



North Sydney Girls High School

2022

HSC TRIAL EXAMINATION

# Mathematics Extension 1

## General Instructions

- Reading Time – 10 minutes
- Working Time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks:**  
**70**

## Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

## Section II – 60 marks (pages 7 – 13)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

NAME: \_\_\_\_\_

TEACHER: \_\_\_\_\_

STUDENT NUMBER:

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| Question | 1-10 | 11  | 12  | 13  | 14  | Total |
|----------|------|-----|-----|-----|-----|-------|
| Mark     | /10  | /15 | /15 | /15 | /15 | /70   |

## Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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- 1 Let  $P(x) = x^3 - 2ax^2 + x - 1$  where  $a \in \mathbb{R}$ . When  $P(x)$  is divided by  $x + 2$ , the remainder is 5. What is the value of  $a$ ?
- A. 2
- B.  $-\frac{7}{4}$
- C.  $\frac{1}{2}$
- D. -2
- 2 The points  $A$  and  $B$  have coordinates  $(-2, 3)$  and  $(2, -5)$  respectively. Which of the following is the vector  $\overrightarrow{AB}$ ?
- A.  $-2\hat{j}$
- B.  $4\hat{i} - 8\hat{j}$
- C.  $-4\hat{i} + 8\hat{j}$
- D.  $2\hat{j}$
- 3 What is the angle between the vectors  $\begin{pmatrix} -7 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ?
- A.  $\cos^{-1}(-0.8)$
- B.  $\cos^{-1}(-0.08)$
- C.  $\cos^{-1}(0.8)$
- D.  $\cos^{-1}(0.08)$

4 Which of the following is the derivative of  $\tan^{-1}(3x)$ ?

A.  $3\tan^{-1} 3x$

B.  $\frac{3}{1+9x^2}$

C.  $\frac{3}{1+3x^2}$

D.  $3\sec^2 3x$

5 What is the equation of the inverse of  $f(x) = \frac{5+e^{2x}}{3}$ ?

A.  $y = \frac{3}{5+e^{2x}}$

B.  $y = e^{5-3x}$

C.  $y = \frac{1}{2}\ln(3x-5)$

D.  $y = \frac{1}{2}\ln(5-3x)$

6 Four female and four male students are to be seated around a circular table. In how many ways can this be done if the males and females must alternate?

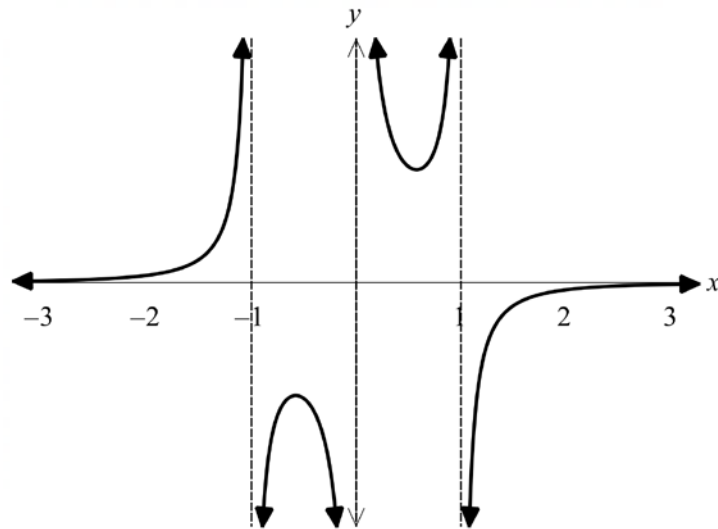
A.  $4! \times 4!$

B.  $3! \times 4!$

C.  $3! \times 3!$

D.  $2 \times 3! \times 3!$

- 7 The graph below shows  $y = \frac{1}{f(x)}$ .



Which of the following best represents the equation of  $f(x)$  ?

- A.  $f(x) = 1 - x^2$
- B.  $f(x) = x(x^2 - 1)$
- C.  $f(x) = x(1 - x^2)$
- D.  $f(x) = x^2(x^2 - 1)$
- 8 What is the vector projection of  $\underline{a} = 2\underline{i} + 3\underline{j}$  in the direction of  $\underline{b} = \underline{i} - 4\underline{j}$ ?

