NESA Number:_____
SW MA RC MN SE GS

Ascham School
Mathematics Advanced
2021 Trial Examination

General Instructions:

Reading time – 10 minutes
Working time – 3 hours
Write using black pen.
Calculators approved by NESA may be used.
A reference sheet is provided.

Section I – 10 marks

Use the SEPARATE Multiple Choice Answer Sheet provided.
Allow about 15 minutes for this section.

Section II – 90 marks

Attempt Questions 11-33
Allow about 2 hours 45 minutes for this section.
For questions in Section II, show relevant mathematical reasoning and/or calculations.
Additional writing space is provided at the end of Section II.

[Blank Page]

NESA Number:_____
SW MA RC MN SE GS

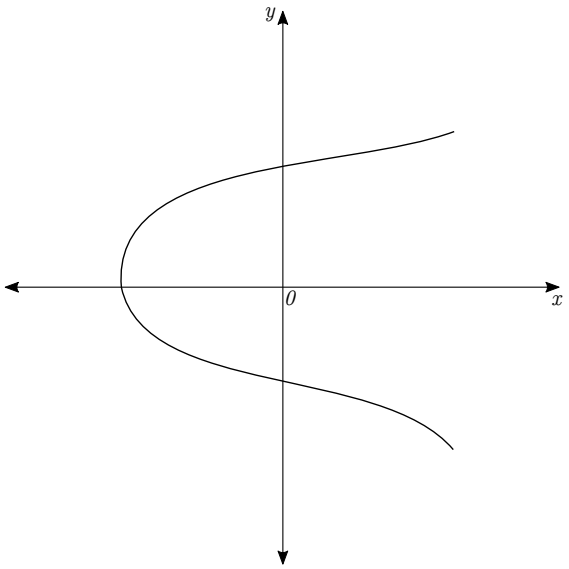
Section I

10 marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the separate multiple choice answer sheet.

Question 1

What type of relation is shown?



A) one-to-one

B) one-to-many

C) many-to-one

D) many-to-many

Question 2

Which interval describes the domain of $y = \dfrac{1}{\sqrt{x-2}}$?

A) $[2, \infty)$

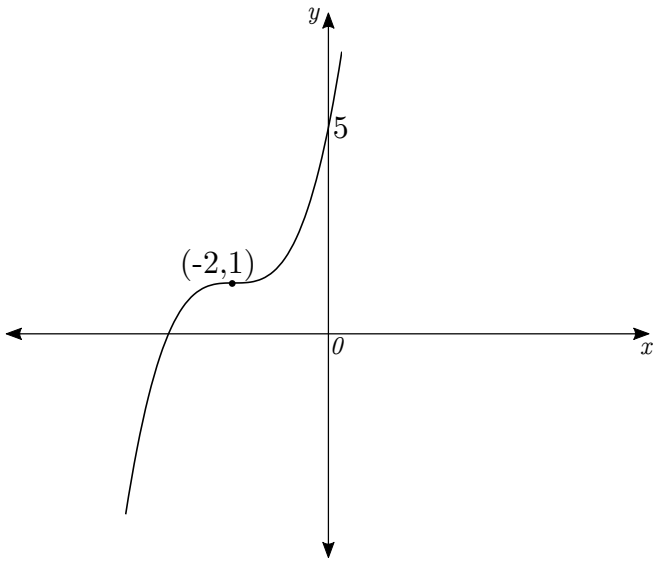
B) $(2, \infty)$

C) $[-\infty, 2)$

D) $(-\infty, 2)$

Question 3

Which is the correct equation of the following graph?



A) $y = (x+2)^3 + 1$

B) $y = \frac{1}{2}(x+2)^3 + 1$

C) $y = (x+2)^3 + 5$

D) $y = \frac{1}{2}(x+2)^3 + 5$

Question 4

Find $\int x^2\left(2x^3-1\right)^4\,dx$.

A) $\frac{\left(2x^3-1\right)^5}{5} + C$

B) $\frac{\left(2x^3-1\right)^5}{30} + C$

C) $\frac{\left(2x^3-1\right)^5}{5x^2} + C$

D) $\frac{\left(2x^3-1\right)^5}{30x^2} + C$

Question 5

Which pair of functions satisfy the condition $f\Big(g(x)\Big) = x$?

