Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial 2022
Solutions

Multiple Choice

Solutions	Marker's Comments
Question 1 D $\cos\theta = \dfrac{1\times 5 + 3(-1)}{\sqrt{10}\times\sqrt{26}}$ $= \dfrac{2}{\sqrt{260}}$ $\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{260}}\right)$ $\approx 82.875°$	
Question 2 D $\alpha+\beta+\gamma = \dfrac{-b}{a} = -2$ $\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{c}{a} = -3$ $\alpha\beta\gamma = \dfrac{-d}{a} = -6$ $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma} = \dfrac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ $= \dfrac{-3}{-6}$ $= \dfrac{1}{2}$	
Question 3 A A. $\underset{\sim}{a} + \underset{\sim}{v} = \underset{\sim}{b}$ $\underset{\sim}{v} = \underset{\sim}{b} - \underset{\sim}{a}$ B. $\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{v}$ C. $\underset{\sim}{b} + \underset{\sim}{a} = -\underset{\sim}{v}$ $\underset{\sim}{v} = -\underset{\sim}{a} - \underset{\sim}{b}$ D. $\underset{\sim}{v} + \underset{\sim}{b} = \underset{\sim}{a}$ $\underset{\sim}{v} = \underset{\sim}{a} - \underset{\sim}{b}$	

Solutions	Marker's Comments
Question 4 C $\sqrt{3}\cos 2\theta - \sin\theta = R\cos(2\theta + \alpha)$ $= R\cos 2\theta\cos\alpha - R\sin 2\theta\sin\alpha$ $R\cos\alpha = \sqrt{3},\ \ R\sin\alpha = 1$ $\tan\alpha = \frac{1}{\sqrt{3}},\ \ \alpha = \frac{\pi}{6}$ $R^2\sin^2\alpha + R^2\cos^2\alpha = 1 + \sqrt{3}^2$ $\mathrm{R}^2 = 4$ $R = 2$	
Question 5 B $\sin(A+B) - \sin(A-B) = 2\sin A\cos B$ $\sin(3x+x) - \sin(3x-x) = 2\sin 3x\cos x$	
Question 6 B $\begin{pmatrix} 4 \\ a+1 \end{pmatrix} \cdot \begin{pmatrix} a \\ -2 \end{pmatrix} = 0$ $4a - 2(a+1) = 0$ $2a - 2 = 0$ $a = 1$	
Question 7 C $-2 \le x \le 2$ $-1 \le \frac{x}{2} \le 1$ $0 \le y \le 2\pi$ $0 \le \frac{y}{2} \le \pi$ $\frac{y}{2} = \cos^{-1}\left(\frac{x}{2}\right)$ $y = 2\cos^{-1}\left(\frac{x}{2}\right)$ $A = 2, B = \frac{1}{2}$	

Solutions	Marker's Comments
Question 8 B $\frac{dy}{dx}=0$ for $y=-x$, exclude C and D $x>0, y>0, \frac{dy}{dx}>0$, exclude A or: choose two points to test, eg at $\left(\frac{1}{2},\frac{1}{2}\right)$, $\frac{dy}{dx}\approx 1$ and at $\left(-\frac{1}{2},\frac{1}{2}\right)$, $\frac{dy}{dx}$=0 only B is true for both points.	

Solutions	Marker's Comments
Question 9 C $y=e^{1-px}$ $\frac{dy}{dx}=-pe^{1-px}$ $\frac{d^2y}{dx^2}=p^2e^{1-px}$ $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$ $p^2e^{1-px}--pe^{1-px}-2\left(e^{1-px}\right)=0$ $\left(p^2+p-2\right)e^{1-px}=0$ $p^2+p-2=0 \quad \left(e^{1-px}\neq 0\right)$ $(p+2)(p-1)=0$ $p=-2,1$	
Question 10 B $1839\div 12=153.25$ All 12 candidates could receive 153 votes each. $153\times 12=1836$ with 3 more members to vote. If they vote for 3 different candidates, there is no clear winner. $\therefore$ 154 is not enough, 155 is needed.	Answer C was a common error

