KNOX
GRAMMAR
SCHOOL

2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided with this paper.
- For questions in Section I, please write your answers on the multiple choice answer sheet with which you have been provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations and write your solutions on the writing paper with which you have been provided and

Total marks: 100

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 6 – 16)
- Attempt Questions 11 – 16.
- Allow about 2 hours and 45 minutes for this section.

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Number of Students in Course: 139

Section I

10 marks
Attempt Questions 1 – 10. Allow about 15 minutes for this section.
Please answer this section on the multiple choice answer sheet with which you have been provided.

Write the letter that corresponds to your answers for Questions 1 – 10.

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1 $\dfrac{(2p)^3}{\sqrt{p}} =$

A. $8\sqrt{p^5}$

B. $8\sqrt[5]{p^2}$

C. $6\sqrt{p^5}$

D. $6\sqrt[5]{p^2}$

2 Which inequality gives the domain of $y = \sqrt{6 - x}$?

A. $x < 6$

B. $x > 6$

C. $x \leq 6$

D. $x \geq 6$

3 The solutions to the quadratic equation $3x^2 - x - 4 = 0$ are,

A. -1 and $-\dfrac{4}{3}$

B. -1 and $\dfrac{4}{3}$

C. 1 and $\dfrac{4}{3}$

D. 1 and $-\dfrac{4}{3}$

4 When evaluated, $\log_3 100$, lies between which pair of consecutive integers ?

A. 0 and 1

B. 1 and 2

C. 4 and 5

D. 33 and 34

5 $\displaystyle\int_0^1 \sqrt{1-x^2}\ dx$

A. 0

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{2}$

D. 1

6 Which of the following is an example of a discrete random variable ?

A. The time needed to run one kilometre.

B. The heights of trees in a local park.

C. The mass of all parcels delivered by Australia Post on a particular day.

D. The number of visits made by 100 patients to their doctor in a year.

7 It is known that for a particular function, $y = f(x)$, that

- $f(3) = -6$
- $f'(3) = 0$
- $f''(3) = 4$

Which statement below is true regarding the graph of $y = f(x)$?

A. It passes through the point (3, 0).

B. There is a local minimum at (3, −6).

C. It is concave down at (3, −6).

D. There is a point of inflexion at (3, 4).

8 The expression $\tan\theta + \sin\theta < 0$ is true for all values of θ in the domain,

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(\pi, \frac{3\pi}{2}\right)$

D. $\left(\frac{3\pi}{2}, 2\pi\right)$

9 The displacement of a particle moving in a straight line is given by the equation $x = 2t - \frac{1}{2}t^2$, where x is in metres, t is in seconds and $t \geq 0$.

Which of the following statements is FALSE ?

A. The particle is initially moving to the left.

B. The particle is at the origin after 4 seconds.

C. The acceleration of the particle is constant.

D. The speed of the particle increases indefinitely.

10 The graph of the function $y = f(x)$ is known to have a minimum turning point at the point $P(-4, -6)$.

Therefore, the graph of $y = -f(2x)$ will have a maximum turning point at,

A. $(-4, 6)$

B. $(-8, 6)$

C. $(-2, 6)$

D. $(2, -6)$

Section II

90 marks
Attempt Questions 11–16. Allow about 2 hours and 45 minutes for this section.
Please write your solutions on the writing paper with which you have been provided and have printed in advance.

Answer each question on lined paper. Label each response with the corresponding question number.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

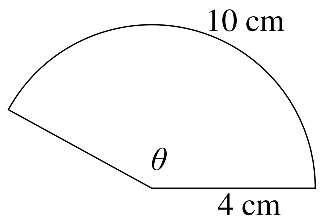
Question 11 (15 marks)

(a) Factorise fully, $x - xy^2$. **2**

(b) Find, $\dfrac{d}{dx}\left(\dfrac{x^2}{x+1}\right)$. **2**

(c) Differentiate the function, $y = x^2 e^{2x}$ leaving your answer in simplest factored form. **2**

(d) Consider the sector drawn below with radius 4 cm and arc length 10 cm.



