NSW GOVERNMENT

NSW Education Standards Authority

2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–36)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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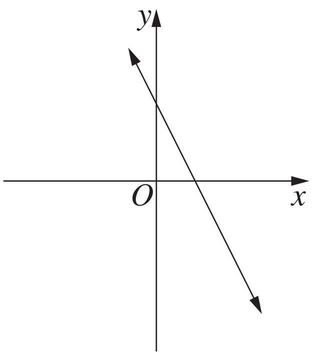
1 Which of the following could be the graph of $y = -2x + 2$?

A.



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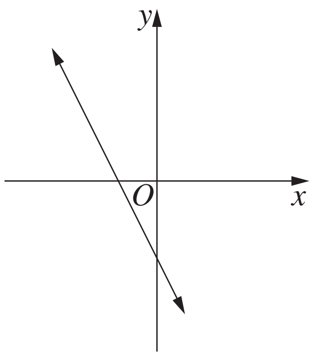

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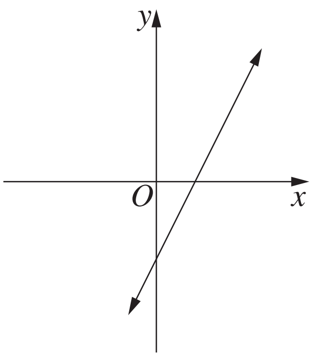

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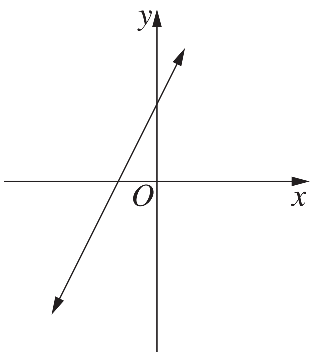


2 Consider the following dataset.

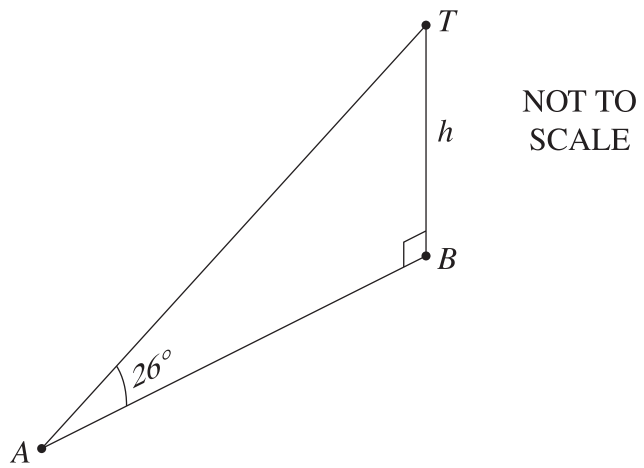
13 16 17 17 21 24

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

	Median	*Mean*
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

3 A tower *BT* has height *h* metres.

From point *A*, the angle of elevation to the top of the tower is 26° as shown.



Which of the following is the correct expression for the length of *AB*?

A. $h \tan 26°$

B. $h \cot 26°$

C. $h \sin 26°$

D. $h \operatorname{cosec} 26°$

4 Which of the following is the range of the function $f(x) = x^2 - 1$?

A. $[-1, \infty)$

B. $(-\infty, 1]$

C. $[-1, 1]$

D. $(-\infty, \infty)$

5 Let $h(x) = \dfrac{f(x)}{g(x)}$, where

$f(1) = 2 \qquad f'(1) = 4$

$g(1) = 8 \qquad g'(1) = 12.$

What is the gradient of the tangent to the graph of $y = h(x)$ at $x = 1$?