A) $f(x) = 3x - 1$, $g(x) = \dfrac{x+1}{3}$

B) $f(x) = 3x - 1$, $g(x) = \dfrac{x}{3} + 1$

C) $f(x) = 3x - 1$, $g(x) = x + \dfrac{1}{3}$

D) $f(x) = 3x - 1$, $g(x) = \dfrac{1}{3} - x$

Question 6

For which value of k does the equation $4x^2 + kx + 9 = 0$ have two distinct (unequal) real roots?

A) $k = 0$

B) $k = 6$

C) $k = 12$

D) $k = 18$

Question 7

The cumulative distribution function (CDF) of a continuous random variable is given by $F(x) = \sqrt{x} - 1$ for the interval $[1, 4]$.

What is the median?

A) 0.5

B) 2.25

C) 2.5

D) 2.75

Question 8

Given the following table of $P(Z \leq z)$ for the standard normal random variable Z.

	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

What is the value of $P(Z \geq -1.5)$?

A) 0.0668

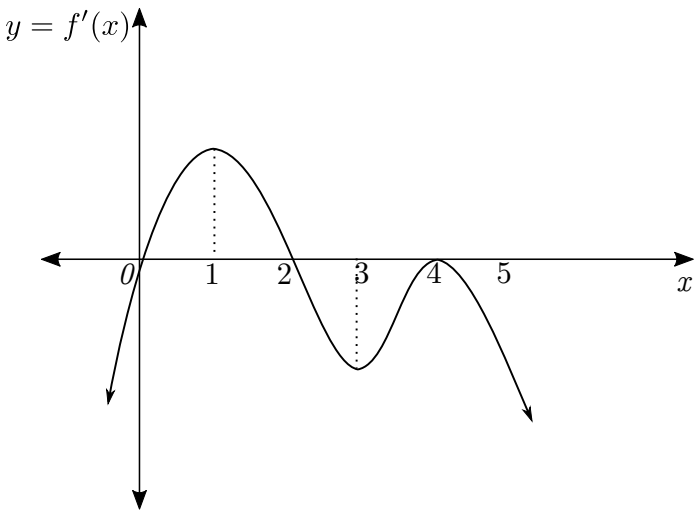
B) 0.4332

C) 0.9332

D) Insufficient table

Question 9

The graph of $y = f'(x)$ is shown below.



For which value of x is there a local maximum on the graph of $y = f(x)$?

A) $x = 0$

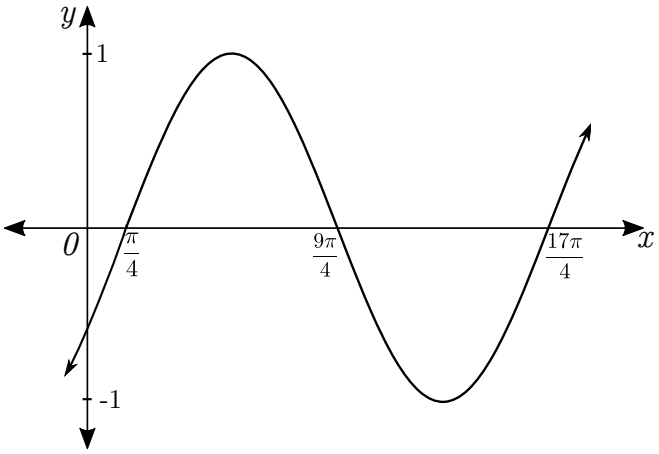
B) $x = 1$

C) $x = 2$

D) $x = 3$

Question 10

Which equation correctly describes the following graph?



A) $y = \sin\left(2x - \frac{\pi}{4}\right)$

B) $y = \sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

C) $y = \sin\left(2x - \frac{\pi}{2}\right)$

D) $y = \sin\left(\frac{1}{2}x - \frac{\pi}{8}\right)$

End of Section I.

Blank Page

Section II

90 marks
Attempt Questions
Allow about 2 hours and 45 minutes for this section.

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of your response.

Your responses should include relevant mathematical reasoning and/or calculations.

If additional space is required, use the additional pages at the end of Section II. Please label them clearly with your NESA number and the question number.

Question 11 (2 marks)

Solve $\left|4x-2\right|=14$. [2]

.....

.....

.....

.....

Question 12 (2 marks)

Differentiate $f(x)=\dfrac{\tan x}{x}$. [2]

.....

.....

.....

.....

.....

.....

Question 13 (3 marks)

Evaluate the series $8+11+14+\ldots+101$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

Question 14 (4 marks)

a) Differentiate $y=(2x+1)e^x$. [2]

.....