SECTION II

Solutions	Marker's Comments
Question 11 (a) (i) $P(b) = b(b-a) - b(b-a)$ $= 0$ $\therefore (x-b)$ is a factor of $P(x)$.	You can use the factor's theorem or factorising in pair to show the factor. Generally well done.
(a) (ii) $$\begin{array}{r} x+(b-a) \\ (x-b)\overline{\big)\, x^2 - ax - b(b-a)} \\ \underline{x^2 - bx} \\ (b-a)x - b(b-a) \\ \underline{(b-a)x - b(b-a)} \\ 0 \end{array}$$ $\therefore$ The other factor is $(x+b-a)$.	You can do long division or use factorising in part a). Generally well done.
(b) $\int_{\sqrt{2}}^{\sqrt{6}} \frac{1}{2+x^2}\,dx = \frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{x}{\sqrt{2}}\right]_{\sqrt{2}}^{\sqrt{6}}$ $= \frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{\sqrt{6}}{\sqrt{2}} - \tan^{-1}\frac{\sqrt{2}}{\sqrt{2}}\right)$ $= \frac{1}{\sqrt{2}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}1\right)$ $= \frac{1}{\sqrt{2}}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ $= \frac{1}{\sqrt{2}} \times \frac{\pi}{12}$ $= \frac{\sqrt{2}}{24}\pi$	Some students didn't find the inverse trig as the integral.

Question 11 (c) $2\cos^2(3x)-1=\cos(6x)$ $2\cos^2(3x)=\frac{1}{2}(\cos(6x)+1)$ $A=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos^2(3x)\,dx$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{2}(\cos(6x)+1)\,dx$ $=\frac{1}{2}\left[\frac{\sin(6x)}{6}+x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\frac{1}{12}\left[\sin(6x)+6x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\frac{1}{12}\left(\sin\left(6\times\frac{\pi}{3}\right)+6\times\frac{\pi}{3}-\sin\left(6\times\frac{\pi}{6}\right)-6\times\frac{\pi}{6}\right)$ $=\frac{1}{12}(\sin 2\pi-\sin\pi+2\pi-\pi)$ $=\frac{1}{12}(0-0+\pi)$ $=\frac{\pi}{12}$	Some students didn't use the double angle formula, hence found the wrong integral.
(d) $x=u^2-1$ $\frac{dx}{du}=2u \therefore dx=2udu$ when $x=0, u=1 (u>0)$ when $x=3, u=2 (u>0)$ $\int_0^3 x\sqrt{x+1}\,dx=\int_1^2 (u^2-1)\sqrt{u^2}\,2udu$ $=2\int_1^2 (u^2-1)u^2du \quad (u>0)$ $=2\left[\frac{u^5}{5}-\frac{u^3}{3}\right]_1^2$ $=2\left(\frac{2^5}{5}-\frac{2^3}{3}-\frac{1^5}{5}+\frac{1^3}{3}\right)$ $=\frac{116}{15}$	**Some mistakes doing the substitution. To integrate in terms of u, the bourndaries need to be converted into the u values.**

Question 11 (e) For $n=0$, $5^{2n+1}+2^{2n+1}=5+2$ $=7$ which is divisible by 7. $\therefore$ Proven true for $n=0$. Assume true for $n=k$, i.e. $5^{2k+1}+2^{2k+1}$ is divisible by 7. i.e. $5^{2k+1}+2^{2k+1}=7P$ where $P\in\mathbb{R}$ Required to prove true for $n=k+1$, i.e. $5^{2(k+1)+1}+2^{2(k+1)+1}$ is divisible by 7. Proof: $5^{2(k+1)+1}+2^{2(k+1)+1}=5^{(2k+1)+2}+2^{(2k+1)+2}$ $=5^2\times5^{2k+1}+2^2\times2^{2k+1}$ $=25(7P-2^{2k+1})+4\times2^{2k+1}$ by assumption $=25\times7P-25\times2^{2k+1}+4\times2^{2k+1}$ $=25\times7P-21\times2^{2k+1}$ $=7(25P-3\times2^{2k+1})$ which is divisible by 7. $\therefore$ Proven true for $n=k+1$. If true for $n=k$, proven true for $n=k+1$. Since true for $n=0$, true for $n=0+1$, $n=1+1$,.... therefore, true for all integer $n(n\geq0)$.	Many missed the initial case of n=0. Some didn't show enough proof for n=k+1 by substitution.