A. -8

B. 8

C. $-\dfrac{1}{8}$

D. $\dfrac{1}{8}$

6 What is $\displaystyle\int \frac{1}{(2x+1)^2}\,dx$?

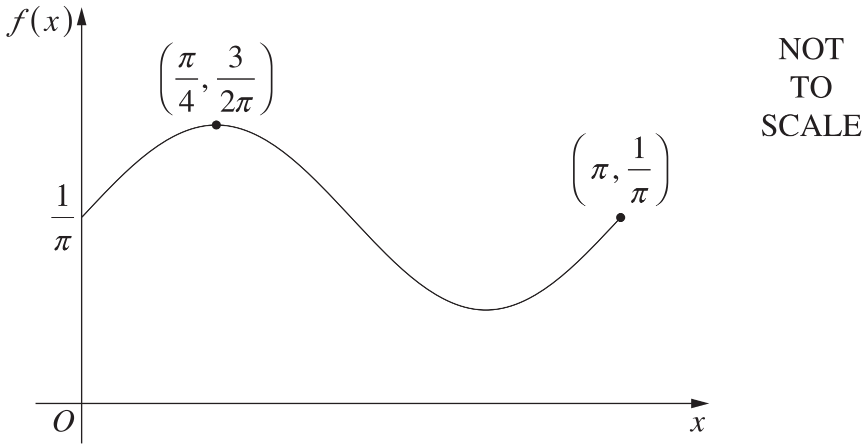
A. $\dfrac{-2}{2x+1} + C$

B. $\dfrac{-1}{2(2x+1)} + C$

C. $2\ln(2x+1) + C$

D. $\dfrac{1}{2}\ln(2x+1) + C$

7 Consider the following graph of a probability density function $f(x)$.



What is the value of the mode?

A. $\frac{1}{\pi}$

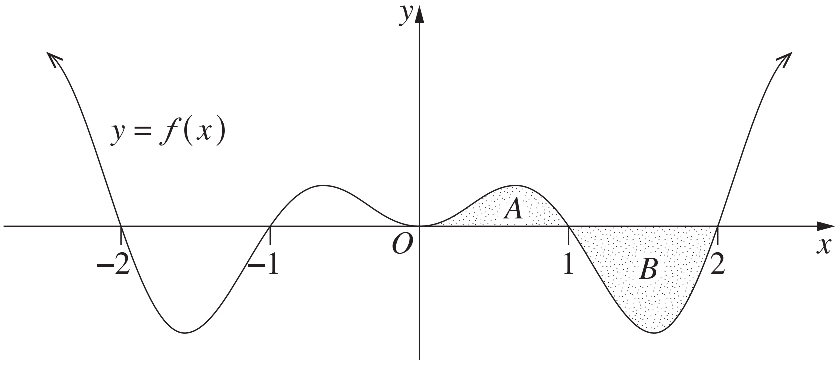
B. $\frac{3}{2\pi}$

C. $\frac{\pi}{4}$

D. π

8 The graph of the even function $y = f(x)$ is shown.

The area of the shaded region A is $\frac{1}{2}$ and the area of the shaded region B is $\frac{3}{2}$.



What is the value of $\int_{-2}^{2} f(x)\,dx$?

A. 4

B. 2

C. −2

D. −4

9 Liam is playing two games. He is equally likely to win each game. The probability that Liam will win at least one of the games is 80%.

Which of the following is closest to the probability that Liam will win both games?