- A.  $-\frac{20}{17}\underline{i} - \frac{30}{17}\underline{j}$
- B.  $-\frac{10}{13}\underline{i} + \frac{40}{13}\underline{j}$
- C.  $-\frac{20}{13}\underline{i} - \frac{30}{13}\underline{j}$
- D.  $-\frac{10}{17}\underline{i} + \frac{40}{17}\underline{j}$

- 9** The radius of a sphere,  $r$ , is increasing at the rate of 0.3 cm per second.  
What is the rate of increase in the volume,  $V$ , in  $\text{cm}^3$  per second, at the instant when the surface area is  $100\pi \text{ cm}^2$ ?
- A.  $10\pi$   
B.  $12\pi$   
C.  $25\pi$   
D.  $30\pi$
- 10** Which of the following is the range of the function  $f(x) = |b \cos^{-1}(x) - a|$ , where  $a > 0$ ,  $b > 0$  and  $a < \frac{b\pi}{2}$ ?
- A.  $[-a, b\pi - a]$   
B.  $[0, b\pi - a]$   
C.  $[a, b\pi - a]$   
D.  $[0, a]$

**End of Section I**

## Section II

**60 marks**

**Attempt Questions 11-14**

**Allow about 1 hour and 45 minutes for this section**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Use a SEPARATE writing booklet

(a) Solve  $|2x - 3| \leq 1$ . **2**

(b) Find  $\int_0^{\frac{1}{2}} \frac{dy}{\sqrt{1-3y^2}}$ . **2**

(c) Let  $\alpha$ ,  $\beta$  and  $\gamma$  be the roots of the equation  $2x^3 - kx^2 - 4x + 12 = 0$ .

(i) Find the value of  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ . **2**

(ii) Given that two of its roots sum to zero, find the third root and hence find the value of  $k$ . **2**

(d) Using the substitution  $t = \tan \frac{\theta}{2}$ , or otherwise, show that **2**

$$\cot \theta + \frac{1}{2} \tan \frac{\theta}{2} = \frac{1}{2} \cot \frac{\theta}{2} \text{ for all } \theta \neq k\pi, k \in \mathbb{Z}.$$

(e) Find the term independent of  $x$  in the expansion of  $\left(3x^2 + \frac{2}{x}\right)^{12}$ . **2**

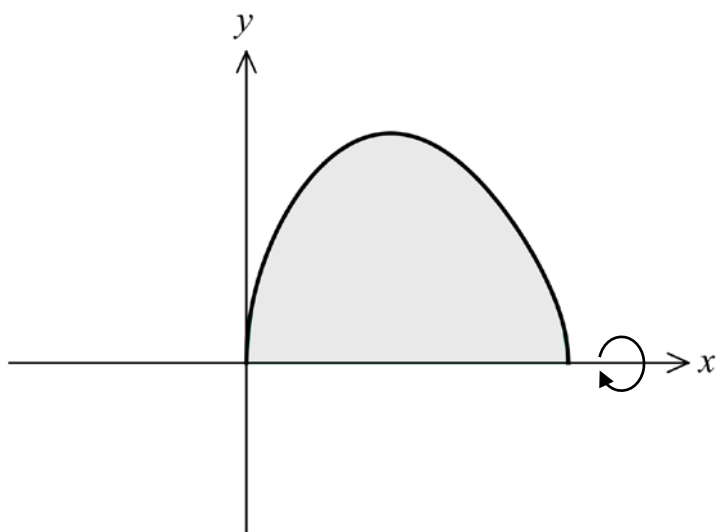
(f) Prove by mathematical induction that  $n^3 + 2n$  is divisible by 3 for all positive integers  $n$ . **3**

**Question 12** (15 marks) Use a SEPARATE writing booklet

(a) Solve  $\frac{x^2 + 6}{x} < 5$ . **3**

(b) By expressing  $\cos x - \sqrt{3} \sin x$  in the form  $A \cos(x + \alpha)$  where  $A > 0$ , solve  $\cos x - \sqrt{3} \sin x + 1 = 0$  for  $0 \leq x \leq 2\pi$ . **4**

(c) A section of the graph of  $y = \sqrt{\sin 3x \cos 2x}$  is shown in the diagram below. **4**

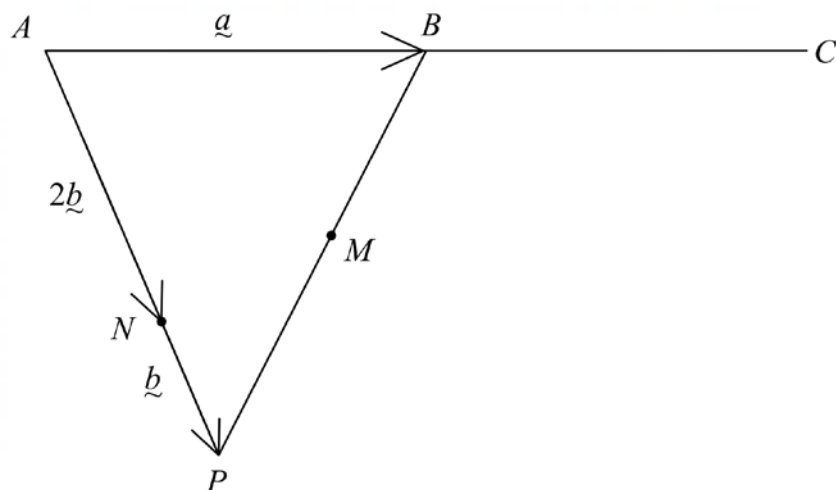


By first finding the smallest positive solution to  $\sin 3x \cos 2x = 0$ , find the volume of the solid formed when the shaded region is rotated about the  $x$ -axis.