.....

.....

.....

.....

.....

b) Hence find an anti-derivative of $f(x)=(6x+9)e^x$. [2]

.....

.....

.....

.....

.....

.....

Question 15 (3 marks)

Solve $2\cos^2 x = 1$ for $-\pi \leq x \leq \pi$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

Question 16 (1 mark)

Simplify $\dfrac{d}{dx}\displaystyle\int_1^x \log_e t \, dt$. [1]

.....

.....

Question 17 (2 marks)

Show that $\dfrac{\cos^2 x}{\sin x} + \sin x = \operatorname{cosec} x$ [2]

.....

.....

.....

.....

.....

.....

Question 18 (4 marks)

A coin is tossed four times. Let X represent the number of tails.

X	0	1	2	3	4
$P(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$		$\frac{4}{16}$	$\frac{1}{16}$

a) Find $P(X = 2)$. [1]

.....

.....

b) Show that $E(X) = 2$. [1]

.....

.....

.....

c) Find $\text{Var}(X)$. [2]

.....

.....

.....

.....

Question 19 (2 marks)

Find $\int \cos 2x \, (1 + \sin 2x)^4 \, dx$. [2]

.....

.....

.....

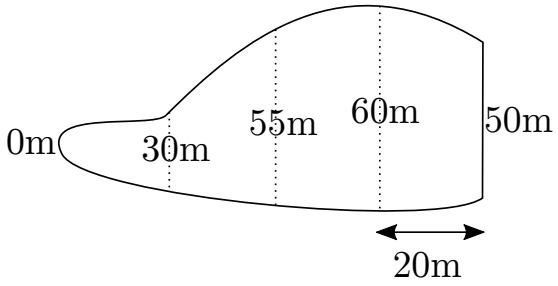
.....

.....

.....

Question 20 (2 marks)

A golf course is partially fenced by a 50 metre wall. A surveyor takes parallel measurements every 20 metres, as shown below.



Use the trapezoidal rule to estimate the area of the golf course. [2]

.....

.....

.....

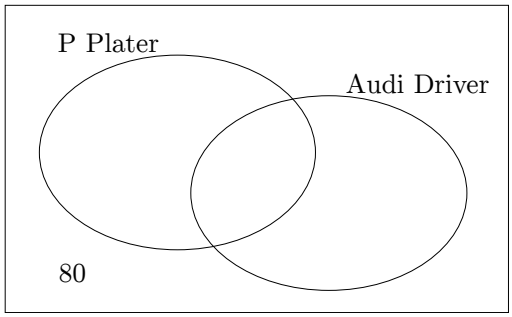
.....

.....

Question 21 (3 marks)

Of 120 drivers that are surveyed, 24 have P plates, 20 drive an Audi, and 80 drivers do neither, as shown in the Venn diagram below.

a) Complete the Venn diagram below. [2]



b) Show that the events "has P plates" and "drives an Audi" are independent. [1]

.....

.....

.....

Question 22 (4 marks)

A particle's motion is modelled by the displacement-time equation $x = 4t - t^2$ where x is the displacement in metres from the origin and t is in seconds.

a) Find the equation for velocity. [1]

.....

.....

.....

b) When is the particle stationary? [1]

.....

.....

.....

.....

c) From $t = 0$ to $t = 6$, when is the particle furthest away from the origin? [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 23 (8 marks)

Consider the function $f(x) = 2x^3 + 9x^2 - 4$

a) Find the coordinates of any stationary points and determine their nature. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

b) Find any points of inflection. [2]

.....

.....

.....