(i) Find the size of the angle θ, in radians. **1**

(ii) Hence, find the exact area of the sector. **1**

(e) Find the exact value of $\displaystyle\int_0^2 \frac{3}{2x+1}\,dx$. **3**

(f) The gradient function of a curve is $f'(x) = 4x + 3$. The curve passes through the point (–2, 5). Find $f(x)$. **2**

(g) The table shows the future value of an annuity of $1 for a selection of interest rates per period and investment terms. The contributions are made at the end of each period.

	Interest Rate Per Period											
Period	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331

(i) Tom invests $750 at the end of each quarter into an account which earns 8% per annum with interest compounded quarterly. **1**

He does this for 3 years.

Find the future value of Tom's investment.

(ii) His brother, Jake, finds an account that will earn him 12% per annum. The interest will also be compounded quarterly. **1**

However, Jake is only prepared to contribute for 2 years.

Find the amount that he must contribute quarterly if he is to achieve the same future value as Tom.

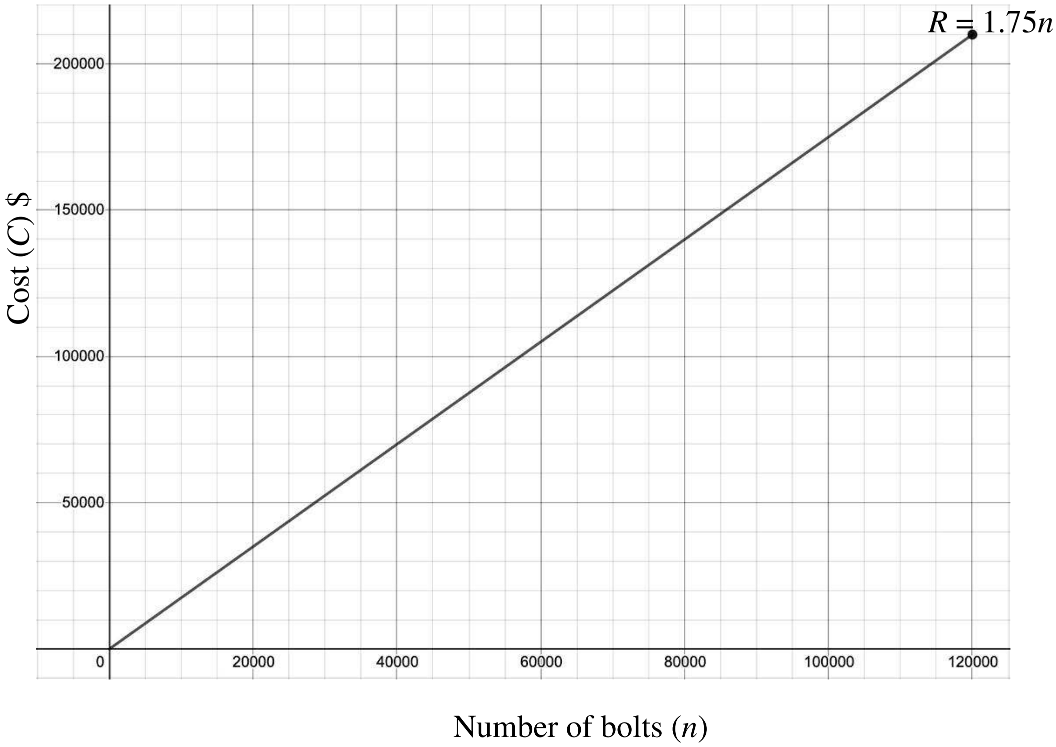
End of Question 11

Question 12 (15 marks)

(a) Jamjet Industries produce galvanised bolts for the roofing industry. Due to manufacturing constraints, the maximum number of bolts they can produce in a month is 120 000.

The cost of production is \$50 000 plus 95 cents per bolt.

The bolts are sold for \$1.75 each and the graph that represents Jamjet's revenue, R, appears on the axes below.



