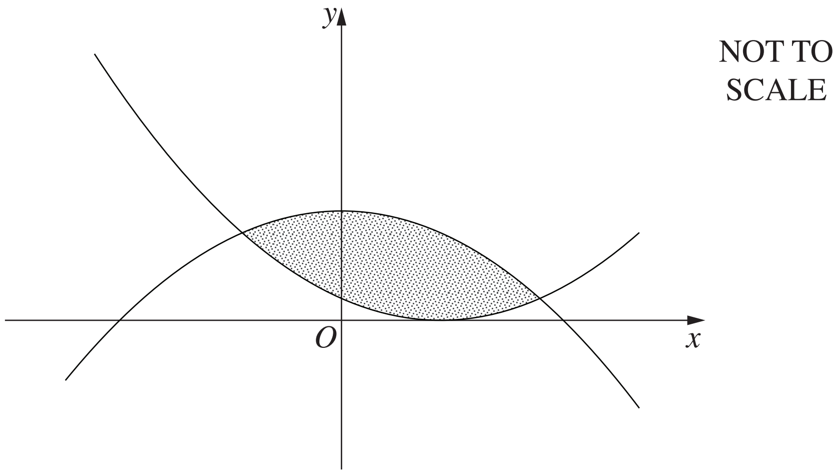


Complete the table using the information provided.

	Population W	*Population K*
Population in 1985	$A = 34$	$B =$
Percentage yearly change in the population		
Predicted population when $x = 50$		61

Question 14 (4 marks)

The curves $y = (x-1)^2$ and $y = 5 - x^2$ intersect at two points, as shown in the diagram.



(a) Find the x-coordinates of the points of intersection of the two curves. **1**

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(b) Find the area enclosed by the two curves. **3**

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Question 15 (3 marks)

Initially there are 350 litres of water in a tank. Water starts flowing into the tank. **3**

The rate of increase of the volume V of water in litres is given by $\frac{dV}{dt} = 300 - 7.5t$, where t is the time in hours.

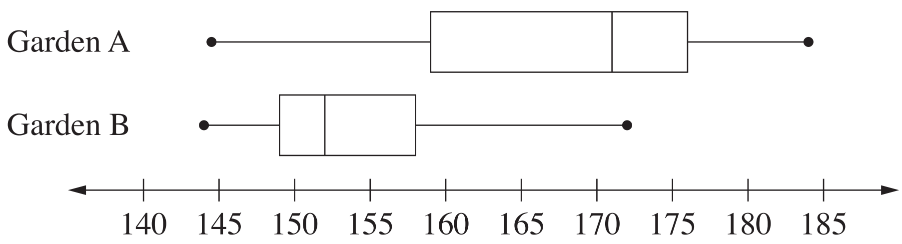
Find the volume of water in the tank when $\frac{dV}{dt} = 0$.

Question 16 (3 marks)

Flowers were planted in two gardens (Garden A and Garden B). **3**

On a particular day, 25 flowers were randomly selected from each garden and their heights measured in millimetres.

The data are represented in parallel box-plots.



Compare the two datasets by examining the skewness of the distributions, and the measures of central tendency and spread.

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Question 17 (6 marks)

In a particular electrical circuit, the voltage V (volts) across a capacitor is given by

$$V(t) = 6.5\left(1 - e^{-kt}\right),$$

where k is a positive constant and t is the number of seconds after the circuit is switched on.

(a) Draw a sketch of the graph of $V(t)$, showing its behaviour as t increases. **2**

(b) When $t = 1$, the voltage across the capacitor is 2.6 volts. **2**

Find the value of k, correct to 3 decimal places.

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(c) Find the rate at which the voltage is increasing when $t = 2$, correct to 3 decimal places. **2**

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Question 18 (3 marks)

In a game, the probability that a particular player scores a goal at each attempt is 0.15.