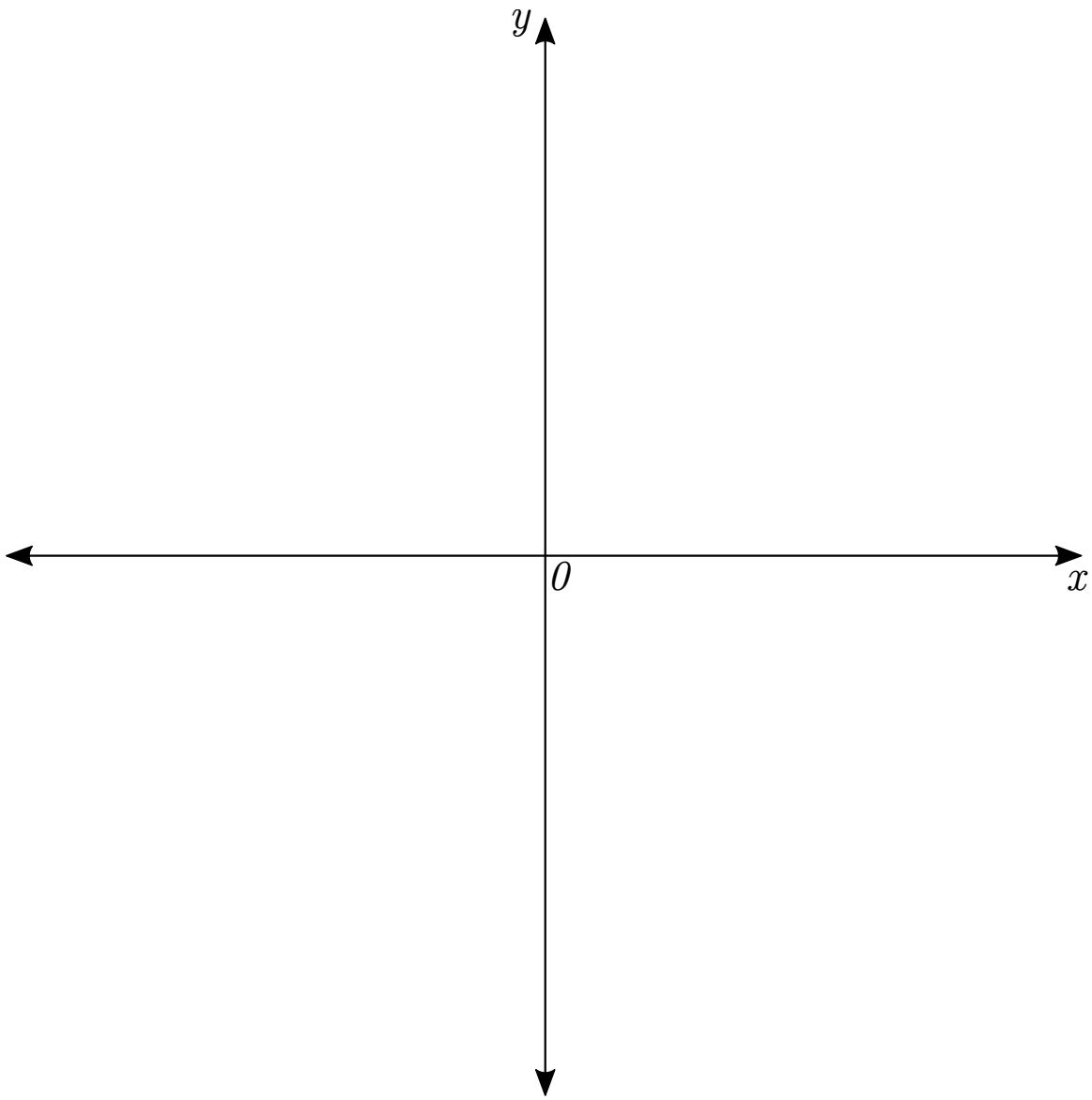
.....

.....

.....

.....

c) Sketch the graph of $y = f(x)$, showing stationary points, points of inflection and the y-intercept. You are not required to label the x-intercepts. [2]

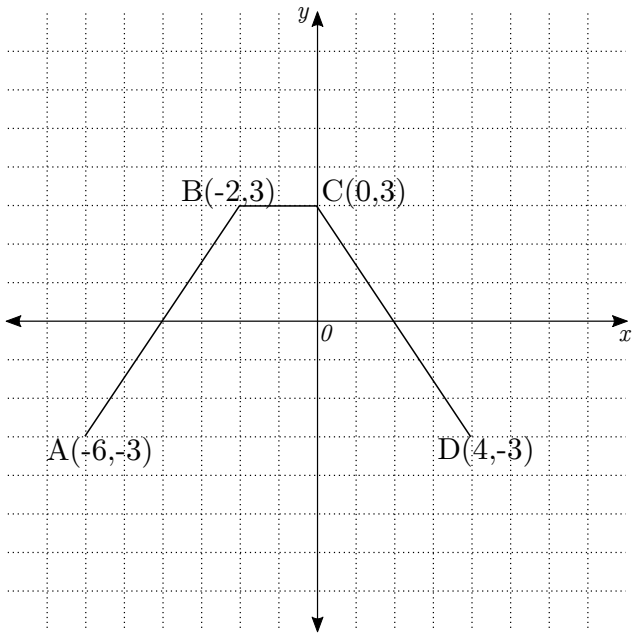


Question 24 (6 marks)