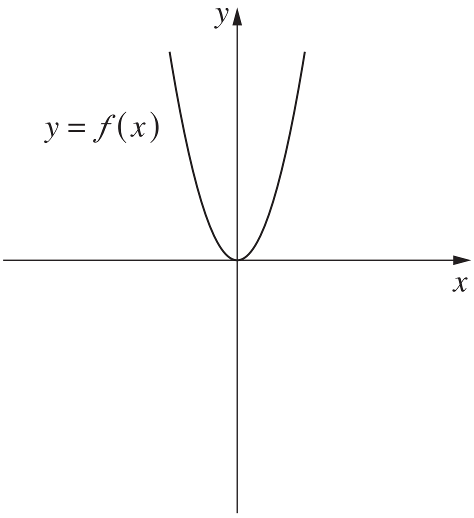
A. 31%

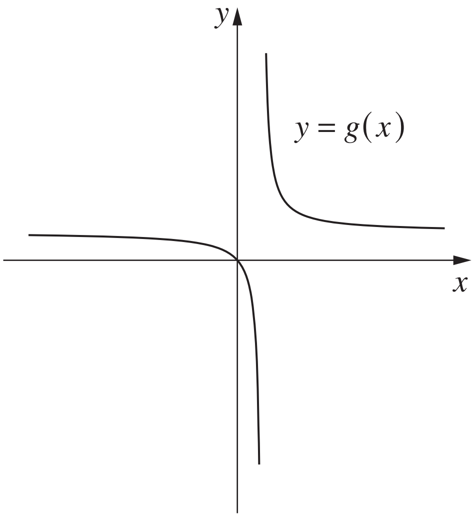
B. 40%

C. 55%

D. 64%

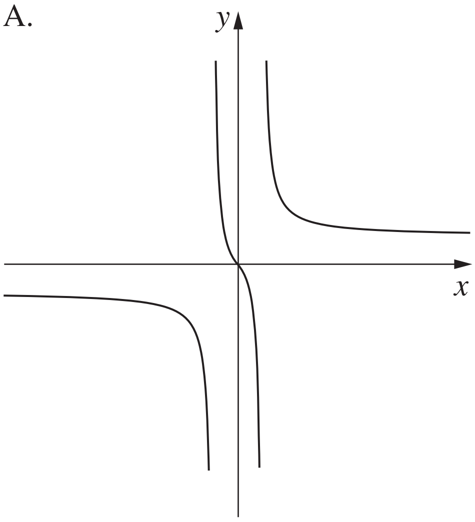
10 The graphs of $y = f(x)$ and $y = g(x)$ are shown.





Which graph best represents $y = g(f(x))$?

A.



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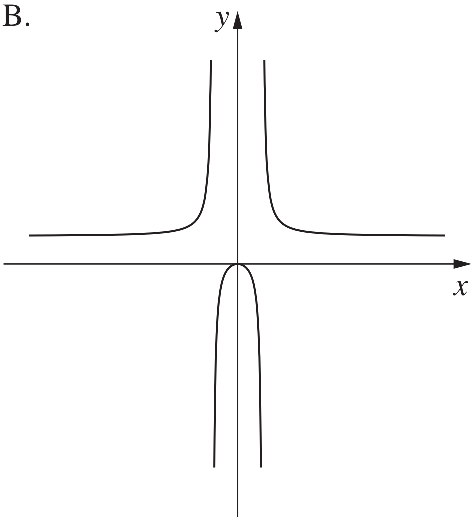

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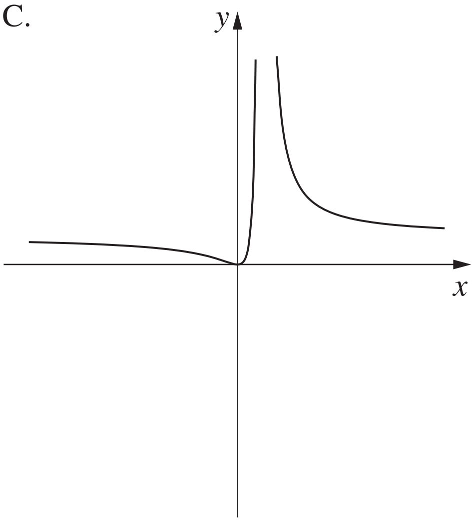

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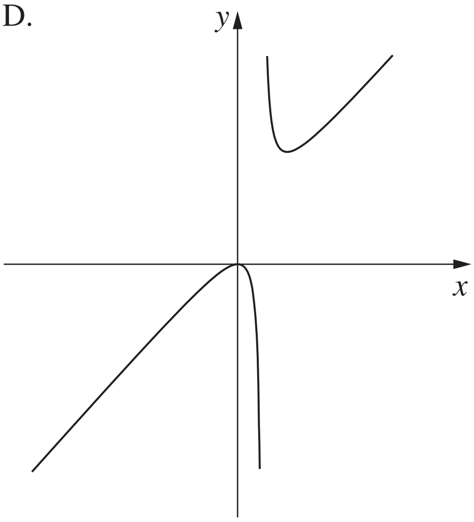

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2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (3 marks)

The table shows the types of customer complaints received by an online business in a month.

Type of complaint	*Frequency*	*Cumulative frequency*	*Cumulative percentage*
Stock shortage	98	98	49
Delivery fee	62	***A***	80
Delivery time	24	184	92
Damaged item	8	192	***B***
Returns policy	6	198	99
Product information	2	200	100
Total	200		

