



NSW Education Standards Authority

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Centre Number

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Student Number

**2023** HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Standard 2

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## General Instructions

- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

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## Total marks: 100

### Section I – 15 marks (pages 2–8)

- Attempt Questions 1–15
- Allow about 25 minutes for this section

### Section II – 85 marks (pages 9–39)

- Attempt Questions 16–38
- Allow about 2 hours and 5 minutes for this section

## Section I

**15 marks**

**Attempt Questions 1–15**

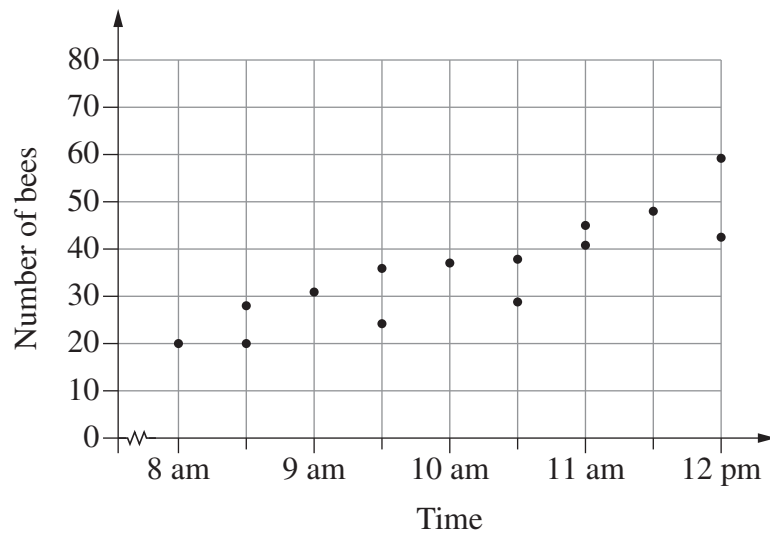
**Allow about 25 minutes for this section**

Use the multiple-choice answer sheet for Questions 1–15.

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- 1** An amount of \$2500 is invested at a simple interest rate of 3% per annum.
- How much interest is earned in the first two years?
- A. \$75
  - B. \$150
  - C. \$2575
  - D. \$2652
- 2** In a normal distribution, what is the approximate percentage of scores with a  $z$ -score less than 1?
- A. 50%
  - B. 68%
  - C. 84%
  - D. 97.5%

- 3 The number of bees leaving a hive was observed and recorded over 14 days at different times of the day.



Which Pearson's correlation coefficient best describes the observations?

- A.  $-0.8$
  - B.  $-0.2$
  - C.  $0.2$
  - D.  $0.8$
- 4 A delivery truck was valued at \$65 000 when new. The value of the truck depreciates at a rate of 22 cents per kilometre travelled.

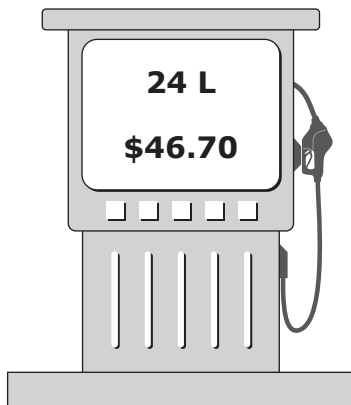
What is the value of the truck after it has travelled a total distance of 132 600 km?

- A. \$35 828
- B. \$29 172
- C. \$14 872
- D. \$14 300

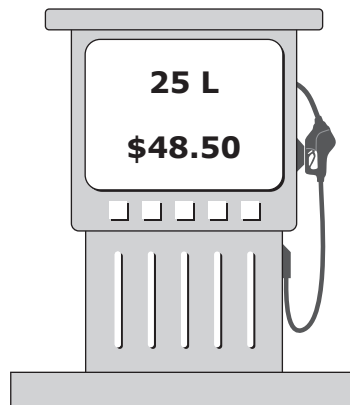
- 5 Four petrol pumps are shown, each with the amount of petrol purchased and its cost.

Which one represents the best value?

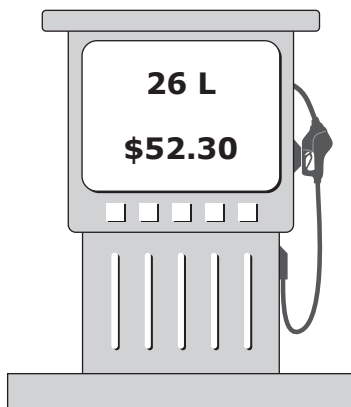
A.



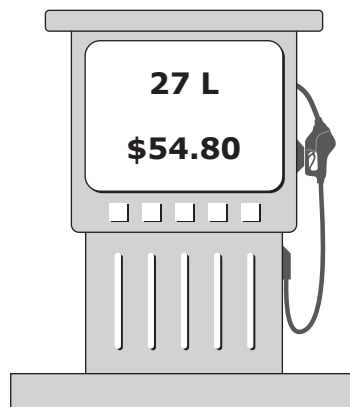
B.



C.



D.



- 6 An item was purchased for a price of \$880, including 10% GST.

What is the amount of GST included in the price?