(i) Find the linear equation for the cost, C, of producing n bolts. **1**

(ii) What is the practical significance of the gradient of the line in (i) ? **1**

(iii) Find the number of bolts Jamjet must sell in a month to break even. **1**

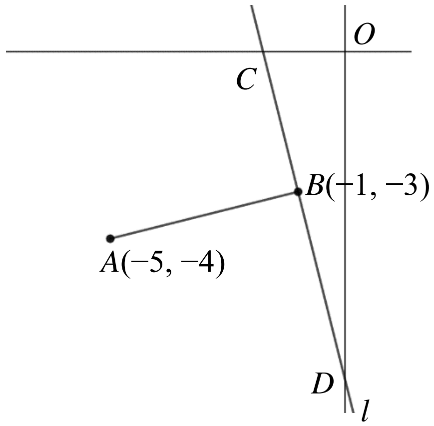
(iv) What is the maximum monthly profit attainable by Jamjet ? **2**

Question 12 continues on page 9

(b) The number plane below shows the interval joining the points $A(-5, -4)$ and $B(-1, -3)$. **5**

The line, l, is drawn through B so as to be perpendicular to AB and meets the x-axis at C and the y-axis at D.

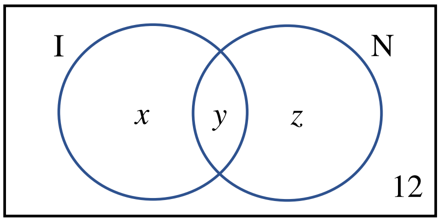
Find the area of triangle COD.



(c) Eighty people were asked if they had holidayed interstate (I) or within NSW (N) in the past five years.

The results showed that,

- 12 people had not holidayed at all.
- 42 people had holidayed interstate (I).
- 36 people had holidayed within NSW (N).



(i) Copy the Venn diagram above onto you answer sheet and complete it by finding values for x, y and z. **1**

(ii) One of the surveyed people is selected at random. Find the probability that the person selected has holidayed both interstate and within NSW. **1**

(iii) A randomly selected person is known to have travelled. Find the probability that this person holidayed within NSW. **1**

(iv) Two people are chosen, in turn, at random. Find the probability that exactly one of those chosen holidayed interstate. **2**

End of Question 12

Question 13 (15 marks)

(a) A particle, initially at the origin, is moving in a straight line. Its velocity, $v\,\text{ms}^{-1}$, after t seconds is given by $v = 2 - 2t$.

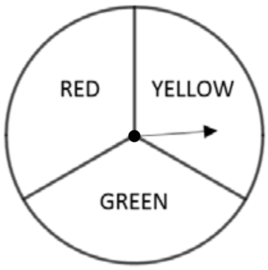
(i) Find the initial velocity. **1**

(ii) Find when the particle is at rest. **1**

(iii) Find when the particle is next at the origin. **2**

(iv) Find the distance travelled in the first 4 seconds. **2**

(b) The spinner below is divided into three sectors of equal area. They are coloured red, yellow and green as shown.



Rachel spins the spinner 5 times and lets the random variable, X, be the number of times the spinner lands on red.

This discrete random variable has the following probability distribution.

x	0	1	2	3	4	5
$P(X = x)$	$\frac{32}{243}$	k	k	$\frac{40}{243}$	$\frac{10}{243}$	$\frac{1}{243}$

(i) Find the value of k. **1**

(ii) Find $E(X)$. **1**

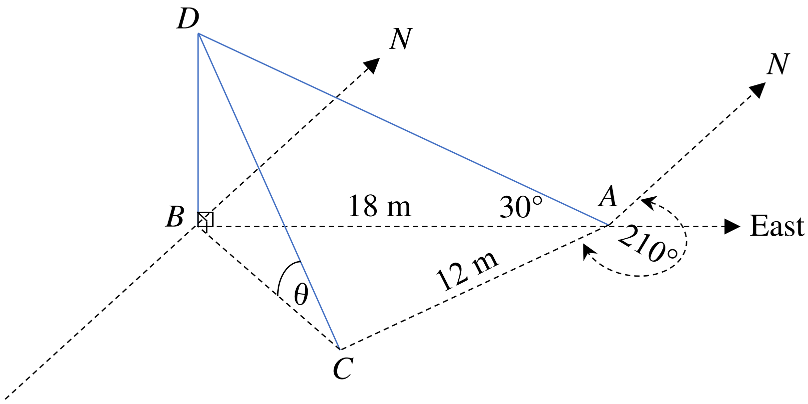
(iii) Find the standard deviation of the distribution. **2**