(a) What are the values of ***A*** and ***B***? **2**

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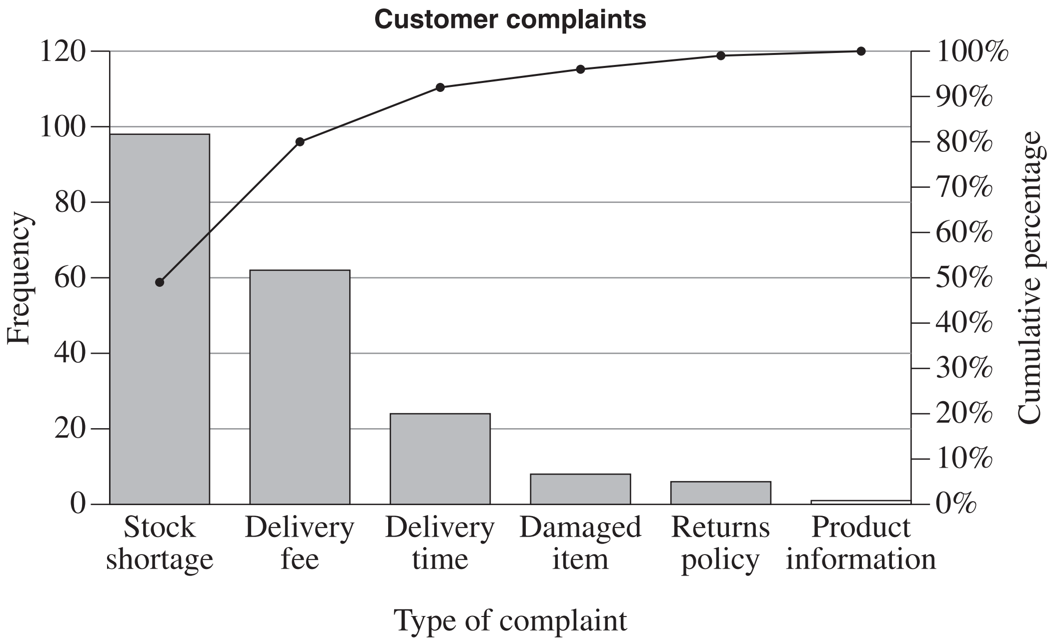
(b) The data from the table are shown in the following Pareto chart. **1**



The manager will address 80% of the complaints.

Which types of complaints will the manager address?

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Question 12 (4 marks)

A student believes that the time it takes for an ice cube to melt (M minutes) varies inversely with the room temperature (T°C). The student observes that at a room temperature of 15°C it takes 12 minutes for an ice cube to melt.

(a) Find the equation relating M and T. **2**

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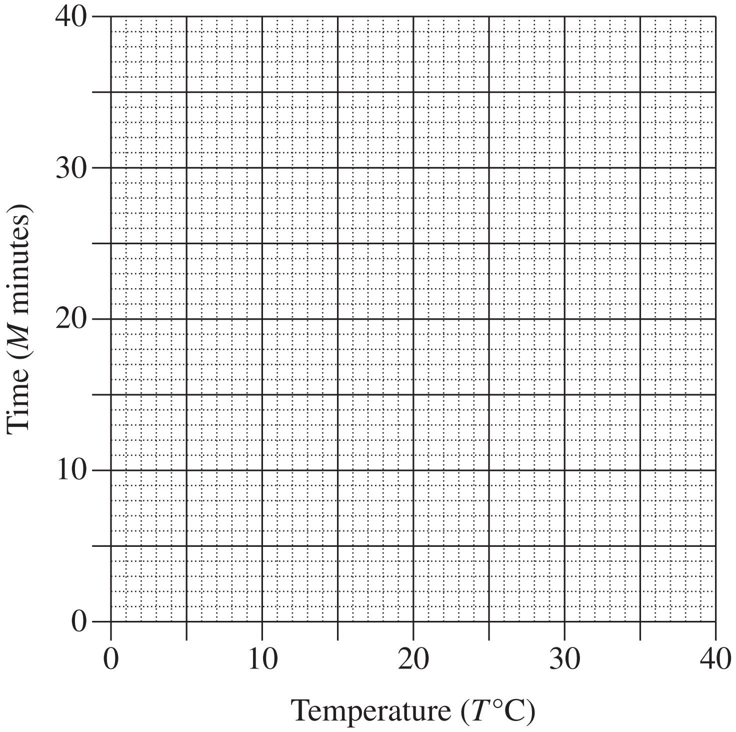
(b) By first completing this table of values, graph the relationship between temperature and time from $T = 5°C$ to $T = 30°C$. **2**

T	5	15	30
M			



Question 13 (2 marks)

Use two applications of the trapezoidal rule to find an approximate value of $\int_0^2 \sqrt{1+x^2}\,dx$. Give your answer correct to 2 decimal places. **2**