(a) What is the probability that this player does NOT score a goal in the first two attempts? **1**

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(b) Determine the least number of attempts that this player must make so that the probability of scoring at least one goal is greater than 0.8. **2**

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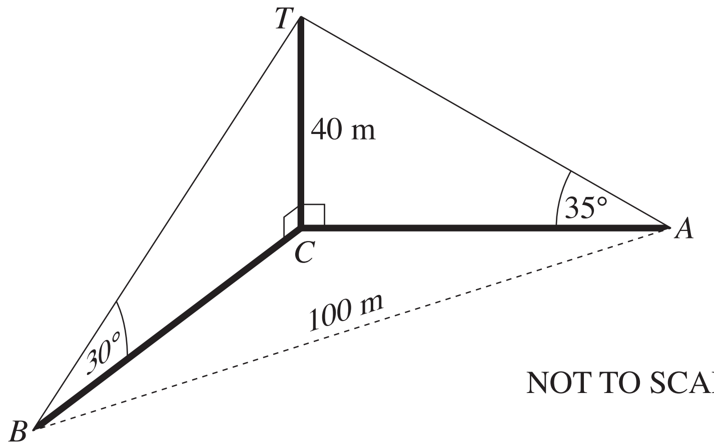
Question 19 (5 marks)

Sketch the curve $y = x^4 - 2x^3 + 2$ by first finding all stationary points, checking their nature, and finding the points of inflection. **5**

Question 20 (4 marks)

A vertical tower TC is 40 metres high. The point A is due east of the base of the tower C. The angle of elevation to the top T of the tower from A is 35°. A second point B is on a different bearing from the tower as shown. The angle of elevation to the top of the tower from B is 30°. The points A and B are 100 metres apart.



(a) Show that distance AC is 57.13 metres, correct to 2 decimal places. **1**

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(b) Find the bearing of B from C to the nearest degree. **3**

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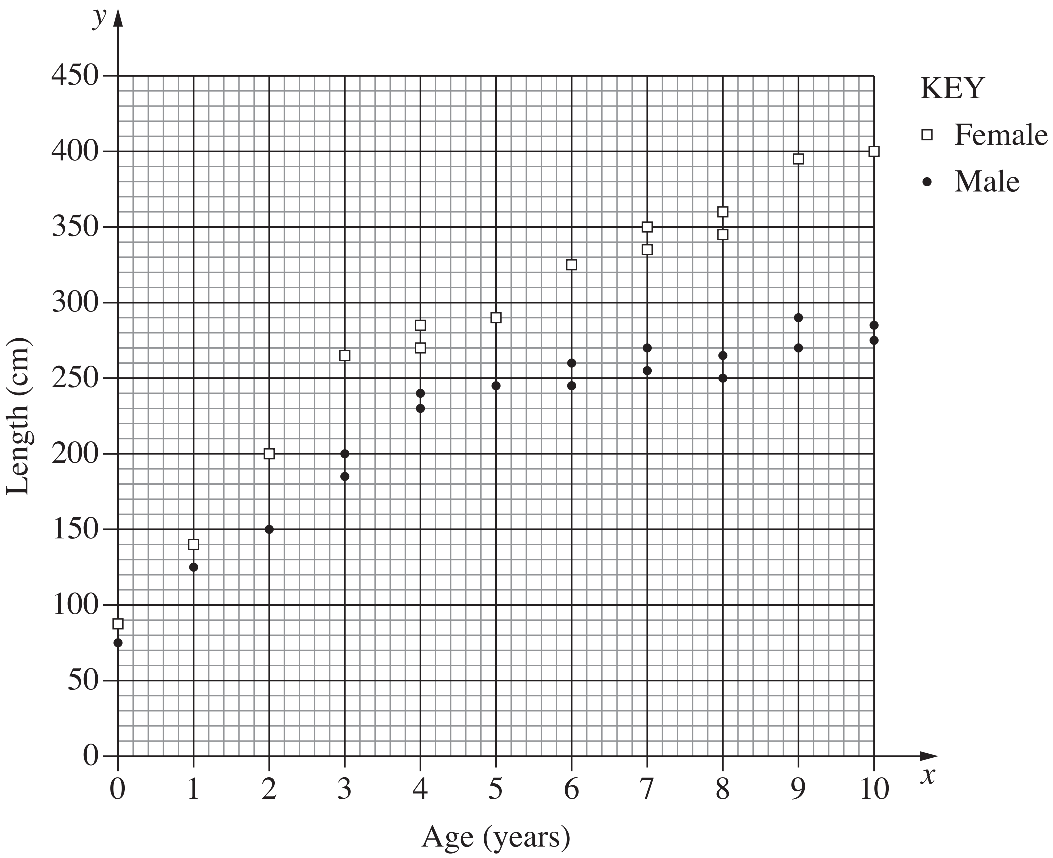
Question 21 (3 marks)