Question 13 continues on page 11

(c) A flagpole, BD, lies on flat level ground. **5**

It is supported by two cables, AD and CD as shown below. The cable AD is inclined at 30° to the horizontal and is tied 18 metres from the foot of the pole.

The second cable is tied at a point, C, 12m from A and on a bearing of 210° from A.



Find θ, correct to the nearest degree.

End of Question 13

Question 14 (15 marks)

(a) The function $f(x) = \dfrac{x^3}{6} - \dfrac{x^2}{4} - 3x + 1$ is defined for x in the domain $[-4, 6]$.

(i) Find the stationary points and determine their nature. **3**

(ii) Find the absolute maximum of $f(x)$. **2**

(iii) Sketch $f(x)$ showing the stationary points and both endpoints. **2**

(b) Given that $f(x) = x^4 - 2x^2$ and $g(x) = \sqrt{x}$, sketch $y = f(g(x))$ over its natural domain. **3**

(c) The percentage results of a university entrance examination are normally distributed with a mean mark is 60 and the standard deviation is 8.

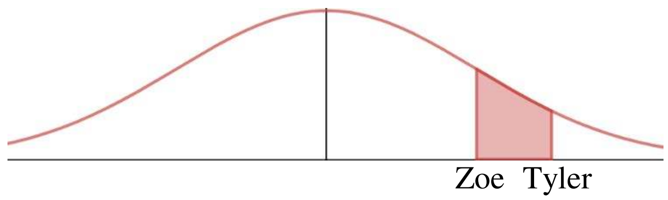
(i) If Jimmy's mark corresponds to a negative z–score in the range, $-1 < z < 0$ give a possible value for Jimmy's mark. **1**

(ii) Tyler obtains a result of 72%. Show that his z-score is 1.5. **1**

(iii) Zoe's z-score was 1. What was her percentage mark? **1**

(iv) When Tyler was sent his results, he was informed that his mark was better than 93.3% of candidates. **2**

The examination was completed by 85000 students. The shaded area below represents those students whose mark was higher than Zoe's but lower than Tyler's. How many students are represented by the shaded area?



End of Question 14

Question 15 (15 marks)

(a) X is a continuous random variable that is defined by the probability density function,

$$f(x) = \begin{cases} k(x-1)^2 & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that $k = \frac{3}{26}$. **2**

(ii) Find the cumulative distribution function. **2**

(iii) Show that the upper quartile is approximately 3.74. **2**

(b) An iron has accidently been left on. Fortunately, it switches itself off after a period of time.

The temperature of the iron can be modelled by the equation,

$$T = 22 + 162\left(1.2\right)^{-0.5t},$$

where T is the temperature of the iron's surface, in degrees Celsius, t minutes after the iron switches itself off.

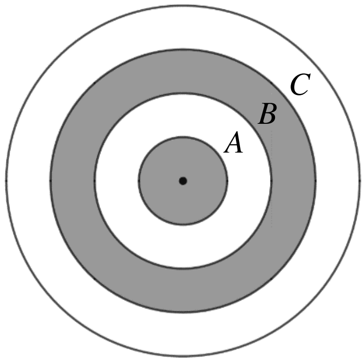
(i) What is the temperature of the iron 6 minutes after it has switched off ? Give your answer to the nearest degree. **1**

(ii) At what rate is the temperature of the iron's surface changing after 6 minutes ? **2**

Question 15 continues on page 14

(c) The diagram shows the beginning of a series of concentric circles. The radius of the inner most circle is 1unit. The radius of each circle moving outwards, is 1 more than the previous circle.