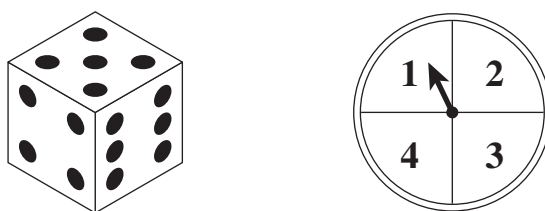
- A. \$8.00
- B. \$8.80
- C. \$80.00
- D. \$88.00

- 7 City *A* is at latitude  $34^{\circ}\text{S}$  and longitude  $151^{\circ}\text{E}$ . City *B* is  $72^{\circ}$  north of City *A* and  $25^{\circ}$  west of City *A*.

What are the latitude and longitude of City *B*?

- A.  $16^{\circ}\text{N}$ ,  $126^{\circ}\text{E}$
- B.  $16^{\circ}\text{N}$ ,  $176^{\circ}\text{E}$
- C.  $38^{\circ}\text{N}$ ,  $126^{\circ}\text{E}$
- D.  $38^{\circ}\text{N}$ ,  $176^{\circ}\text{E}$

- 8 A game involves throwing a die and spinning a spinner.



The sum of the two numbers obtained is the score.

The table of scores below is partially completed.

		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

What is the probability of getting a score of 7 or more?

- A.  $\frac{1}{6}$
- B.  $\frac{1}{4}$
- C.  $\frac{5}{18}$
- D.  $\frac{5}{12}$

- 9** The length and width of a rectangle are measured to be 8 cm and 5 cm respectively, to the nearest centimetre.

What are the lower and upper bounds for the area of the rectangle?

- A.  $28 \text{ cm}^2$  and  $54 \text{ cm}^2$
  - B.  $36 \text{ cm}^2$  and  $42 \text{ cm}^2$
  - C.  $38.25 \text{ cm}^2$  and  $41.25 \text{ cm}^2$
  - D.  $33.75 \text{ cm}^2$  and  $46.75 \text{ cm}^2$
- 10** An amount of \$25 000 is invested for six years. Interest is earned at a rate of 8% per annum, compounding quarterly.
- Which expression gives the value of the investment after 6 years, in dollars?
- A.  $25\,000 \times 1.02^{24}$
  - B.  $25\,000 \times 1.02^6$
  - C.  $25\,000 \times 1.08^{24}$
  - D.  $25\,000 \times 1.08^6$
- 11** A bag contains 150 jelly beans. Some of them are red and the rest are blue. The ratio of red to blue jelly beans is 2 : 3.

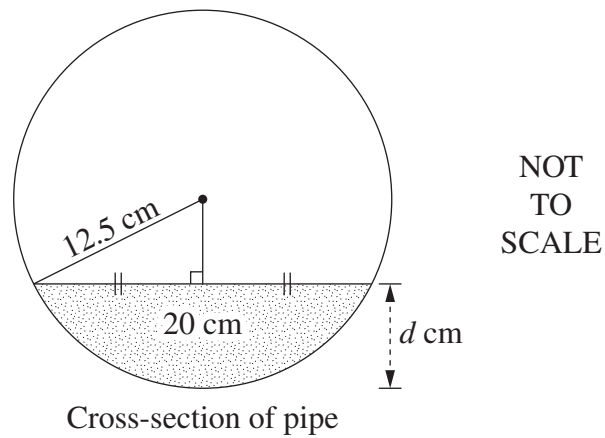
Sophie eats 10 of each colour.

What is the new ratio of red to blue jelly beans?

- A. 2 : 3
- B. 4 : 9
- C. 5 : 8
- D. 11 : 17

- 12** A cylindrical pipe with a radius of 12.5 cm is filled with water to a depth,  $d$  cm, as shown.

The surface of the water has a width of 20 cm.



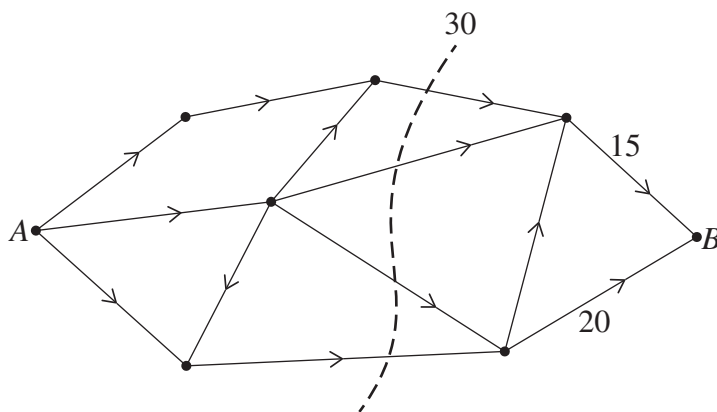
What is the depth of water in the pipe?

- A. 2.5 cm
  - B. 5.0 cm
  - C. 7.5 cm
  - D. 12.5 cm
- 13** An item is discounted by 30% and then a further discount of 20% is applied to the reduced price.

What is the total percentage discount?

- A. 25%
- B. 44%
- C. 50%
- D. 56%

- 14** A network with source  $A$  and sink  $B$  is shown. The capacities of two paths are labelled. The cut shown on the diagram has a capacity of 30.



Which of the following statements is correct?

- A. The maximum flow is 30.
  - B. The maximum flow is 35.
  - C. The maximum flow is 30 or less.
  - D. The maximum flow is 30 or more.
- 15** Ashan's mathematics class needs to complete six tests, each worth 100 marks.

After completing the first five tests, Ashan calculated that he would need a mark of 90 in the final test in order to have a mean mark of 80 for the six tests.

What was Ashan's mean mark after completing the first five tests?