**Question 12 continues on page 9**

Question 12 (continued)

- (d) In the diagram below  $APB$  is a triangle.  $N$  is a point on  $AP$ .  
 $\overrightarrow{AB} = \underline{a}$        $\overrightarrow{AN} = 2\underline{b}$        $\overrightarrow{NP} = \underline{b}$



- (i) Find the vector  $\overrightarrow{PB}$  in terms of  $\underline{a}$  and  $\underline{b}$ . **1**
- (ii)  $B$  is the midpoint of  $AC$ .  $M$  is the midpoint of  $PB$ . **3**  
 Show that  $NMC$  is a straight line.

**End of Question 12**



**Question 13** (15 marks) Use a SEPARATE writing booklet

- (a) A netball team's record for the 2022 season was 16 wins and 4 losses. 2  
None of their games were drawn. Prove that the team must have won at least 4 games in a row somewhere during the season.

- (b) The letters of the word REORDER are arranged randomly in a line.

- (i) Use a combinatorial argument to explain why 2

$$\binom{7}{3}\binom{4}{2}\binom{2}{1}\binom{1}{1} = \binom{7}{1}\binom{6}{1}\binom{5}{2}\binom{3}{3}.$$

- (ii) Hence, or otherwise, find the probability that a random rearrangement has all the consonants grouped together. 3

- (c) A pilot is performing at an air show. The position of her aeroplane at time  $t$  relative to a fixed origin  $O$  is given by  $\underline{r}(t) = \left(450 - 150\sin\left(\frac{\pi t}{6}\right)\right)\underline{i} + \left(400 - 200\cos\left(\frac{\pi t}{6}\right)\right)\underline{j}$ , where  $\underline{i}$  is a unit vector in a horizontal direction and  $\underline{j}$  is a unit vector vertically up. Displacement components are measured in metres and time  $t$  is measured in seconds where  $t \geq 0$ .

- (i) Show that the cartesian equation of the path of the aeroplane is given by: 2

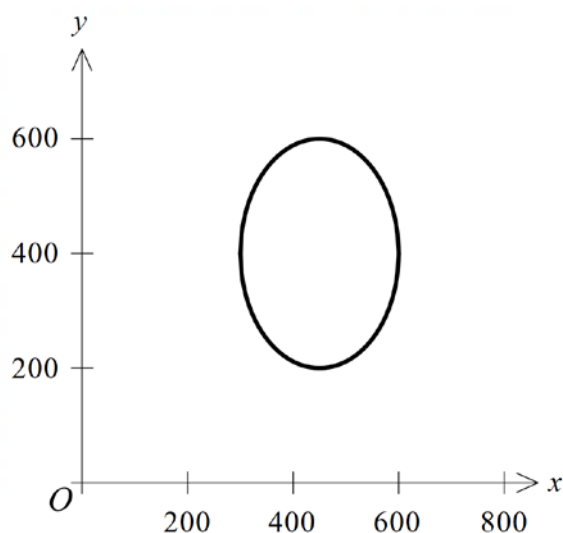
$$\frac{(x-450)^2}{22500} + \frac{(y-400)^2}{40000} = 1.$$

**Question 13 continues on page 11**

Question 13 (continued)

The path of the aeroplane is shown in the diagram below. At the same time that the pilot begins performing, a firework is fired from  $O$  with a velocity of 80 metres per second at an angle of inclination of  $\theta$ . The position of the firework at time  $t$  relative to the fixed origin is given by  $\underline{s}(t) = (80t \cos \theta) \underline{i} + (80t \sin \theta - 5t^2) \underline{j}$ .

(Do NOT prove this).