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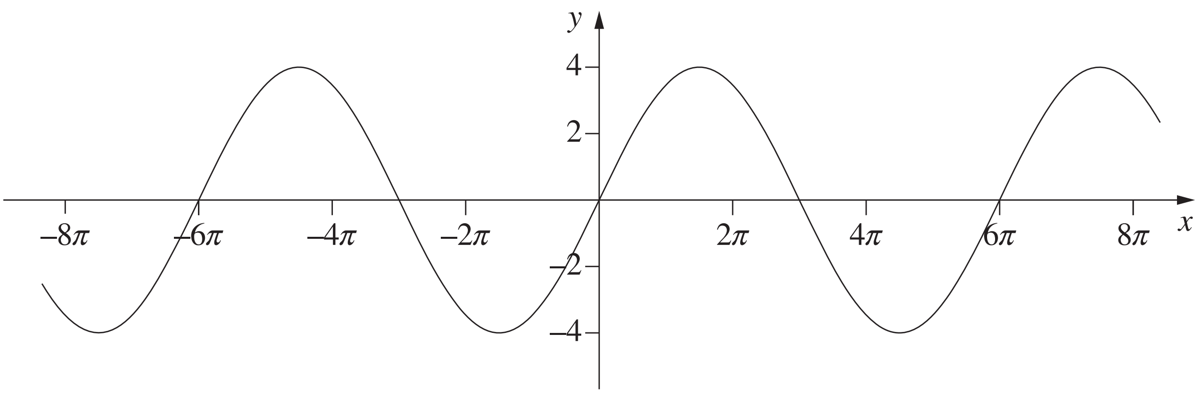
Question 14 (2 marks)

The graph of $y = k\sin(ax)$ is shown. **2**



What are the values of k and a?

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Question 15 (2 marks)

In a bag there are 3 six-sided dice. Two of the dice have faces marked 1, 2, 3, 4, 5, 6. The other is a special die with faces marked 1, 2, 3, 5, 5, 5.

One die is randomly selected and tossed.

(a) What is the probability that the die shows a 5? **1**

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(b) Given that the die shows a 5, what is the probability that it is the special die? **1**

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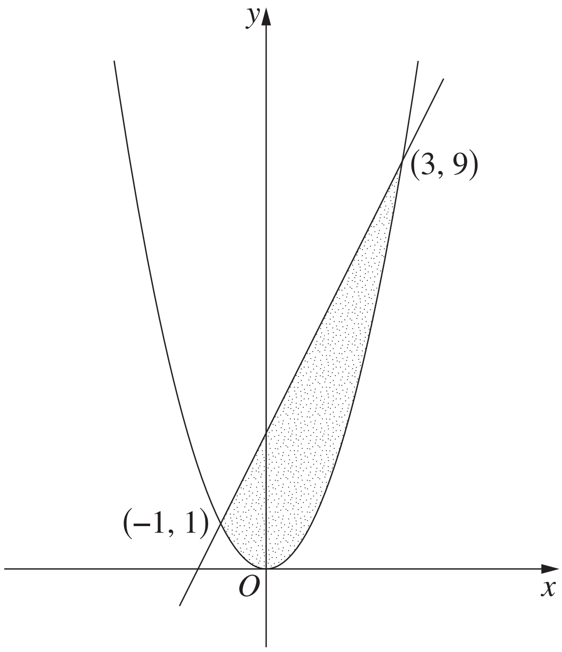
Please turn over

Question 16 (3 marks)

The parabola $y = x^2$ meets the line $y = 2x + 3$ at the points $(-1, 1)$ and $(3, 9)$ as shown in the diagram. **3**



Find the area enclosed by the parabola and the line.

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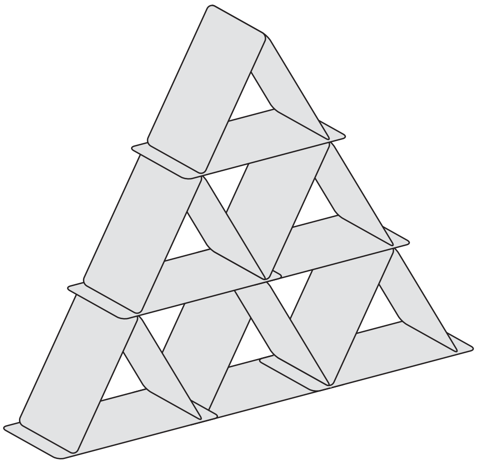
Question 17 (5 marks)

Cards are stacked to build a 'house of cards'. A house of cards with 3 rows is shown.