The area between two concentric circles is called an annulus. The area of the annulus marked A is 3π square units. The area of the annulus marked B is 5π square units. The area of the annulus marked C is 7π square units and so on.



(i) Find the area of the 10th annulus. **1**

(ii) Show that the sum of the first n annuli is found by $S_n = \pi n^2 + 2n\pi$ **2**

(iii) What is the minimum number of annuli needed for a combined area of 300 u^2 ? **3**

End of Question 15

Question 16 (15 marks)

(a) Nick chooses an integer, x, between 1 and 20 at random.

He reduces this number by 1, 4 and 9 in turn and then multiplies to find the product, P.

(i) Show that, $P = x^3 - 14x^2 + 49x - 36$ **1**

(ii) Find the minimum value of P. **3**

(b) Jelena retires with a balance in her superannuation account of \$700 000. **3**

The fund earns interest at the rate of 3.6% p.a. compounded monthly.

At the end of each month, Jelena withdraws \$4000 from the account.

The amount, A_n, left in her fund after the n^{th} withdrawal is given by,

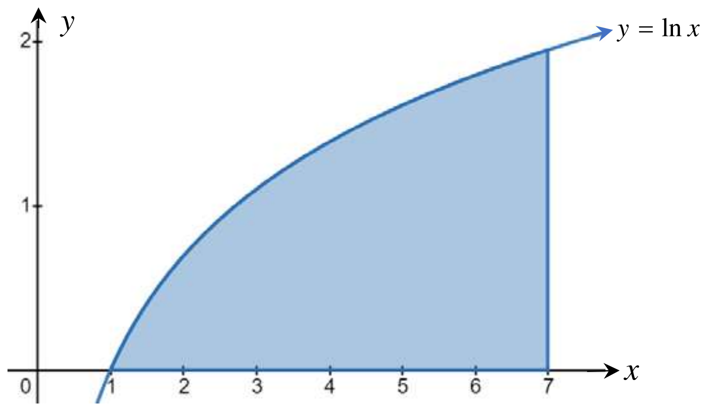
$$A_n = 700\,000(1.003)^n - 4000\left\{1 + 1.003 + (1.003)^2 + \ldots + (1.003)^{n-1}\right\}$$

(Do NOT prove this result)

Find, correct to the nearest month, the time taken for the balance in the fund to reduce to \$400 000.

Question 16 continues on page 16

(c) The diagram below shows the graph of $y = \ln x$. The region bounded by the graph, the x-axis and the line $x = 7$ has been shaded.



(i) Use the trapezoidal rule with 3 function values to approximate the shaded area. Give your answer to 3 decimal places. **2**

(ii) Find $\frac{d}{dx}(x\ln x)$. **2**

(iii) Hence, or otherwise, find, $\int_1^7 \ln x \, dx$ **3**

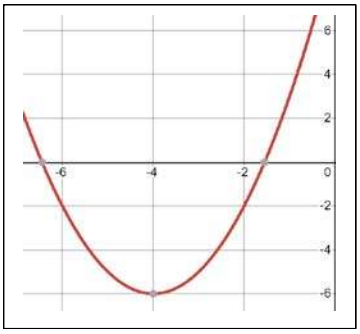
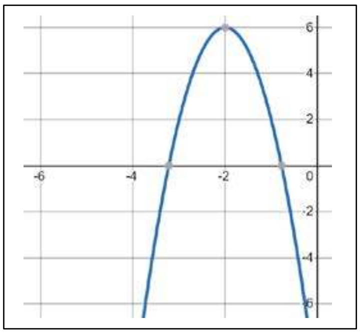
(iv) Hence, find an approximation for $\ln 7$. **1**