- A. 78
- B. 74
- C. 70
- D. 65



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Centre Number

# Mathematics Standard 2

## Section II Answer Booklet

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Student Number

**85 marks**

**Attempt Questions 16–38**

**Allow about 2 hours and 5 minutes for this section**

### Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

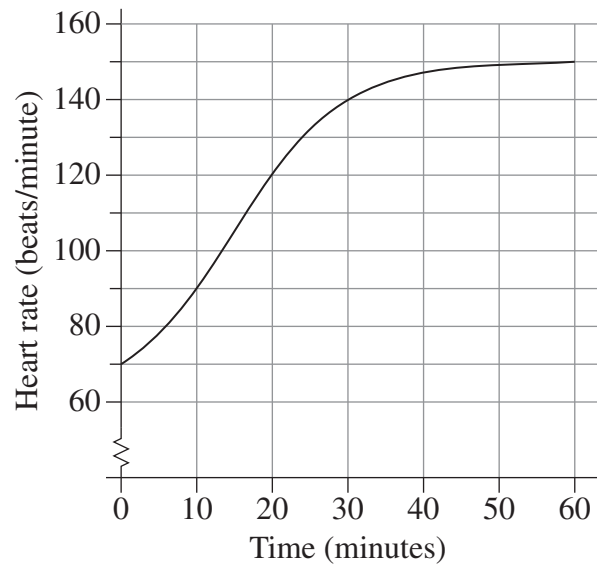
**Please turn over**

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**Question 16** (2 marks)

The graph shows Peta's heart rate, in beats per minute, during the first 60 minutes of a marathon.



- (a) What was Peta's heart rate 20 minutes after she started her marathon? 1

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- (b) Peta started the marathon at 10 am. At what time would her heart rate first reach 140 beats/minute? 1

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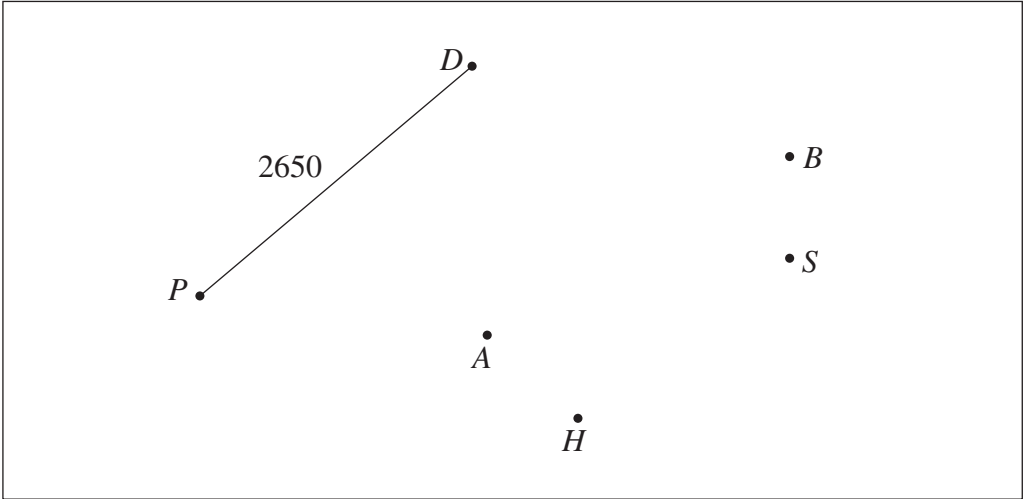
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Question 17 (3 marks)

The table shows some of the flight distances (rounded to the nearest 10 km) between various Australian cities.

City	Adelaide (A)	Brisbane (B)	Darwin (D)	Hobart (H)	Perth (P)	Sydney (S)
Adelaide		–	–	1170	2120	–
Brisbane	–		2850	–	–	750
Darwin	–	2850		–	2650	3150
Hobart	1170	–	–		–	1040
Perth	2120	–	2650	–		3270
Sydney	–	750	3150	1040	3270	

- (a) Use the information in the table to complete the network diagram where the edges are labelled with distances. 2



- (b) Mahsa wants to travel from Hobart to Darwin. She wants to change planes only once. 1

Using the network diagram, calculate how many kilometres she will travel by plane.

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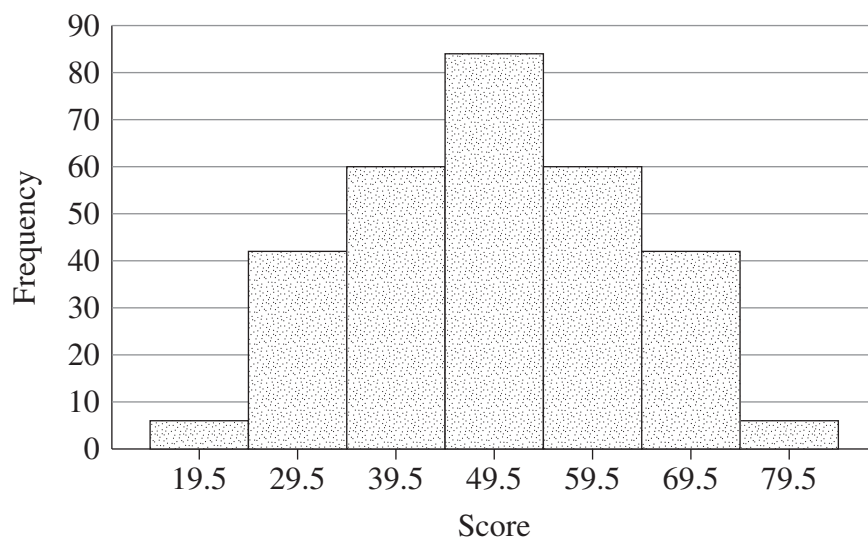
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**Question 18** (2 marks)

The histogram shows a summary of scores on a test.