- (ii) Find the value of  $\theta$  given that the firework explodes when it reaches its maximum height of 160 m. **3**
- (iii) By first finding a vector that represents the displacement of the aeroplane from the firework at time  $t$ , find how far the aeroplane is from the firework when it explodes. Give your answer to the nearest metre. **3**

**End of Question 13**

**Question 14** (15 marks) Use a SEPARATE writing booklet

(a) Use the substitution  $x = \sin \theta$  to find  $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}} dx$ . **3**

(b) (i) Write  $2 \sin x \sin((2k+1)x)$  as the difference of two cosine functions. **1**

(ii) Prove by mathematical induction that for all integers  $n \geq 1$ , **3**

$$\sin x + \sin 3x + \sin 5x + \dots + \sin(2n-1)x = \frac{1 - \cos 2nx}{2 \sin x}.$$

(c) (i) The graph of  $f(x) = -\frac{1}{x^2}$  is shown on the separate Response Sheet for Question 14 c (i). **2**

On the Response Sheet, use addition of ordinates to sketch the graph of  $g(x) = x^2 - \frac{1}{x^2}$  for  $y \in [-10, 10]$  clearly showing the location of the  $x$ -intercepts.

**You do not need to find the  $x$ -coordinates at the endpoints of the range.**

(ii) Show that  $g(x)$  may be rearranged to give **2**

$$x^2 = \frac{y + \sqrt{y^2 + 4}}{2}.$$

A glass with a hollow stem, and with base at  $y = -10$  is made by rotating the part of  $g(x)$  where  $x > 0$  and  $y \in [-10, 10]$  about the  $y$ -axis to form a solid of revolution, where length units are in centimetres.

(iii) Write down a definite integral which, when evaluated, would give the volume of the glass. **1**

**Do not attempt to evaluate this integral.**

**Question 14(c) continues on page 13**

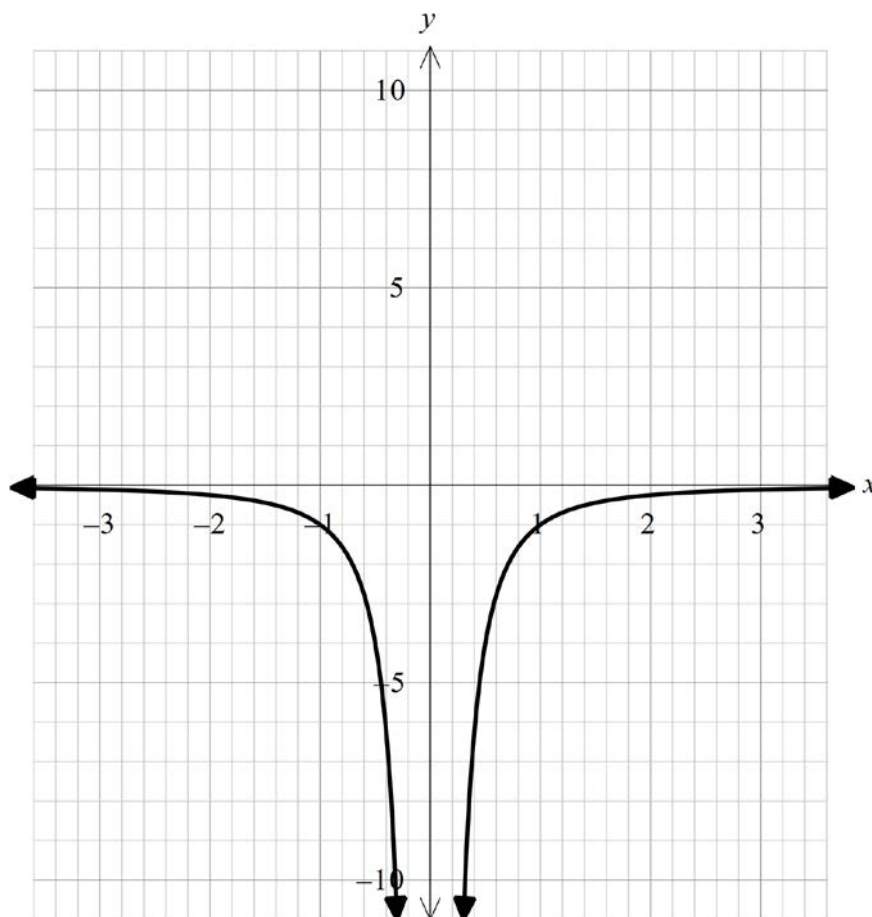
Question 14 (continued)

- (iv) Liquid is poured into the glass at a rate of  $1.5 \text{ cm}^3$  per second. **3**  
Find the rate at which the surface of the liquid is rising when it is 6 cm from the top of the glass.

**End of paper**

**Question 14 c) (i) – Response Sheet**

14 c) The graph of  $f(x) = -\frac{1}{x^2}$  is shown below.



- (i) On this Response Sheet provided, use addition of ordinates to sketch the graph of the  $g(x) = x^2 - \frac{1}{x^2}$  for  $y \in [-10, 10]$  clearly showing the location of the  $x$ -intercepts. **2**
- You do not need to find the  $x$ -coordinates at the endpoints of the range.**

**Place this sheet inside your Question 14 answer booklet.**