Solutions	Marker's Comments
Question 12 (a) (i) $2\sqrt{x} = -2x + 4$ $4x = (-2x+4)^2$ $4x = 4x^2 - 16x + 16$ $4x^2 - 20x + 16 = 0$ $x^2 - 5x + 4 = 0$ $(x-4)(x-1) = 0$ $x = 1, 4$ sub into $y = -2x + 4$ $x = 1, y = -2(1) + 4 = 2$ $x = 4, y = -2(4) + 4 = -4$ since $y = 2\sqrt{x} > 0$, $\therefore (1,2)$ is the only point of intersection. **Alternatively, just show (1,2) satisfies both equations**	**1 mark** for a correct method with correct working. Most took the long path to answer this question rather than just sub (1,2) to show it's a solution.
(a) (ii) $y = 2\sqrt{x},\ x = \left(\frac{y}{2}\right)^2$ $y = -2x + 4,\ x = \frac{4-y}{2}$ $V = \pi\int_0^2 x^2 dy + \pi\int_2^4 x^2 dy$ $= \pi\int_0^2 \left(\frac{y}{2}\right)^4 dy + \pi\int_2^4 \left(\frac{4-y}{2}\right)^2 dy$ $= \frac{\pi}{16}\left[\frac{y^5}{5}\right]_0^2 + \frac{\pi}{4}\left[\frac{(4-y)^3}{-3}\right]_2^4$ $= \frac{\pi}{80}(32-0) - \frac{\pi}{12}(0-8)$ $= \frac{16\pi}{15}$	4 marks. This question was a rotation about the Y AXIS. Many did this about the x axis. When students did do it around the y axis the most common mistake was not using the correct boundaries for each part of the integral. Some students subtracted the integrals instead of adding them as well. Marks were awarded for change of subject to x bounds correct set up correct integrations correct substitution/solution
(b) (i) for $f(x) = \sec x$: Range: $y \geq 1$ for $y = f^{-1}(x)$: Domain: $x \geq 1$	Many failed to recognise there was a domain for the question and did not know what sec x looked like in this domain.

Solutions	**Marker's Comments**
Question 12 (b) (ii) $y = \sec x$ for $f^{-1}x$: $x = \sec y$ $= \frac{1}{\cos y}$ $\cos y = \frac{1}{x}$ $\therefore y = \cos^{-1}\left(\frac{1}{x}\right)$	1 mark Cosy=1/x was the crucial step in this question. Could have been done a lot better
(b) (iii) $\frac{dy}{dx} = \frac{-(-x^{-2})}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$ $= \frac{x^{-2}}{\sqrt{\frac{x^2-1}{x^2}}}$ $= \frac{1}{x^2 \frac{1}{\|x\|}\sqrt{x^2-1}}$ ***but*** $\|x\| = x$ ***for*** $x > 1$ $= \frac{1}{x\sqrt{x^2-1}}$	Generally well done question except for proper simplification by quite a few students. Some integrated the 1/x to get ln(x) rather than differentiating to get -1/x^2 in the chain rule. The use of the absolute of x was ignored in the simplification Marks Correct use of chain rule with inverse trig Correct simplification
(c) $\frac{{}^{10}C_3 \times {}^{8}C_2}{{}^{18}C_5} = \frac{120 \times 28}{8568}$ $= \frac{20}{51}$	Generally well done question. A few silly mistakes caused loss of marks here. Some used addition in the numerator rather than multiplication Marks Correct numerator Correct denominator
(d) (i) $\overrightarrow{OB} - \overrightarrow{OA} = -4\underset{\sim}{i} - \underset{\sim}{j} - (2\underset{\sim}{i} - 4\underset{\sim}{j})$ $= -6\underset{\sim}{i} + 3\underset{\sim}{j}$ $\sqrt{36+9} = \sqrt{45}$ $= 3\sqrt{5}$	Generally, well done. No issues here. Marks Correct k vector Correct magnitude of k vector