A house of cards requires 3 cards in the top row, 6 cards in the next row, and each successive row has 3 more cards than the previous row.



(a) Show that a house of cards with 12 rows has a total of 234 cards. **2**

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(b) Another house of cards has a total of 828 cards. **3**

How many rows are in this house of cards?

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Question 18 (3 marks)

(a) Differentiate $y = \left(x^2 + 1\right)^4$. **2**

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(b) Hence, or otherwise, find $\int x\left(x^2 + 1\right)^3 dx$. **1**

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Question 19 (3 marks)

The graph of the function $f(x) = x^2$ is translated m units to the right, dilated vertically by a scale factor of k and then translated 5 units down. The equation of the transformed function is $g(x) = 3x^2 - 12x + 7$. **3**

Find the values of m and k.

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Question 20 (4 marks)

A scientist is studying the growth of bacteria. The scientist models the number of bacteria, N, by the equation

$$N(t) = 200e^{0.013t},$$

where t is the number of hours after starting the experiment.

(a) What is the initial number of bacteria in the experiment? **1**

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(b) What is the number of bacteria 24 hours after starting the experiment? **1**

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(c) What is the rate of increase in the number of bacteria 24 hours after starting the experiment? **2**

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Question 21 (4 marks)

Eli is choosing between two investment options.

Option 1: Depositing a single amount of \$40 000 today, earning interest of 1.2% per annum, compounded monthly.

Option 2: Depositing \$1000 at the end of each quarter, earning interest of 2.4% per annum, compounded quarterly.

A table of future value interest factors for an annuity of \$1 is shown.

r / N	*Interest rate per period as a decimal*					
	0.002	0.006	0.020	0.024	0.060	0.240
10	10.09048	10.27437	10.94972	11.15211	13.18079	31.64344
20	20.38460	21.18211	24.29737	25.28909	36.78559	303.60062
30	30.88646	32.76227	40.56808	43.20983	79.05819	2640.91639
40	41.60026	45.05630	60.40198	65.92708	154.76197	22 728.80260

(a) What is the value of Eli's investment after 10 years using Option 1? **2**

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(b) What is the difference between the future values after 10 years using Option 1 and Option 2? **2**

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Question 22 (4 marks)

Find the global maximum and minimum values of $y = x^3 - 6x^2 + 8,$ where $-1 \leq x \leq 7.$ **4**

Please turn over

Question 23 (6 marks)

The depth of water in a bay rises and falls with the tide. On a particular day the depth of the water, d metres, can be modelled by the equation

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right),$$

where t is the time in hours since low tide.

(a) Find the depth of water at low tide and at high tide. **2**

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(b) What is the time interval, in hours, between two successive low tides? **1**

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(c) For how long between successive low tides will the depth of water be at least 1 metre? **3**

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Questions 11–23 are worth 45 marks in total

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Question 24 (4 marks)

Jo is researching the relationship between the ages of teenage characters in television series and the ages of actors playing these characters. **4**

After collecting the data, Jo finds that the correlation coefficient is 0.4564.

A scatterplot showing the data is drawn. The line of best fit with equation $y = -7.51 + 1.85x$, is also drawn.

Describe and interpret the data and other information provided, with reference to the context given.

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Question 25 (3 marks)

Let $f(x) = \sin(2x)$. **3**

Find the value of x, for $0 < x < \pi$, for which $f'(x) = -\sqrt{3}$ AND $f''(x) = 2$.

Question 26 (3 marks)

The life span of batteries from a particular factory is normally distributed with a mean of 840 hours and a standard deviation of 80 hours. **3**

It is known from statistical tables that for this distribution approximately 60% of the batteries have a life span of less than 860 hours.

What is the approximate percentage of batteries with a life span between 820 and 920 hours?

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Please turn over

Question 27 (7 marks)

Let $f(x) = xe^{-2x}$.

It is given that $f'(x) = e^{-2x} - 2xe^{-2x}$.