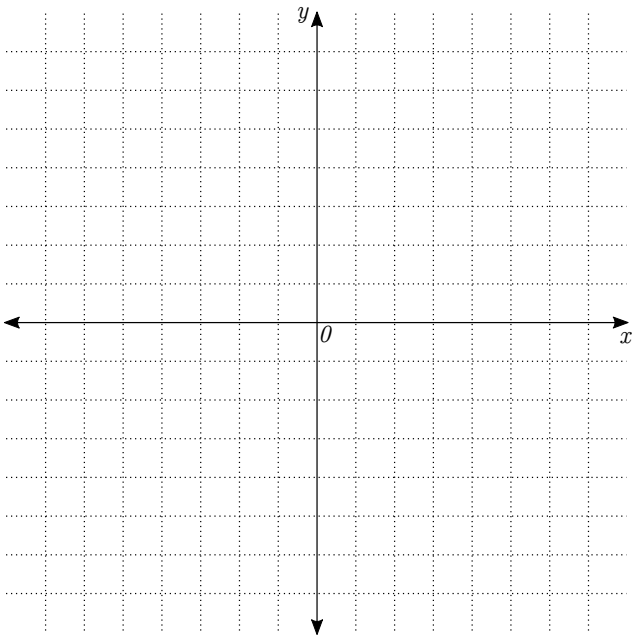
Consider the graph of $y = f(x)$ for $-6 \leq x \leq 4$ shown below.

Sketch the following, indicating the images A', B',C', and D' where applicable.

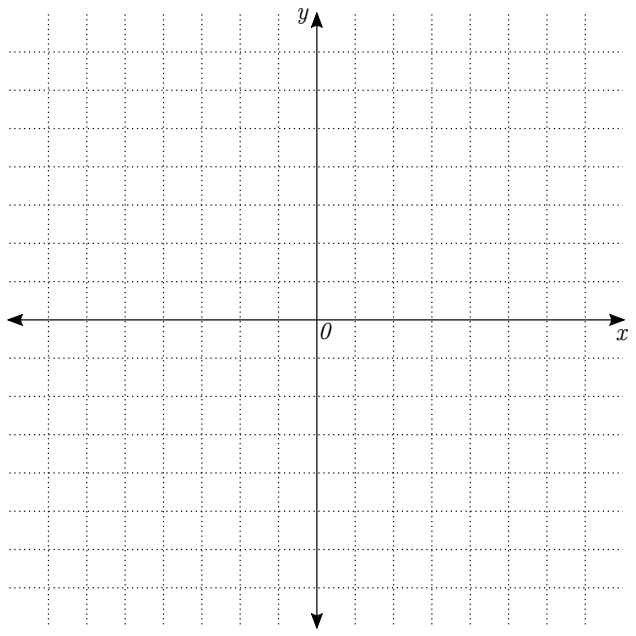
$y = f(x)$



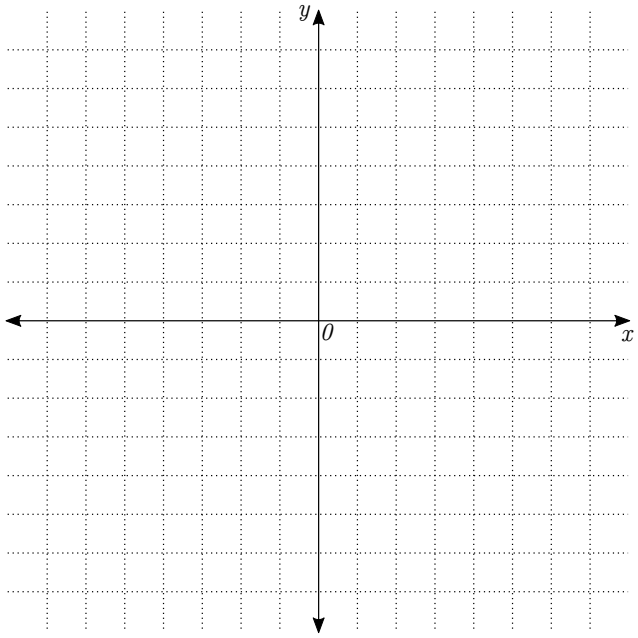
a) $y = 3 - f(x)$ [2]



b) $y = f(2x - 2)$ [2]