Solutions	Marker's Comments
Question 12 (d) (ii) $\tan\theta = \frac{3}{6}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ direction in bearing: $270 + \tan^{-1}\left(\frac{1}{2}\right) = 296.5650...^\circ$ $\approx 297^\circ T$	This on the whole was well done, however there were quite a few who attempted to find an angle between 2 vectors for some reason. This does not give a bearing that was required. Some found the correct angle of 27 degrees but misused it in finding the bearing. DRAW a diagram to help. Marks Correct angle Correct bearing
Question 13 (a) (i) $\frac{dt}{dP} = \frac{1}{k(P-6)}$ $dt = \frac{dP}{k(P-6)}$ $\int dt = \int \frac{dP}{k(P-6)}$ $t = \frac{1}{k}\ln\|P-6\| + C$ $t = 0, P = 10$ $0 = \frac{1}{k}\ln\|10-6\| + C$ $C = -\frac{1}{k}\ln 4$ $t = \frac{1}{k}\ln\|P-6\| - \frac{1}{k}\ln 4$ $kt = \ln\left(\frac{\|P-6\|}{4}\right)$ $\frac{P-6}{4} = e^{kt}$ $P = 4e^{kt} + 6$ OR	1 for integrating 1 for finding c 1 for correct rearrangement

$$\frac{dt}{dP} = \frac{1}{k(P-6)}$$

$$kdt = \frac{dP}{(P-6)}$$

$$\int kdt = \int \frac{1}{P-6} dP$$

$$kt = \ln|P-6| + C$$

$$when, t = 0, P = 10$$

$$0 = \ln|10-6| + C$$

$$C = -\ln 4$$

$$kt = \ln|P-6| - \frac{1}{k}\ln 4$$

$$kt = \ln\left(\frac{|P-6|}{4}\right)$$

$$\frac{P-6}{4} = e^{kt}$$

$$P = 4e^{kt} + 6$$

OR

$$\ln|P-6| = kt$$

$$P - 6 = e^{kt}$$

$$P - 6 = e^{kt} \times e^{c}$$

$$P - 6 = Ae^{kt} where A = \pm e^{c}$$

$$p = 6 + Ae^{kt}$$

$$When, t = 0, P = 10$$

$$10 = 6 + Ae^{0}$$

$$10 = 6 + A$$

$$A = 4$$

$$P = 6 + 4e^{kt}$$

Some students did not solve the differential equation $\frac{dP}{dt} = k(P-6)$ to show that $P = 4e^{kt} + 6$, rather, they verified that $P = 4e^{kt} + 6$ is a solution of the DE, which is easier but NOT what the question asked for, so only one mark was awarded.