(a) Show that $f''(x) = 4(x-1)e^{-2x}$. **2**

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(b) Find any stationary points of $f(x)$ and determine their nature. **2**

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Question 27 continues on page 25

Question 27 (continued)

(c) Sketch the curve $y = xe^{-2x}$, showing any stationary points, points of inflection and intercepts with the axes. **3**

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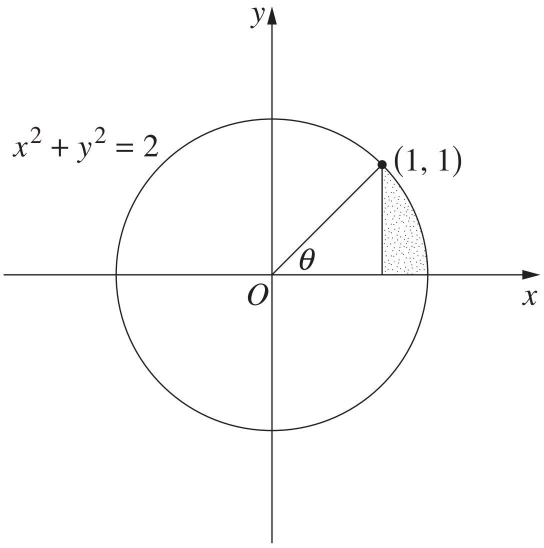
End of Question 27

Question 28 (7 marks)

The graph of the circle $x^2 + y^2 = 2$ is shown.

The interval connecting the origin, O, and the point $(1, 1)$ makes an angle θ with the positive x-axis.



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(a) By considering the value of θ, find the exact area of the shaded region, as shown on the diagram. **2**

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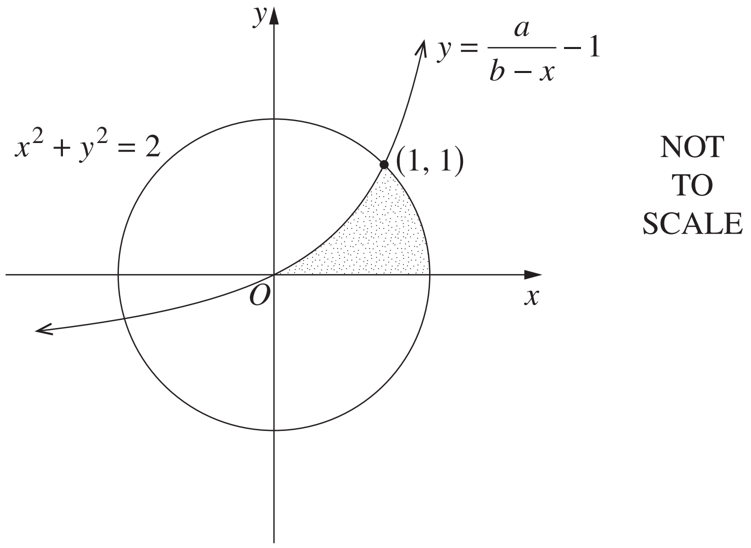
Question 28 continues on page 27

Question 28 (continued)

Part of the hyperbola $y = \dfrac{a}{b - x} - 1$ which passes through the points $(0, 0)$ and $(1, 1)$ is drawn with the circle $x^2 + y^2 = 2$ as shown.



(b) Show that $a = b = 2$. **2**

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(c) Using parts (a) and (b), find the exact area of the region bounded by the hyperbola, the positive x-axis and the circle as shown on the diagram. **3**

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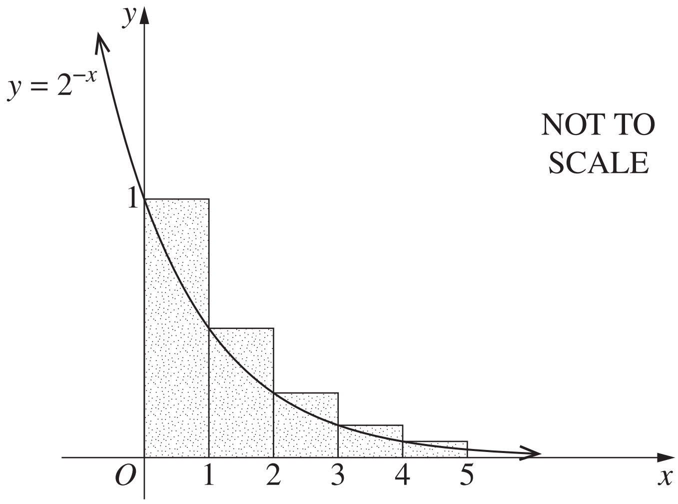
End of Question 28

Question 29 (5 marks)

(a) The diagram shows the graph of $y = 2^{-x}$. Also shown on the diagram are the first 5 of an infinite number of rectangular strips of width 1 unit and height $y = 2^{-x}$ for non-negative integer values of x. For example, the second rectangle shown has width 1 and height $\frac{1}{2}$. **1**



The sum of the areas of the rectangles forms a geometric series.

