HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate

Trial Examination Term 3 2022

STUDENT NUMBER: ____

STUDENT NAME: ______________________ **TEACHER NAME:** ______________________

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–12)

- Attempt Questions 11–14
- Start each question in a new writing booklet
- Write your student number on every writing booklet
- Allow about 1 hour and 45 minutes for this section

Question	*1-10*	*11*	*12*	*13*	*14*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

Question 1

What is the angle between the vectors $\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 5\underset{\sim}{i} - \underset{\sim}{j}$?

(A) $58 \cdot 909^\circ$

(B) $60 \cdot 255^\circ$

(C) $82 \cdot 582^\circ$

(D) $82 \cdot 875^\circ$

Question 2

The equation $x^3 + 2x^2 - 3x + 6 = 0$ has roots α, β and γ .
What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{3}$

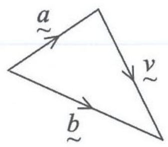
(C) $\frac{1}{3}$

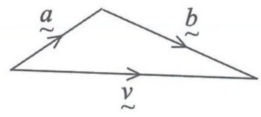
(D) $\frac{1}{2}$

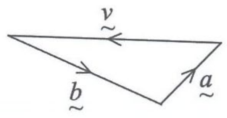
Question 3

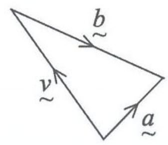
The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown.

Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{b} - \underset{\sim}{a}$

A. 

B. 

C. 

D. 

Question 4

$\sqrt{3}\cos 2\theta - \sin 2\theta$ in the form $R\cos(2\theta + \alpha)$ where R is positive and α is in radians is

(A) $\frac{1}{2}\cos\left(2\theta + \frac{\pi}{6}\right)$

(B) $\frac{1}{2}\cos\left(2\theta + \frac{\pi}{3}\right)$

(C) $2\cos\left(2\theta + \frac{\pi}{6}\right)$

(D) $2\cos\left(2\theta + \frac{\pi}{3}\right)$

Question 5

$\sin(3x + x) - \sin(3x - x) =$

(A) $-2\sin 3x \sin x$

(B) $2\cos 3x \sin x$

(C) $2\cos 3x \cos x$

(D) $2\sin 3x \sin x$

Question 6

The vectors $\vec{p}=\begin{pmatrix}4\\a+1\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}a\\-2\end{pmatrix}$ are perpendicular. What is the value of a ?

(A) -1

(B) 1

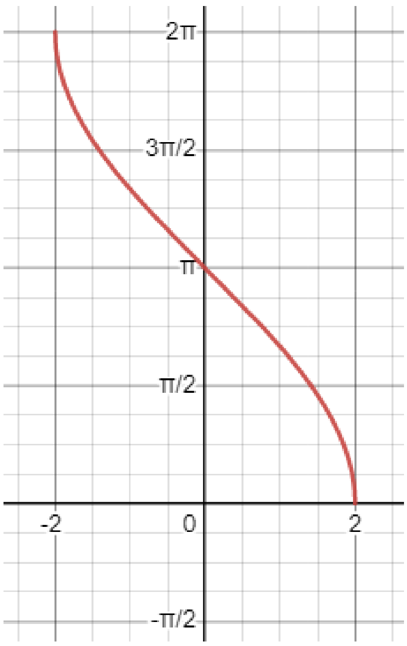
(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

Question 7

The function shown in the diagram below has equation $y = A\cos^{-1} Bx$.

Which of the following is true?