A researcher is studying anacondas (a type of snake). **3**

A dataset recording the age (in years) and length (in cm) of female and male anacondas is displayed on the graph.

Anacondas reach maturity at about 4 years of age.



Write THREE observations about anacondas that may be made from the scatterplot. (Note: No calculations are required.)

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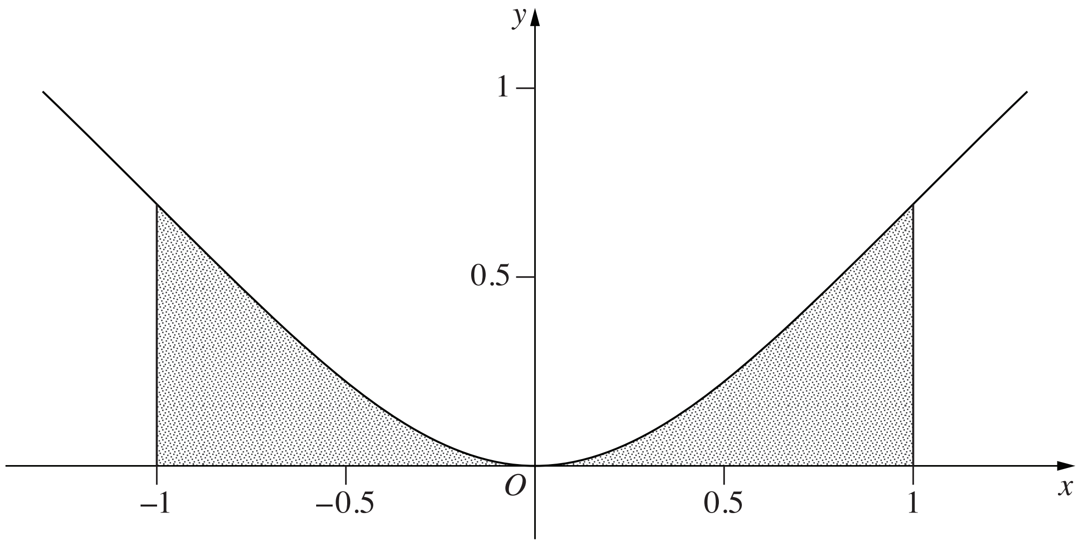
Question 22 (6 marks)

The graph of the function $f(x) = \ln(1 + x^2)$ is shown.



(a) Prove that $f(x)$ is concave up for $-1 < x < 1$. **3**

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Question 22 continues on page 23

Question 22 (continued)

(b) A table of function values, correct to 4 decimal places, for some x values is provided. **2**

x	0	0.25	0.5	0.75	1
$\ln(1+x^2)$	0	0.0606	0.2231	0.4463	0.6931

Using the function values provided and the trapezoidal rule, estimate the shaded area in the diagram.