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Provide TWO features of the histogram that indicate that the data comes from a normal distribution.

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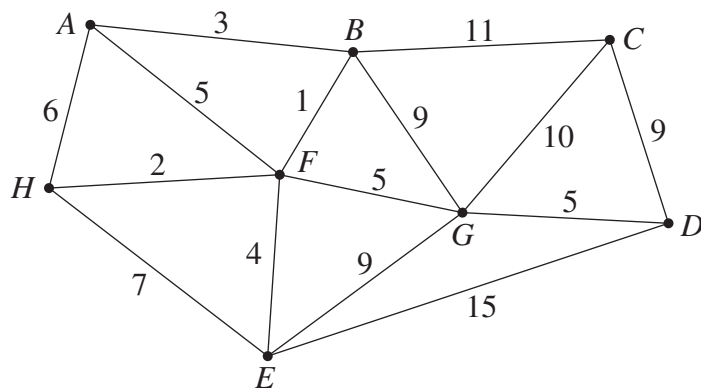
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**Question 19** (4 marks)

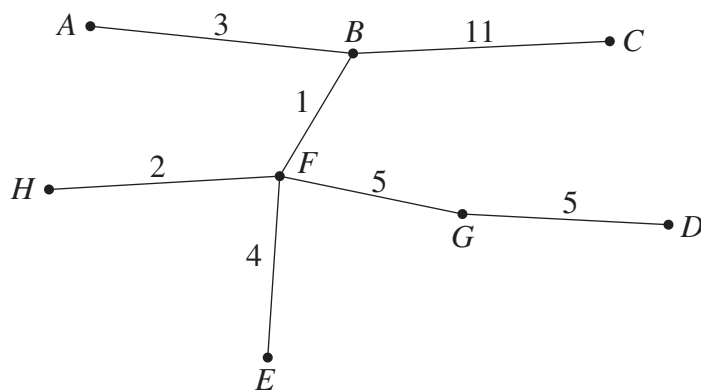
A network of running tracks connects the points  $A, B, C, D, E, F, G, H$ , as shown. The number on each edge represents the time, in minutes, that a typical runner should take to run along each track.



- (a) Which path could a typical runner take to run from point  $A$  to point  $D$  in the shortest time? 2

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- (b) A spanning tree of the network above is shown. 2



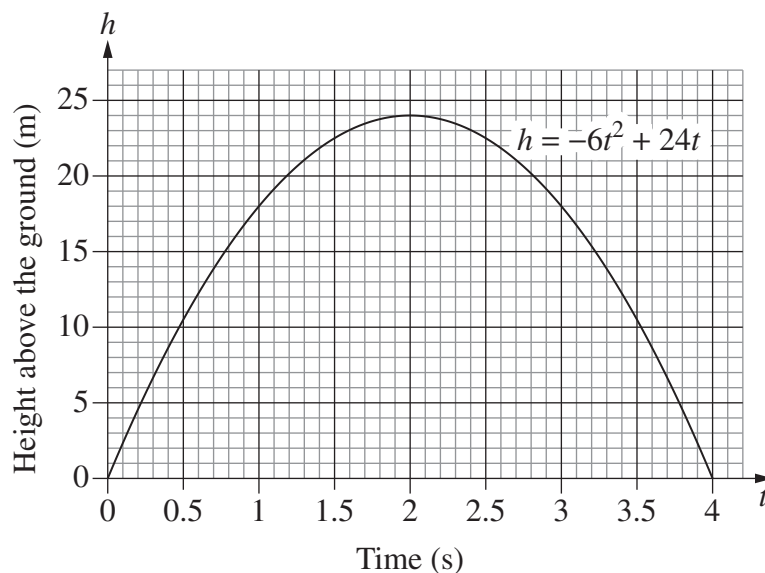
Is it a minimum spanning tree? Give a reason for your answer.

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**Question 20** (3 marks)

On another planet, a ball is launched vertically into the air from the ground.

The height above the ground,  $h$  metres, can be modelled using the function  $h = -6t^2 + 24t$ , where  $t$  is measured in seconds. The graph of the function is shown.



- (a) Based on the graph, what is the maximum height reached by the ball?

1

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- (b) Based on the graph, at what TWO times is the ball at  $\frac{3}{4}$  of its maximum height?