Solutions	Marker's Comments
Question 13 (a) (ii) $t=4, P=16$ $16=4e^{4k}+6$ $10=4e^{4k}$ $e^{4k}=\frac{10}{4}=\frac{5}{2}$ $4k=\ln\left(\frac{5}{2}\right)$ $k=\frac{1}{4}\ln\left(\frac{5}{2}\right)$	1 for finding value of k
$P=4e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)t}+6=4e^{kt}+6$ $\frac{dP}{dt}=4\left(\frac{1}{4}\ln\left(\frac{5}{2}\right)\right)e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)t}=4ke^{kt}$ $=\ln\left(\frac{5}{2}\right)e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)t}$ The initial rate of change is: $t=0, \frac{dP}{dt}=\ln\left(\frac{5}{2}\right)=4k\cong 0.916$	Some students rounded off too early. Keep exact until final calculation.
$when, t=T, \frac{dP}{dt}=10\times\ln\left(\frac{5}{2}\right)=40k$ $\therefore findTwhen$ $40k=4ke^{kT}$ $10\ln\left(\frac{5}{2}\right)=\ln\left(\frac{5}{2}\right)e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)T}$ $10=e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)T}$ $\ln 10=\frac{1}{4}\ln\left(\frac{5}{2}\right)T$	1 for understanding initial rate and $\frac{dP}{dt}=40k$ when $t=T$ $\frac{dP}{dt}=10\times$ initial RATE of increase, NOT initial price 10x10=100. This was a common error and gave an answer of 13.78
$T=\frac{\ln 10}{\frac{1}{4}\ln\left(\frac{5}{2}\right)}$ $=10.051766...$ ≈ 10.05 months OR	1 for solving

$\frac{dP}{dt} = k(P-6)$ When $t = 0, P = 10$ $\frac{dP}{dt} = k(10-6)$ $\frac{dP}{dt} = 4k, initially$ Aim: to find T when $\frac{dP}{dt} = 10 \times 4k$ $\frac{dP}{dt} = 10 \times 4 \times \frac{1}{4} \ln \frac{5}{2}$ $\frac{dP}{dt} = 10 \ln \frac{5}{2}$ $10 \ln \frac{5}{2} = \frac{1}{4} \ln \frac{5}{2} (P-6)$ $40 = P - 6$ $P = 46$ Sub into $P = 4e^{kT} + 6$ $46 = 4e^{kT} + 6$ $40 = 4e^{kT}$ $e^{kT} = 10$ $kT = \ln 10$ $T = \ln 10 \div k$ $T = \ln 10 \div \frac{1}{4} \ln \frac{5}{2}$ $T \cong 10 \cdot 05176638...$ $T \cong 10.05 months$	
(b) (i) $\frac{dV}{dt} = -2, V = x^3,$ $\frac{dV}{dx} = 3x^2$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ $-2 = 3x^2 \times \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{-2}{3x^2}$	

Solutions	Marker's Comments
Question 13 (b) (ii) $\frac{dt}{dx}=\frac{3x^2}{-2}$ $-2dt=3x^2dx$ $\int -2dt=\int 3x^2dx$ $-2t=x^3+C$ $t=0, x=8$ $0=8^3+C$ $C=-512$ $-2t=x^3-512$ $x^3=512-2t$ $x=\sqrt[3]{512-2t}$	
(c) (i) $\frac{d}{dx}\left(\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}\right)=\frac{(4+x^2)\times 2-2x(2x)}{(4+x^2)^2}+\frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}$ $=\frac{8+2x^2-4x^2}{(4+x^2)^2}+\frac{2}{4+x^2}$ $=\frac{8-2x^2+2(4+x^2)}{(4+x^2)^2}$ $=\frac{8-2x^2+8+2x^2)}{(4+x^2)^2}$ $=\frac{16}{\left(4+x^2\right)^2}$	1 for <u>each</u> differentiation =2 1 for **showing** the required result
(c) (ii) $\int \frac{16}{\left(4+x^2\right)^2}dx=\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}+C$ $\int_0^{2\sqrt{3}}\frac{1}{\left(4+x^2\right)^2}dx=\frac{1}{16}\left[\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}\right]_0^{2\sqrt{3}}$ $=\frac{1}{16}\left(\frac{2(2\sqrt{3})}{4+(2\sqrt{3})^2}+\tan^{-1}\frac{2\sqrt{3}}{2}-\frac{2(0)}{4+(0)^2}-\tan^{-1}\frac{0}{2}\right)$ $=\frac{1}{16}\left(\frac{4\sqrt{3}}{16}+\frac{\pi}{3}-0-0\right)$ $=\frac{1}{16}\left(\frac{4\sqrt{3}}{16}+\frac{\pi}{3}\right)$ $=\frac{1}{16}\left(\frac{\sqrt{3}}{4}+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{64}+\frac{\pi}{48}=\frac{3\sqrt{3}+4\pi}{192}$	Some students missed the point of 'hence' ie. use the result from part (a) MUST be in radians. An answer of 3.78 comes from using degrees. Leave answer in exact form