End of Paper

Topic Test 11 – Trial HSC 2 – Solutions and Marking Scheme

Section I

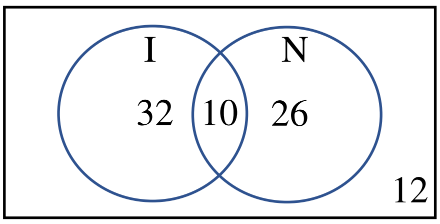
Question	Working	Notes	Ans
1	$\dfrac{(2p)^3}{\sqrt{p}} = \dfrac{8p^3}{p^{\frac{1}{2}}}$ $= 8p^{\frac{5}{2}}$ $= 8\sqrt{p^5}$		**A**
2	$6 - x \geq 0$ $\therefore x \leq 6$		**C**
3	$3x^2 - x - 4 = 0$ $\therefore (3x-4)(x+1) = 0$ $\therefore x = -1$ and $\dfrac{4}{3}$		**B**
4	Since $3^4 = 81$ and $3^5 = 243$ for $3^x = 100$, x must be between 4 and 5. Alternatively, $\log_3 100 = \dfrac{\ln 100}{\ln 3}$ (by the change of base law). ≈ 4.2		**C**
5	The graph of $f(x) = \sqrt{1 - x^2}$ is a semi-circle of radius 1. So $\int_0^1 \sqrt{1 - x^2}\ dx$ is equivalent to half the area of the semi-circle. That is, $\dfrac{\pi}{4}$.		**B**
6	A discrete variable is one that can only take on specific numerical values; normally integers. Generally, it would be a quantity that was counted rather than measured. So, in this case, th e number of visits made by 100 patients to their doctor in a year.		**D**
7	Since $f(3) = -6$, the graph of $y = f(x)$ passes through the point (3, −6). Since $f'(3) = 0$, there is some kind of stationary point at (3, −6). Since $f''(3) = 4$, the graph is concave up at (3, −6) meaning the stationary point must be a local minimum.		**B**

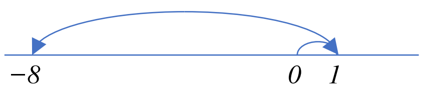
8	In order to be certain that $\tan\theta + \sin\theta < 0$, both $\tan\theta$ and $\sin\theta$ must be less than 0. This only happens in the fourth quadrant (think ASTC) and so the statement is true for all values of θ in the domain $\left(\frac{3\pi}{2}, 2\pi\right)$.		**D**
9	Check A: $x = 2t - \frac{1}{2}t^2$ When $t = 4$, $x = 2(4) - \frac{1}{2}(4)^2 = 0$ $\therefore$ True Check B: $x = 2t - \frac{1}{2}t^2$ so $v = 2 - t$ When $t = 0$, $v = 2 - 0 = 2 > 0$ This means the particle is initially moving right making statement B, False.		**A**
10	The diagram shows a **possible** graph of $y = f(x)$.  The graph of $y = -f(x)$ will be a reflection in the x-axis producing a maximum turning point at $(-4, 6)$. The dilation experienced by $y = -f(2x)$ will halve the distance of all points on $y = -f(x)$ from the y–axis. Therefore, the maximum turning point at $(-4, 6)$ on the graph of $y = -f(x)$, will move to $(-2, 6)$ on the graph of $y = -f(2x)$. 		**C**