2

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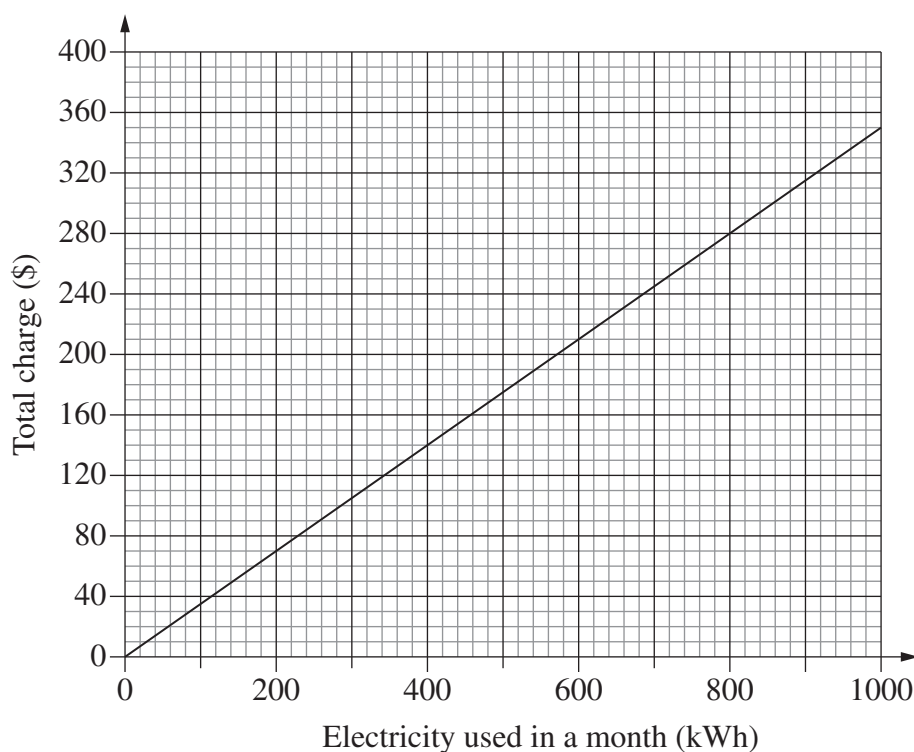
**Question 21** (5 marks)

Electricity provider A charges 25 cents per kilowatt hour (kWh) for electricity, plus a fixed monthly charge of \$40.

- (a) Complete the table showing Provider A's monthly charges for different levels of electricity usage. 1

<i>Electricity used in a month (kWh)</i>	0	400	1000
<i>Monthly charge (\$)</i>	40		290

Provider B charges 35 cents per kWh, with no fixed monthly charge. The graph shows how Provider B's charges vary with the amount of electricity used in a month.



**Question 21 continues on page 17**



Question 21 (continued)

(b) On the grid on the previous page, graph Provider *A*'s charges from the table in part (a). 1

(c) Use the two graphs to determine the number of kilowatt hours per month for which Provider *A* and Provider *B* charge the same amount. 1

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(d) A customer uses an average of 800 kWh per month. 2

Which provider, *A* or *B*, would be the cheaper option and by how much?

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**End of Question 21**

**Please turn over**

**Question 22** (3 marks)

The braking distance of a car, in metres, is directly proportional to the square of its speed in km/h, and can be represented by the equation

$$\text{braking distance} = k \times (\text{speed})^2$$

where  $k$  is the constant of variation.

The braking distance for a car travelling at 50 km/h is 20 m.

- (a) Find the value of  $k$ .

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- (b) What is the braking distance when the speed of the car is 90 km/h?

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**Question 23** (2 marks)

One hundred tickets are sold in a raffle which offers two prizes. Hazel buys five of the tickets.

**2**

A ticket is drawn at random for the first prize. A second ticket is drawn from the remaining tickets for the other prize.

What is the probability that Hazel wins both prizes?

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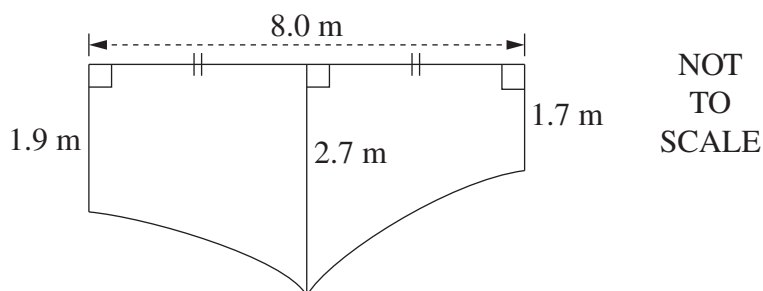
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**Question 24** (5 marks)

The diagram shows the cross-section of a wall across a creek.



- (a) Use two applications of the trapezoidal rule to estimate the area of the cross-section of the wall. 2

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- (b) The wall has a uniform thickness of 0.80 m. The weight of  $1 \text{ m}^3$  of concrete is 3.52 tonnes. 3

How many tonnes of concrete are in the wall? Give the answer to two significant figures.

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**Question 25** (5 marks)

A table of future value interest factors for an annuity of \$1 is shown.

<i>Rate</i> <i>Period</i>	1.5%	3%	4.5%	6%
5	5.152	5.309	5.471	5.637
10	10.703	11.464	12.288	13.181
20	23.124	26.870	31.371	36.786
40	54.268	75.401	107.030	154.762

- (a) Micky wants to save \$450 000 over the next 10 years.

**2**

If the interest rate is 6% per annum compounding annually, how much should Micky contribute each year? Give your answer to the nearest dollar.

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- (b) Instead, Micky decides to contribute \$8535 every three months for 10 years to an annuity paying 6% per annum, compounding quarterly.

**3**

How much will Micky have at the end of 10 years?

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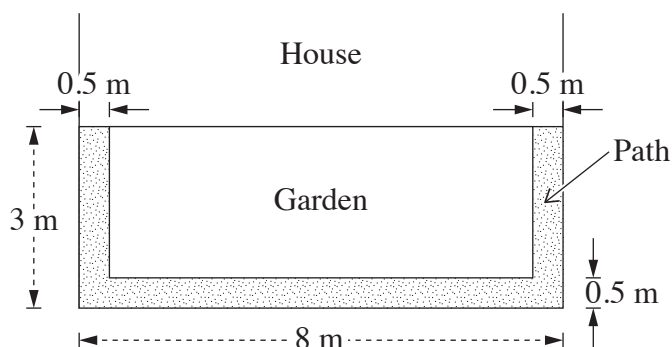
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**Question 26** (5 marks)

Kim is building a path around a garden at the back of a house, as shown. The path is 0.5 m wide.