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(c) Is the answer to part (b) an overestimate or underestimate? Give a reason for your answer. **1**

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End of Question 22

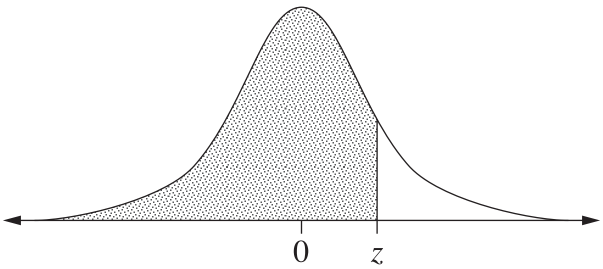
Questions 11–22 are worth 46 marks in total

Question 23 (5 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than z.

z	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Probability	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849	0.9032	0.9192

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



The scores in a university examination with a large number of candidates are normally distributed with mean 58 and standard deviation 15.

(a) By calculating a z-score, find the percentage of scores that are between 58 and 70. **2**

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(b) Explain why the percentage of scores between 46 and 70 is twice your answer to part (a). **1**

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(c) By using the values in the table above, find an approximate minimum score that a candidate would need to be placed in the top 10% of the candidates. **2**

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Question 24 (4 marks)

Jack intends to deposit $80 into a savings account on the first day of each month for 24 months. The interest rate during this time is 6% per annum, compounded monthly.

(a) Calculate how much money Jack will have in his account at the end of the 24 months. **3**

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(b) A table of future value interest factors could have been used to calculate how much money Jack would have in his account at the end of the 24 months. **1**

Part of the table is shown.

Table of future value interest factors for an annuity of $1

Number of periods	*Interest rate per period*		
		0.5%	
24		A	

What is the value of A in the table? Give your answer correct to 3 decimal places.

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Question 25 (6 marks)

A function $f(x)$ is defined as

$$f(x)=\begin{cases}0, & \text{for } x<0\\ 1-\dfrac{x}{h}, & \text{for } 0\le x\le h\\ 0, & \text{for } x>h\end{cases},$$

where h is a constant.

(a) Find the value of h such that $f(x)$ is a probability density function. **2**

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(b) By first finding a formula for the cumulative distribution function, sketch its graph. **2**

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Question 25 continues on page 27

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Question 25 (continued)

(c) Find the value of the median of the probability density function $f(x)$. Give your answer correct to 3 decimal places. **2**

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End of Question 25

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Question 26 (4 marks)

Twenty-five years ago, Phoenix deposited a single sum of money into a new bank account, earning 2.4% interest per annum compounding monthly. **4**

Present value interest factors for an annuity of \$1 for various interest rates (r) and numbers of periods (n) are given in the table.

Period (n) \ *Rate (r)*	*0.001*	*0.002*	*0.003*	*0.004*
60	58.207	56.487	54.835	53.249
120	113.026	106.592	100.649	95.156
180	164.655	151.036	138.927	128.137
240	213.278	190.460	170.908	154.093
300	259.071	225.430	197.627	174.521

Phoenix made the following withdrawals from this account.

- \$2000 at the end of each month for the first 15 years, starting at the end of the first month.
- \$1200 at the end of each month for the next 10 years, starting at the end of the 181st month after the account was opened.

Calculate the minimum sum that Phoenix could have deposited in order to make these withdrawals.

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Question 27 (5 marks)

(a) Find the derivative of $x^2 \tan x$. **2**

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(b) Hence, find $\int (x \tan x + 1)^2 \, dx$. **3**

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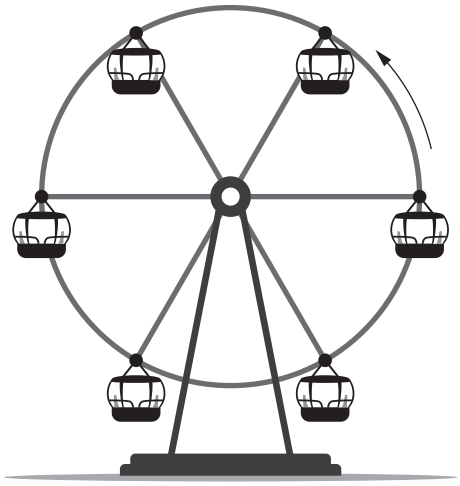
Question 28 (7 marks)

Anna is sitting in a carriage of a Ferris wheel which is revolving. The height, $A(t)$, in metres above the ground of the top of her carriage is given by

$$A(t) = c - k\cos\left(\frac{\pi t}{24}\right),$$

where t is the time in seconds after Anna's carriage first reaches the bottom of its revolution and c and k are constants.