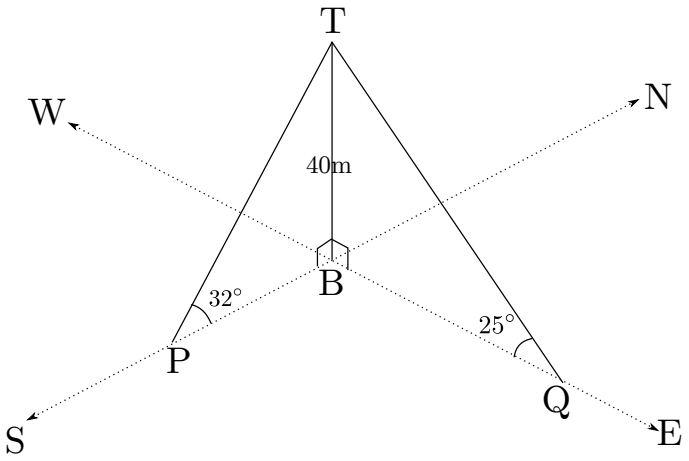


c) $y = f'(x)$ [2]



Question 25 (5 marks)

The top T of a 40 metre tall tree is observed from station P south of the tree with an angle of elevation of $32°$, and also observed from station Q east of the tree with an angle of elevation of $25°$.



a) Show that the distance from P to the base B of the tree is $40\cot 32°$ [2]

.....

.....

.....

.....

b) Hence find the distance between the stations P and Q to the nearest metre. [3]

.....

.....

.....

.....

.....

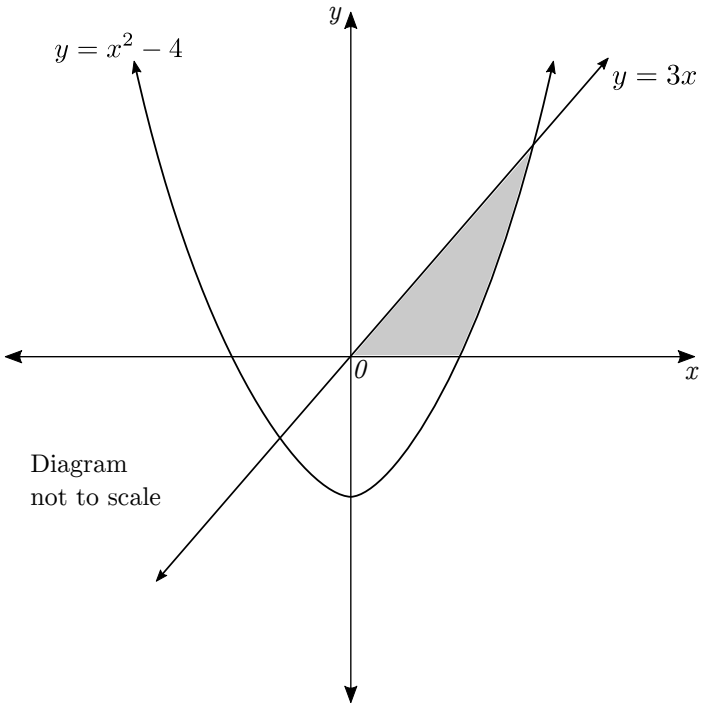
.....

.....

.....

Question 26 (4 marks)

Find the shaded area.



.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 27 (6 marks)

A new crypto-token called Meowcoin decreases in value after its release based on the formula $P = 1200 + 3000\left(1.2\right)^{-t}$, where P is its price in dollars and t is time since release in years.

a) Find Meowcoin's initial price. [1]

.....

.....

b) Sketch $P = 1200 + 3000\left(1.2\right)^{-t}$ below. [2]

c) Find the rate of decrease of Meowcoin's price 5 years after its release. [2]

.....

.....

.....

.....

d) What value will Meowcoin eventually reach? [1]

.....

.....

Question 28 (5 marks)

A farm produces eggs whose weights are normally distributed with a mean of 60 grams and a standard deviation of 2.4 grams.

a) What percentage of eggs will weigh:

i) between 57.6 g and 62.4 g? [1]

.....

.....

.....

ii) less than 55.2 g? [1]

.....

.....

.....

b) Consider the following table of $P(Z \leq z)$ for a standard normal variable Z.