- (a) Find the area of the path.

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- (b) Kim is mixing some concrete for the path. The concrete mix is made up of crushed rock, sand and cement in the ratio of 4 : 2 : 1 by weight.

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Kim needs 2.1 tonnes of concrete mix in the correct ratio.

Calculate how many 15 kg bags of cement Kim needs to buy.

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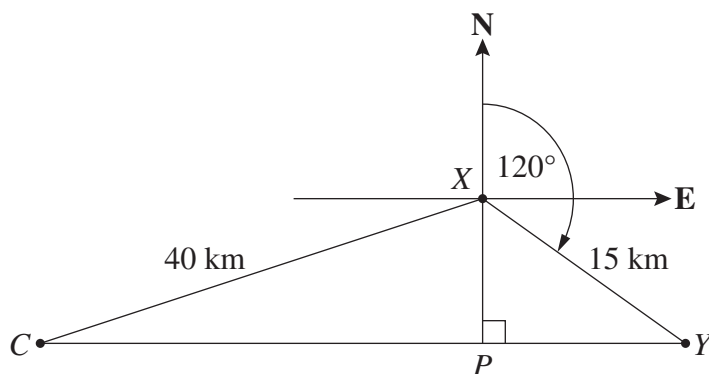
**Question 27** (4 marks)

The diagram shows the location of three places  $X$ ,  $Y$  and  $C$ .

$Y$  is on a bearing of  $120^\circ$  and 15 km from  $X$ .

$C$  is 40 km from  $X$  and lies due west of  $Y$ .

$P$  lies on the line joining  $C$  and  $Y$  and is due south of  $X$ .



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- (a) Find the distance from  $X$  to  $P$ .

2

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- (b) What is the bearing of  $C$  from  $X$ , to the nearest degree?

2

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**Questions 16–27 are worth 43 marks in total**

**Question 28** (3 marks)

A plumber leases equipment which is valued at \$60 000.

3

The salvage value of the equipment at any time can be calculated using either of the two methods of depreciation shown in the table.

<i>Method of depreciation</i>	<i>Rate of depreciation</i>
Straight-line method	\$3500 per annum
Declining-balance method	12% per annum

Under which method of depreciation would the salvage value of the equipment be lower at the end of 3 years? Justify your answer with appropriate mathematical calculations.

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**Question 29** (4 marks)

The table shows monthly repayments for each \$1000 borrowed.

**Monthly repayment table**

Principal and Interest per \$1000 borrowed						
<i>Interest rate</i> (per annum)	<i>Term of loan (years)</i>					
	5	10	15	20	25	30
6.5%	19.57	11.35	8.71	7.46	6.75	6.32
7.0%	19.80	11.61	8.99	7.75	7.07	6.65
7.5%	20.04	11.87	9.27	8.06	7.39	6.99
8.0%	20.28	12.13	9.56	8.36	7.72	7.34

- (a) A couple borrows \$520 000 to buy a house at 8% per annum over 25 years.

**3**

How much does the couple repay in total for this loan?

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- (b) Chris borrows some money at 7% per annum. Chris will repay the loan over 15 years, paying \$3596 per month.

**1**

How much money does Chris borrow?

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**Question 30** (3 marks)

A receipt from a supermarket shows a total of \$124.87. The GST shown on the receipt is \$3.86.

**3**

GST, at a rate of 10%, is only charged on some items.

What was the value of the items which did NOT have GST charged?

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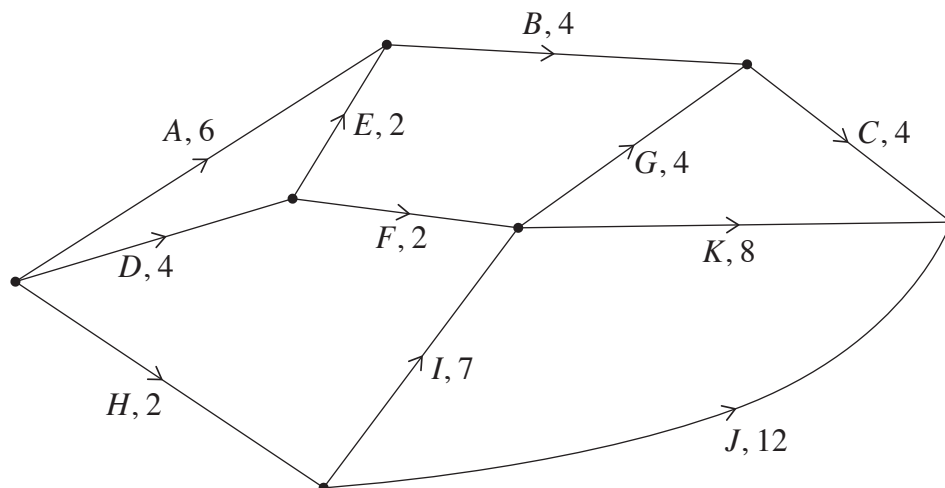
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**Question 31** (4 marks)

A function centre employs staff so that all necessary tasks can be completed between the end of one function and the beginning of the next function.