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The top of each carriage reaches a greatest height of 39 metres and a smallest height of 3 metres.

(a) Find the value of c and k. **2**

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(b) How many seconds does it take for one complete revolution of the Ferris wheel? **1**

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Question 28 continues on page 31

Question 28 (continued)

(c) Billie is in another carriage. The height, $B(t)$, in metres above the ground of the top of her carriage is given by **4**

$$B(t) = c - k\cos\left(\frac{\pi}{24}(t-6)\right),$$

where c and k are as found in part (a).

During each revolution, there are two occasions when Anna's and Billie's carriages are at the same heights. At what two heights does this occur? Give your answer correct to 2 decimal places.

End of Question 28

Question 29 (4 marks)

Consider the curve $y = ax^2 + bx + c$, where $a \neq 0$. **4**

At a particular point, the tangent and normal to the curve are given by $t(x) = 2x + 3$ and $n(x) = -\frac{1}{2}x - 2$ respectively.

The curve has a minimum turning point at $x = -4$.

Find the values of a, b and c.

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Question 30 (3 marks)

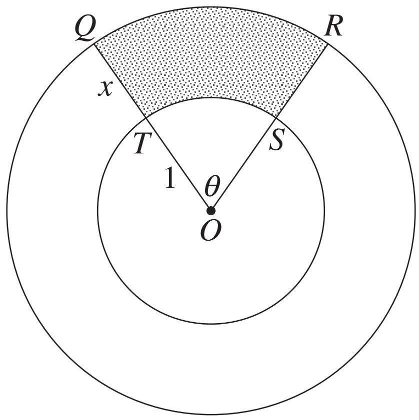
Suppose the geometric series $x + x^2 + x^3 + \cdots$ has a limiting sum, S. **3**

By considering the graph $y = -1 - \dfrac{1}{x-1}$, or otherwise, find the range of possible values of S.

Question 31 (6 marks)

Two circles have the same centre O. The smaller circle has radius 1 cm, while the larger circle has radius $(1 + x)$ cm. The circles enclose a region $QRST$, which is subtended by an angle θ at O, as shaded.

The area of $QRST$ is A cm^2, where A is a constant and $A > 0$.



Let P cm be the perimeter of $QRST$.

(a) By finding expressions for the area and perimeter of $QRST$, show that $P(x) = 2x + \dfrac{2A}{x}$. **3**

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Question 31 continues on page 35

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Question 31 (continued)

(b) Show that if the perimeter, $P(x)$, is minimised, then θ must be less than 2. **3**

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NSW Education Standards Authority

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

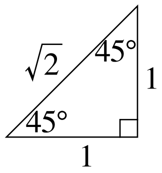
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

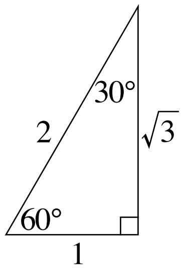
$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

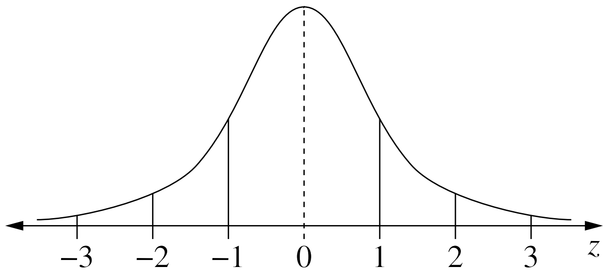
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left| \underset{\sim}{u} \right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left| \underset{\sim}{u} \right| \left| \underset{\sim}{v} \right| \cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$