(A) $A=\frac{1}{2},\ B=\frac{1}{2}$

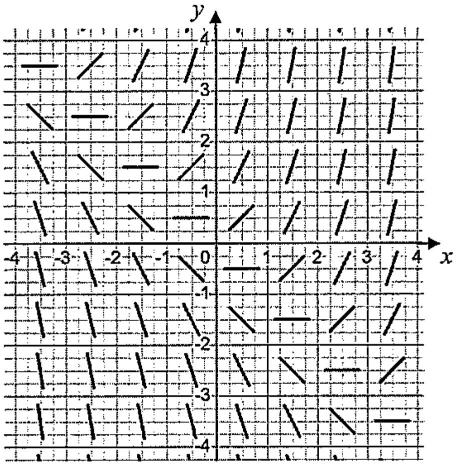
(B) $A=\frac{1}{2},\ B=2$

(C) $A=2,\ B=\frac{1}{2}$

(D) $A=2,\ B=2$

Question 8

Which differential equation is represented by the following slope field?



(A) $\frac{dy}{dx}+x+y=0$

(B) $\frac{dy}{dx}-x-y=0$

(C) $\frac{dy}{dx}+x-y=0$

(D) $\frac{dy}{dx}-x+y=0$

Question 9

What are the values of p for which $y=e^{1-px}$ satisfies the equation $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$?

(A) $p=1,\ p=2$

(B) $p=-1,\ p=2$

(C) $p=1,\ p=-2$

(D) $p=-1,\ p=-2$

Question 10

The members of a club votes for a new president. There were 12 candidates for the position of president and 1839 members voted. Each member voted for one candidate only.

One candidate received more votes than any other candidate and so became the new president.

What is the smallest number of votes the new president could have received?

(A) 156

(B) 155

(C) 154

(D) 153

Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section
Instructions

- Answer the questions in the appropriate writing booklet.
- In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) Consider the polynomial $P(x)=x(x-a)-b(b-a)$.

(i) Show that $(x-b)$ is a factor of $P(x)$. **1**

(ii) By division or otherwise, find the other factor. **2**

(b) Evaluate and find the exact value of $\int_{\sqrt{2}}^{\sqrt{6}} \frac{1}{2+x^2}\,dx$. **3**

(c) Find the area bounded by the $y=\cos^2(3x)$, the x-axis and the ordinates at $x=\frac{\pi}{6}$ and $x=\frac{\pi}{3}$. **3**

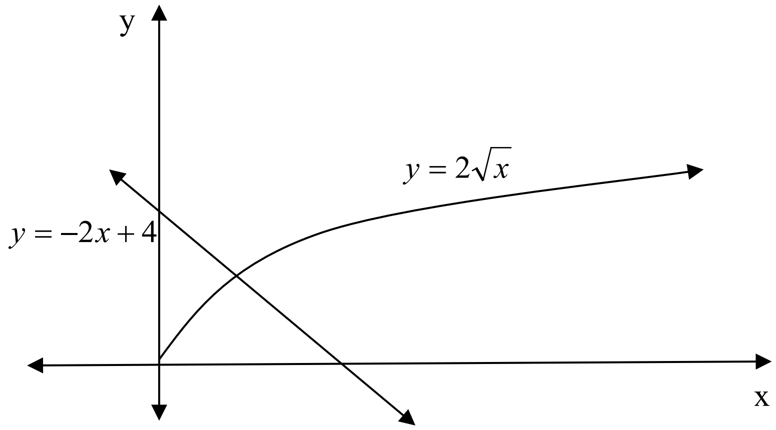
(d) Use the substitution $x=u^2-1 \quad (u>0)$ to evaluate $\int_0^3 x\sqrt{x+1}\,dx$. **3**

(e) Prove, by Mathematical Induction, that $5^{2n+1}+2^{2n+1}$ is divisible by 7 for all integers $n \geq 0$. **3**

End of Question 11

Question 12 (15 marks) Start a new writing booklet.

(a) The area enclosed by the curve $y = 2\sqrt{x}$, the line $y = -2x + 4$, and the y axis is revolved about the y axis.