	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

The family that owns the farm keeps the eggs that weigh more than 65.28 grams. Out of 10000 eggs produced, approximately how many eggs will they keep? [3]

.....

.....

.....

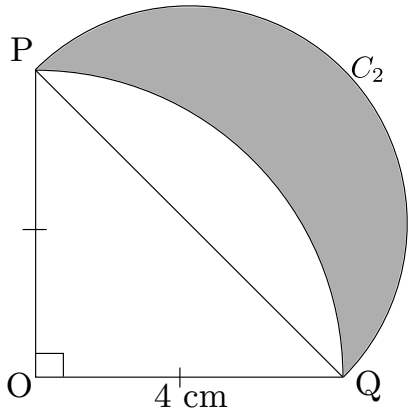
.....

.....

.....

Question 29 (4 marks)

A right-angled isosceles triangle is shown below, and the unshaded sector has centre O and radius 4 cm. A semicircle C_2 is drawn with PQ as the diameter.



Show that the shaded area is the same as the area of $\triangle OPQ$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 30 (4 marks)

The probability density function of a continuous random variable X is given by:

$$f(x) = \begin{cases} ke^{\frac{x}{2}}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Show that $k = \dfrac{1}{2(e-1)}$. [2]

.....

.....

.....

.....

.....

.....

.....

b) Find the 3rd quartile correct to 3 decimal places. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 31 (6 marks)

Jack inherits \$300,000 and deposits it into a bank account that earns interest at a rate of 2.4% p.a. compounded monthly. Jack plans to withdraw \$5000 at the end of each month to supplement his lifestyle.

Let A_n be his account balance after n months, just after each withdrawal.

a) Show that $A_2 = 300\,000 \times (1.002)^2 - 5000 \times (1.002 + 1)$. [1]

.....

.....

.....

.....

.....

.....

.....

b) Show that at the end of the first year Jack has \$246615.31 remaining in the account. [2]

.....

.....

.....

.....

.....

.....

.....

.....

c) After the first year, Jack decides to withdraw \$10,000 at the end of each month instead (or part thereof if there is not enough money). How many *more* months does it take for the account to completely run out of funds? [3]

Question 32 (4 marks)

Beth the chess prodigy has a 85% chance of winning a game when playing against non-master level players, but only a 35% chance of winning when playing against master level players. At a competition there are 80 other players and 10 of them are master level.

One of those 80 players is selected at random to play against Beth.

By considering a probability tree, or otherwise, find the probability that Beth plays a master level player given that she wins the game. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

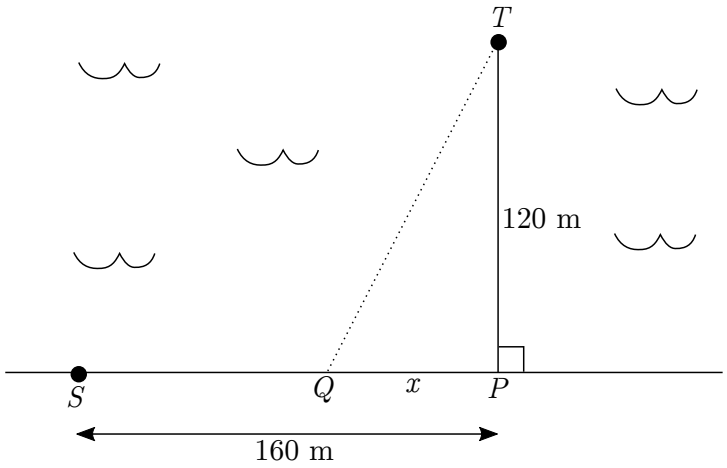
.....

.....

.....

Question 33 (6 marks)

NBNCo is planning to connect its substation S to an offshore tower T, which is 120 metres from the closest point P on the shore. The cost of running cable in the water is \$500 per metre, and the cost of running cable along the shore is \$350 per metre.



Let x be the distance NBNCo chooses for PQ so that cabling runs from S to Q then Q to T.

a) Show that the total cost $\$C$ of cabling is given by: [2]

$$C = 500\sqrt{x^2 + 14400} - 350x + 56000$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

b) Find the value of x to the nearest metre that minimises the total cost. [4]

End of Section II.

Additional Writing Space

Clearly label with question/part number.

Additional Writing Space

Clearly label with question/part number.

Additional Writing Space

Clearly label with question/part number.