Section II

Question	Working	Marking Scheme
11(a)	$x - xy^2 = x\left(1 - y^2\right)$ $= x\left(1 - y\right)\left(1 + y\right)$	Award 1 for removing the x Award 2 for then using difference of two squares. Award 2 for bald answer.
11(b)	$\dfrac{d}{dx}\left(\dfrac{x^2}{x+1}\right) = \dfrac{(x+1)2x - x^2.1}{(x+1)^2}$ $= \dfrac{2x^2 + 2x - x^2}{(x+1)^2}$ $= \dfrac{x^2 + 2x}{(x+1)^2}$	Award 1 for an incorrect attempt to use the quotient rule (the "1" need not be shown. Award to for correctly using the quotient rule. Simplification is not required. Ignore any subsequent error.
11(c)	$y = x^2 e^{2x}$ $\therefore\ y' = x^2(2e^{2x}) + e^{2x}(2x)$ $= 2xe^{2x}(x+1)$	Award 1 for each line of the solution. (See placement of ticks)
11(d)(i)	$l = r\theta$ $\therefore 10 = 4\theta$ $\therefore \theta = 2.5$ radians	Award 1 for correct answer.
11(d)(ii)	$A = \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 4^2 \times 2.5$ $= 20\ \text{cm}^2$.	Award 1 for correct answer.
11(e)	$\displaystyle\int_0^2 \frac{3}{2x+1}\,dx$ $= \dfrac{3}{2}\displaystyle\int_0^2 \frac{2}{2x+1}\,dx$ $= \left[\dfrac{3}{2}\ln\left(2x+1\right)\right]_0^1$ $= \dfrac{3}{2}\ln 5 - \dfrac{3}{2}\ln 1$ or $= \dfrac{3}{2}\ln 5 - 0$ $= \dfrac{3}{2}\ln 5$	Aw 1 for, $\dfrac{3}{2}\displaystyle\int_0^2 \frac{2}{2x+1}\,dx$ a primitive that is an "ln" Aw 2 for, $\left[\dfrac{3}{2}\ln\left(2x+1\right)\right]_0^1$ $\dfrac{3}{2}\ln 5$ without previous line. Aw 3 for, $\dfrac{3}{2}\ln 5$ with either expression on previous line.

11(f)	$f'(x) = 4x + 3$ $\therefore f(x) = 2x^2 + 3x + C$ Since the curve passes through $(-2, 5)$, $f(-2) = 5$. $\therefore 5 = 2(-2)^2 + 3(-2) + C$ $5 = 8 - 6 + C$ $\therefore C = 3$ $\therefore f(x) = 2x^2 + 3x + 3$	Aw 1 for, correct primitive including with or without + C. substituting (−2, 5) into an incorrect primitive and correctly completing the question. Aw 2 for, correct calculation of C.
11(g)(i)	As interest is compound quarterly, $r = 2\%$ and $n = 12$ $\therefore$ Future Value $= 750 \times 13.4121$ $= \$10059.08$	Aw 1 for correct answer.
11(g)(ii)	For Jake, $r = 3\%$ and $n = 8$. Let the amount of his quarterly contribution be \$M. Then, $10059.68 = M \times 8.8923$ $\therefore M = \$1131.21$	Aw 1 for correct answer.
12(a)(i)	$C = 50000 + 0.95n$	Aw 1 for correct answer.
12(a)(ii)	The gradient represents the cost of production per bolt.	Aw 1 for correct answer. Must reference rate in some way.
12(a)(iii)	$1.75n = 50000 + 0.95n$ $0.8n = 50000$ $\therefore n = 62500$	Aw 1 for correct answer.
12(a)(iv)	Profit $= 1.75n - (50000 + 0.95n)$ $= 1.75n - 50000 - 0.95n$ $= 0.8n - 50000$ when $n = 120000$ $= \$46000$	Aw 1 for, correct profit expression **or** substituting $n = 120000$ into their expression for profit. Aw 2 for, correct solution.