The network diagram shows the time taken in hours for the tasks that need to be completed.



- (a) Find the TWO critical paths.

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- (b) The function centre wants to decrease the length of each critical path by 3 hours. They can do this by hiring more staff to do ONE of the tasks so it takes less time to complete.

2

For which task should the centre hire more staff, and how long should that task take to ensure all tasks can be completed in 14 hours?

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**Question 32** (4 marks)

Ali has a credit card which has no interest-free period. Interest is charged at 13.5% per annum, compounding daily, on the amount owing.

During the month, Ali made only one purchase of \$450 using the credit card. The full amount owing was repaid 21 days later.

- (a) Calculate the amount of interest charged on the purchase, assuming that interest is charged for the 21 days. 2

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- (b) What percentage of the full amount repaid is the interest? Give the answer to two decimal places. 2

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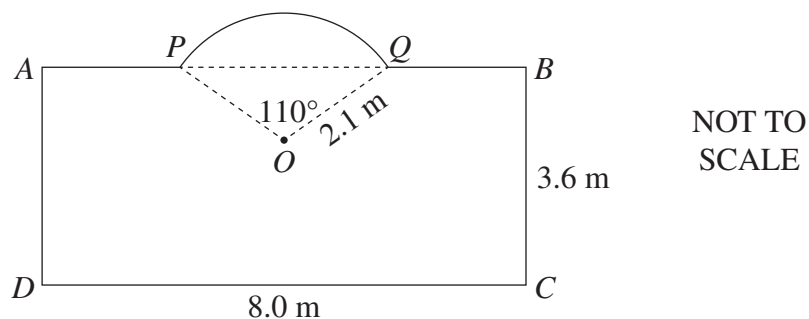
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**Question 33** (4 marks)

The diagram shows a shape  $APQBCD$ . The shape consists of a rectangle  $ABCD$  with an arc  $PQ$  on side  $AB$  and with side lengths  $BC = 3.6$  m and  $CD = 8.0$  m.

4

The arc  $PQ$  is an arc of a circle with centre  $O$  and radius  $2.1$  m and  $\angle POQ = 110^\circ$ .



What is the perimeter of the shape  $APQBCD$ ? Give your answer correct to one decimal place.

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**Question 34** (6 marks)

A university uses gas to heat its buildings. Over a period of 10 weekdays during winter, the gas used each day was measured in megawatts (MW) and the average outside temperature each day was recorded in degrees Celsius ( $^{\circ}\text{C}$ ).

Using  $x$  as the average daily outside temperature and  $y$  as the total daily gas usage, the equation of the least-squares regression line was found.

The equation of the regression line predicts that when the temperature is  $0^{\circ}\text{C}$ , the daily gas usage is 236 MW.

The ten temperatures measured were:  $0^{\circ}$ ,  $0^{\circ}$ ,  $0^{\circ}$ ,  $2^{\circ}$ ,  $5^{\circ}$ ,  $7^{\circ}$ ,  $8^{\circ}$ ,  $9^{\circ}$ ,  $9^{\circ}$ ,  $10^{\circ}$ .

The total gas usage for the ten weekdays was 1840 MW.

In any bivariate dataset, the least-squares regression line passes through the point  $(\bar{x}, \bar{y})$ , where  $\bar{x}$  is the sample mean of the  $x$ -values and  $\bar{y}$  is the sample mean of the  $y$ -values.

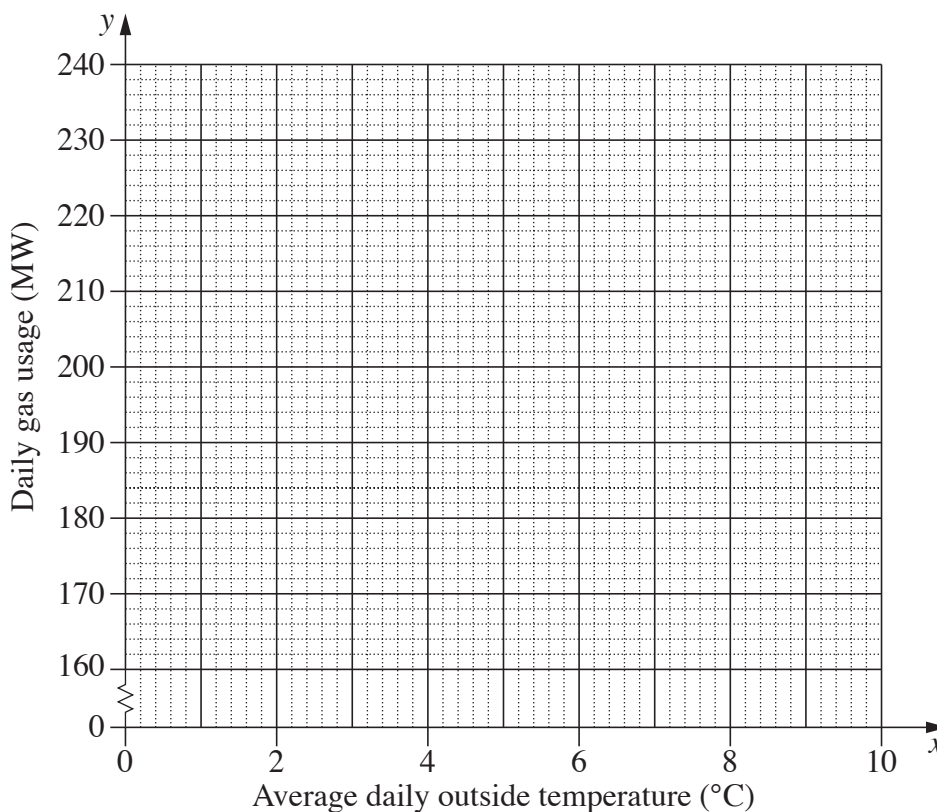
- (a) Using the information provided, plot the point  $(\bar{x}, \bar{y})$  and the  $y$ -intercept of the least-squares regression line on the grid.