Show that the limiting sum of this series is 2.

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(b) Show that $\int_0^4 2^{-x}\,dx = \frac{15}{16\ln 2}$. **2**

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Question 29 continues on page 29

Question 29 (continued)

(c) Use parts (a) and (b) to show that $e^{15} < 2^{32}$. **2**

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Question 30 (3 marks)

A continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 1 & x > e^3 \\ \frac{1}{k}\ln x & 1 \le x \le e^3 \\ 0 & x < 1 \end{cases}.$$

(a) Show that $k = 3$. **1**

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(b) Given that $P(X < c) = 2P(X > c)$, find the exact value of c. **2**

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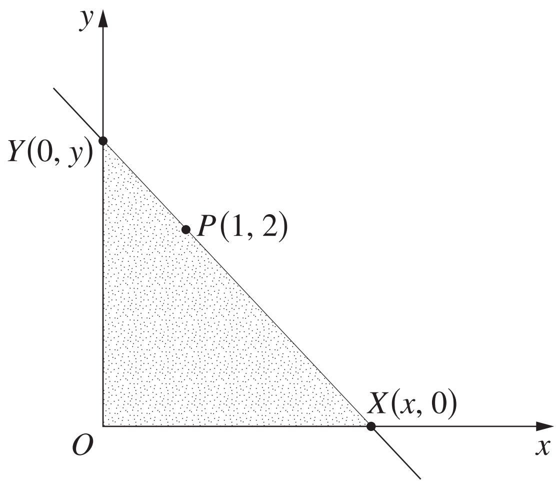
Question 31 (6 marks)

A line passes through the point $P(1, 2)$ and meets the axes at $X(x, 0)$ and $Y(0, y)$, where $x > 1$.



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(a) Show that $y = \dfrac{2x}{x-1}$. **2**

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Question 31 continues on page 31

Question 31 (continued)

(b) Find the minimum value of the area of triangle XOY. **4**

End of Question 31

Question 32 (7 marks)

In a reducing-balance loan, an amount $\$P$ is borrowed for a period of n months at an interest rate of 0.25% per month, compounded monthly. At the end of each month, a repayment of $\$M$ is made. After the nth repayment has been made, the amount owing, $\$A_n$, is given by

$$A_n = P(1.0025)^n - M\left(1 + (1.0025)^1 + (1.0025)^2 + \cdots + (1.0025)^{n-1}\right).$$

(Do NOT prove this.)

(a) Jane borrows \$200 000 in a reducing-balance loan as described. **2**

The loan is to be repaid in 180 monthly repayments.

Show that $M = 1381.16$, when rounded to the nearest cent.

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Question 32 continues on page 33

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Question 32 (continued)

(b) After 100 repayments of \$1381.16 have been made, the interest rate changes to 0.35% per month. **3**

At this stage, the amount owing to the nearest dollar is \$100 032. (Do NOT prove this.)

Jane continues to make the same monthly repayments.

For how many more months will Jane need to make full monthly payments of \$1381.16?

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(c) The final repayment will be less than \$1381.16. **2**

How much will Jane need to pay in the final payment in order to pay off the loan?

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End of paper

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NSW Education Standards Authority

2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

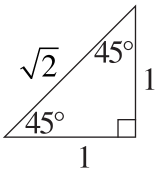
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

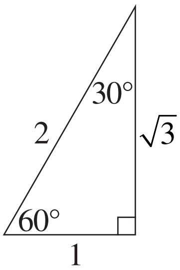
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

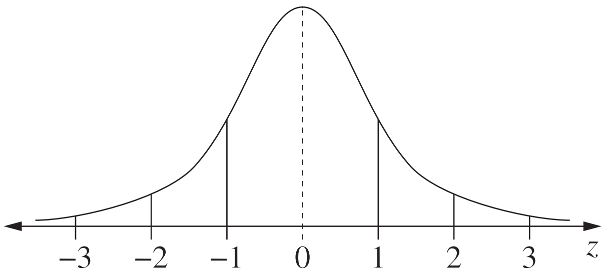
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^ne^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$