Solutions	Marker's Comments
Question 14 (a) $f'(x)=\cot x+x$ $f(x)=\int\left(\frac{\cos x}{\sin x}+x\right)dx$ $=\int\left(\frac{\cos x}{\sin x}\right)dx+\int xdx$ $=\ln\|\sin x\|+\frac{x^2}{2}+C$ $f\left(\frac{\pi}{2}\right)=0$ $\ln\|\sin\frac{\pi}{2}\|+\frac{\left(\frac{\pi}{2}\right)^2}{2}+C=0$ $\ln 1+\frac{\pi^2}{8}+C=0$ $C=\frac{-\pi^2}{8}$ $f(x)=\ln\|\sin x\|+\frac{x^2}{2}-\frac{\pi^2}{8}$	Done well by most students. 37 got full marks, 9 got zero marks 1 for each integration Should have absolute value but no penalty was applied A few students incorrectly introduced a negative 1 for correct c and answer
(b) (i) $LHS=\cos 4x$ $=\cos x(2\times 2x)$ $=2\cos^2(2x)-1$ $=2(\cos 2x)^2-1$ $=2\left(2\cos^2 x-1\right)^2-1$ $=2\left(4\cos^4 x-4\cos^2 x+1\right)-1$ $=8\cos^4 x-8\cos^2 x+2-1$ $=8\left(\cos^4 x-\cos^2 x\right)+1$ $=RHS$	Various methods accepted

Solutions	Marker's Comments
Question 14 (b) (ii) $\cos 4x = 8\left(\cos^4 x - \cos^2 x\right) + 1$ $\frac{\cos 4x - 1}{8} = \cos^4 x - \cos^2 x$ $\cos^2 x - \cos^4 x = \frac{1}{16}$ $\frac{1 - \cos 4x}{8} = \frac{1}{16}$ $1 - \cos 4x = \frac{1}{2}$ $\cos 4x = \frac{1}{2}, 0 \le x \le 2\pi$ $4x = \frac{\pi}{3}, \frac{5\pi}{3}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}$	NOT $1 - \frac{\cos 4x}{8} = \frac{1}{16}$ common error 1 mark 1 mark
(c) $a = 1, r = \frac{2x}{x+1}$ The series has a limiting sum when $\lvert r \rvert < 1$ or $-1 < r < 1$ $\left\lvert \frac{2x}{x+1} \right\rvert < 1$ $\lvert 2x \rvert < \lvert x+1 \rvert$ $4x^2 < (x+1)^2$ $3x^2 - 2x - 1 < 0$ $(3x+1)(x-1) < 0$ $-\frac{1}{3} < x < 1, x \ne 0$ **OR** $-1 < \frac{2x}{x+1} < 1$ $-1 < \frac{2x}{x+1}$ *and* $\frac{2x}{x+1} < 1$ $-\left(x+1\right)^2 < \frac{2x}{x+1}\left(x+1\right)^2$ $-x^2 - 2x - 1 < 2x(x+1)$ $0 < 3x^2 + 4x + 1$ $(3x+1)(x+1) > 0$ $x < -1, x > \frac{-1}{3}$	Poorly done Many students tried to find the limiting sum rather than finding the values of x where a limiting sum will exist. Over 30 students did not even state the initial condition $\left\lvert \frac{2x}{x+1} \right\rvert < 1$ Various methods of solving the inequality were accepted