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**Question 34 continues on page 31**

Question 34 (continued)

- (b) What is the equation of the regression line?

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- (c) In the context of the dataset, identify ONE problem with using the regression line to predict gas usage when the average outside temperature is 23°C.

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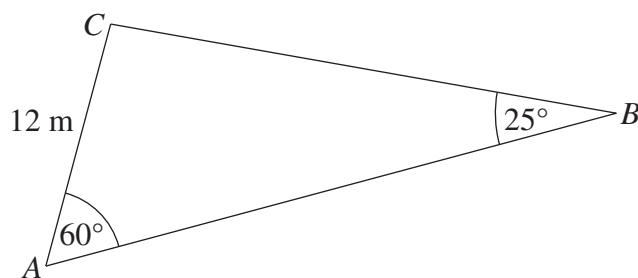
**End of Question 34**

**Please turn over**

**Question 35** (3 marks)

The diagram shows triangle  $ABC$ .

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Calculate the area of the triangle, to the nearest square metre.

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**Question 36** (4 marks)

The following formula can be used to calculate an estimate for blood alcohol content (BAC) for males.

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$$BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$$

$N$  is the number of standard drinks consumed

$M$  is the person's weight in kilograms

$H$  is the number of hours of drinking

Cameron weighs 75 kg. His BAC was zero when he began drinking alcohol. At 9:00 pm, after consuming 3 standard drinks, his BAC was 0.02.

Using the formula, estimate at what time Cameron began drinking alcohol, to the nearest minute.

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**Question 37** (3 marks)

The table shows personal income tax rates for different taxable incomes for a particular country.

3

<i>Taxable income</i>	<i>Tax payable</i>
\$0 – \$11 000	Nil
\$11 001 – \$42 400	20 cents for each \$1 over \$11 000
\$42 401 – \$78 800	\$6280 plus 33 cents for each \$1 over \$42 400
\$78 801 – \$108 400	\$18 292 plus <span style="border: 1px solid black; padding: 0 5px;"><i>X</i></span> cents for each \$1 over \$78 800
\$108 401 and over	\$31 316 plus 48 cents for each \$1 over \$108 400

A person with a taxable income of \$90 000 pays 25.8% of that income in tax (excluding any levies).

What is the value of *X* in the table?

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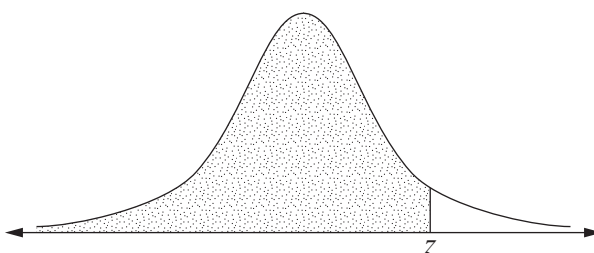
**Question 38** (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below  $z$  for some positive values of  $z$ .

**4**

$z$	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

The probability values given in the table are represented by the shaded area in the following diagram.



The weights of adult male koalas form a normal distribution with mean  $\mu = 10.40$  kg, and standard deviation  $\sigma = 1.15$  kg.

In a group of 400 adult male koalas, how many would be expected to weigh more than 11.93 kg?

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# Mathematics Standard 1

# Mathematics Standard 2

## REFERENCE SHEET

### Measurement

#### Limits of accuracy

$$\text{Absolute error} = \frac{1}{2} \times \text{precision}$$

$$\text{Upper bound} = \text{measurement} + \text{absolute error}$$

$$\text{Lower bound} = \text{measurement} - \text{absolute error}$$

#### Length

$$l = \frac{\theta}{360} \times 2\pi r$$

#### Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

#### Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

#### Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

#### Trigonometry

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

### Financial Mathematics

$$FV = PV(1 + r)^n$$

#### Straight-line method of depreciation

$$S = V_0 - Dn$$

#### Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

### Statistical Analysis

An outlier is a score

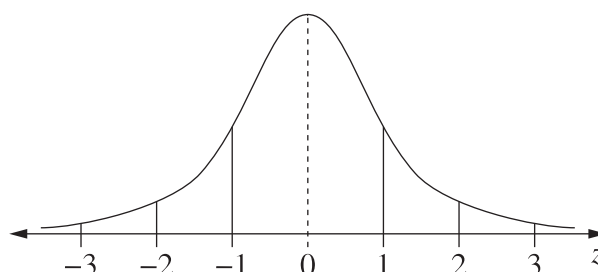
$$\text{less than } Q_1 - 1.5 \times IQR$$

or

$$\text{more than } Q_3 + 1.5 \times IQR$$

$$z = \frac{x - \mu}{\sigma}$$

#### Normal distribution



- approximately 68% of scores have  $z$ -scores between  $-1$  and  $1$
- approximately 95% of scores have  $z$ -scores between  $-2$  and  $2$
- approximately 99.7% of scores have  $z$ -scores between  $-3$  and  $3$

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