(i) Show $y = 2\sqrt{x}$ and $y = -2x + 4$ intersect at the coordinate $(1, 2)$. **1**

(ii) Find the volume of the solid of revolution about the y axis. **4**

(b) Consider the function $f(x) = \sec x$ over the domain $0 \le x < \dfrac{\pi}{2}$

(i) State the domain of $f^{-1}(x)$ **1**

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right)$ **1**

(iii) Hence find $\dfrac{d}{dx} f^{-1}(x)$ **2**

(c) A class consists of 10 girls and 8 boys. What is the probability of selecting a committee of 5, that contains 3 girls and 2 boys. **2**

(d) If $\overrightarrow{OA} = 2\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\overrightarrow{OB} = -4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{k} = \overrightarrow{OB} - \overrightarrow{OA}$ find the

(i) magnitude of $\underset{\sim}{k}$. **2**

(ii) direction of $\underset{\sim}{k}$ as a bearing, to the nearest degree. **2**

End of Question 12

Question 13 (15 marks) Start a new writing booklet.

(a) The rate of change of the share price $\$P$ of company $\boldsymbol{X}$ over a twelve month period can be modelled by the differential equation $\frac{dP}{dt} = k\left(P-6\right)$ where k is a constant and t is the time in months.

The price increases from \$10 per share to \$16 per share over 4 months.

(i) **By solving the differential equation** show that $P = 6 + 4e^{kt}$. **3**

(ii) After T months the share price is increasing at ten times the initial rate of increase. Find T correct to two decimal places. **3**

(b) A cubic block of ice of side length $8\ \text{cm}$ is melting at a constant rate of $2\ \text{cm}^3/\text{min}$.
After time t minutes the cubic block of ice has edge length x cm and volume V cm^3.

(i) Show that $\frac{dx}{dt} = \frac{-2}{3x^2}$ **1**

(ii) Hence find an expression for x as a function of t. **3**

(c) (i) Show that $\frac{d}{dx}\left(\frac{2x}{4+x^2} + \tan^{-1}\frac{x}{2}\right) = \frac{16}{\left(4+x^2\right)^2}$ **3**

(ii) Hence, evaluate $\int_0^{2\sqrt{3}} \frac{dx}{\left(4+x^2\right)^2}$ **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet.

(a) If $f'(x) = \cot x + x$ and $f\left(\frac{\pi}{2}\right) = 0$, find an expression for $f(x)$. **3**

(b)

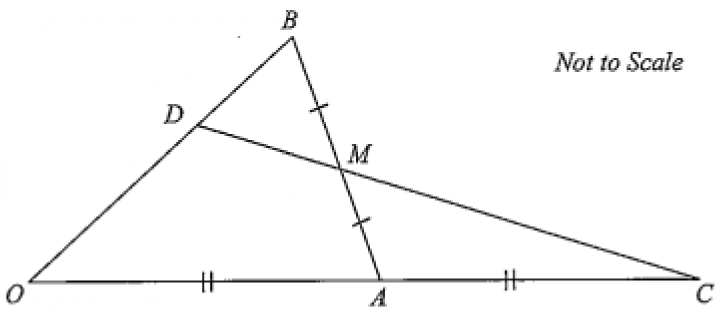
(i) Show that $\cos 4x = 8\left(\cos^4 x - \cos^2 x\right) + 1$. **2**

(ii) Hence, or otherwise, solve $\cos^2 x - \cos^4 x = \frac{1}{16}$ for $0 \le x \le \frac{\pi}{2}$. **2**

(c) For what values of x $(x \ne 0)$ does the geometric series

$1 + \frac{2x}{x+1} + \left(\frac{2x}{x+1}\right)^2 + \ldots$ have a limiting sum? **3**

(d)



In the diagram, side OA of ΔOAB is produced to C so that $AC = OA$.

M is the midpoint of AB and CM produced meets OB in D.

Given $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{CD} = \lambda \overrightarrow{CM}$ for some constant λ.

(i) Show that $\overrightarrow{OD} = \left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda \underset{\sim}{b}$. **3**

(ii) Hence show that $OD : DB = 2:1$ using $\overrightarrow{OD} = \mu \overrightarrow{OB}$ for some constant μ. **2**

End of paper