Solutions	Marker's Comments
Question 14 (d) (i) $\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{CD}$ $=2\underset{\sim}{a}+\overrightarrow{CD}$ $=2\underset{\sim}{a}+\lambda\overrightarrow{CM}$ $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$ $\underset{\sim}{b}=\underset{\sim}{a}+2\overrightarrow{AM}$ $\overrightarrow{AM}=\dfrac{\underset{\sim}{b}-\underset{\sim}{a}}{2}$ $\overrightarrow{CM}=\overrightarrow{AM}-\overrightarrow{AC}$ $\overrightarrow{CM}=\overrightarrow{AM}-\underset{\sim}{a}$ $=\dfrac{\underset{\sim}{b}-\underset{\sim}{a}}{2}-\underset{\sim}{a}$ $=\dfrac{\underset{\sim}{b}}{2}-\dfrac{3}{2}\underset{\sim}{a}$ $\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{CD}$ $\overrightarrow{OD}=2\underset{\sim}{a}+\lambda\overrightarrow{CM}$ $=2\underset{\sim}{a}+\lambda\left(\dfrac{\underset{\sim}{b}}{2}-\dfrac{3}{2}\underset{\sim}{a}\right)$ $=\left(2-\tfrac{3}{2}\lambda\right)\underset{\sim}{a}+\tfrac{1}{2}\lambda\underset{\sim}{b}.$	Reasonably well done Students need to ensure that they give all working for 'show that' questions

(d) (ii) $\overrightarrow{OD} = \mu \underset{\sim}{b}$ $\overrightarrow{DB} = \underset{\sim}{b} - \mu \underset{\sim}{b}$ $\quad = (1-\mu)\underset{\sim}{b}$ $\overrightarrow{OD} + \overrightarrow{DB} = \underset{\sim}{b}$ $\left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda\underset{\sim}{b} + (1-\mu)\underset{\sim}{b} = \underset{\sim}{b}$ $\left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} = \underset{\sim}{b} - \frac{1}{2}\lambda\underset{\sim}{b} - (1-\mu)\underset{\sim}{b}$ $\left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} = \left(\mu - \frac{1}{2}\lambda\right)\underset{\sim}{b}$ since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are not parallel or overlapping, for $\underset{\sim}{a} = \underset{\sim}{b}$, $\left(2 - \frac{3}{2}\lambda\right) = 0$ and $\left(\mu - \frac{1}{2}\lambda\right) = 0$ $2 - \frac{3}{2}\lambda = 0, \lambda = \frac{4}{3}$ sub $\lambda = \frac{4}{3}$ into $\left(\mu - \frac{1}{2}\lambda\right) = 0$ $\mu - \frac{1}{2} \times \frac{4}{3} = 0$ $\mu = \frac{2}{3}$ $\overrightarrow{OD} = \frac{2}{3}\underset{\sim}{b}$ $\overrightarrow{OD} = \frac{2}{3}\overrightarrow{OB}$ $\frac{\overrightarrow{OD}}{\overrightarrow{OB}} = \frac{2}{3}$ $\frac{\overrightarrow{OD}}{\overrightarrow{OD} + \overrightarrow{DB}} = \frac{2}{3} = \frac{2}{2+1}$ $\therefore \overrightarrow{OD} : \overrightarrow{DB} = 2:1$	Poorly done Only 9 students got full marks for Q14d OR $\overrightarrow{OD} = \left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda\underset{\sim}{b}$ from part (i) $\overrightarrow{OD} = \mu.\overrightarrow{OB} = \mu.\underset{\sim}{b}$ from part (ii) So $\left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda\underset{\sim}{b} = 0\underset{\sim}{a} + \mu\underset{\sim}{b}$ $\left(2 - \frac{3}{2}\lambda\right) = 0$ and $\frac{1}{2}\lambda = \mu$ 1 for two equations linking λ and μ 1 for showing result