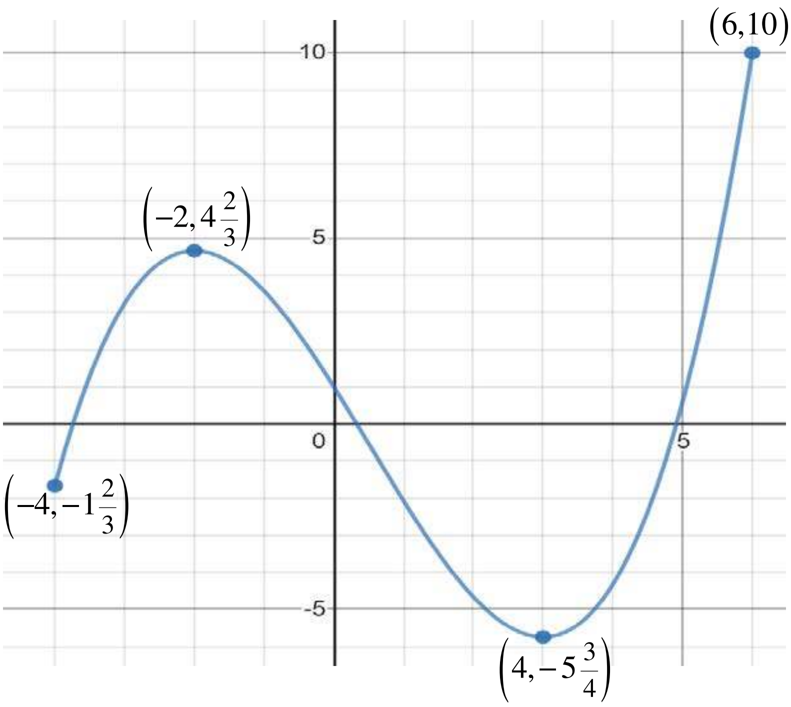
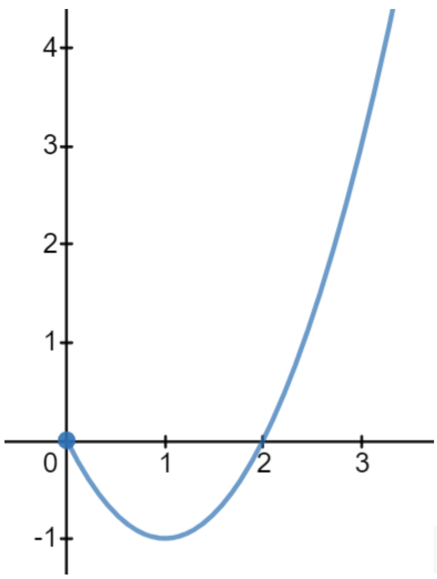
12(b)	$m_{AB} = \dfrac{-3--4}{-1--5} = \dfrac{1}{4}$ $\therefore \quad m_l = -4$ $y - y_1 = m(x - x_1)$ $y + 3 = -4(x + 1)$ $y + 3 = -4x - 4$ $y = -4x - 7$ $\therefore$ D is $(0, -7)$ Set $y = 0$. $0 = 4x - 7$ $4x = -7$ $\therefore \; x = -\dfrac{7}{4}$ so C is the point $\left(-\dfrac{7}{4}, 0\right)$. So $OD = 7$ and $OC = \dfrac{7}{4}$ Area $= \dfrac{1}{2} \times \dfrac{7}{4} \times 7$ $= \dfrac{49}{8} u^2$	Aw 1 for gradient of AB. Aw 2 for gradient of l. Aw 3 for equation of l. Aw 4 for point C. Aw 5 for correct answer.
12(c)(i)		Aw 1 for correct answer.
12(c)(ii)	$P(\mathrm{I} \cup \mathrm{N}) = \dfrac{10}{80} = \dfrac{1}{8}$	Aw 1 for correct answer. Simplification unnecessary.
12(c)(iii)	$P(\mathrm{N} \mid \text{traveller}) = \dfrac{36}{68} = \dfrac{9}{17}$	Aw 1 for correct answer. Simplification unnecessary.
12(c)(iv)	$P(\text{exactly 1 interstate traveller}) = P(\mathrm{I},\tilde{\mathrm{I}}) + P(\tilde{\mathrm{I}},\mathrm{I})$ $= \left(\dfrac{42}{80} \times \dfrac{38}{79}\right) + \left(\dfrac{36}{80} \times \dfrac{42}{79}\right)$ $= \dfrac{399}{790}$	Aw 1 for one of the brackets. Aw 2 for answer.

13(a)(i)	$v = 2 - 2(0)$ $= 2$ m/s. ✓	Aw 1 for correct answer.
13(a)(ii)	$0 = 2 - 2t$ $\therefore t = 1$ ✓	Aw 1 for correct answer.
13(a)(iii)	$v = 2 - 2t$ $\therefore x = 2t - t^2 + C$ ✓ When $t = 0,\ x = 0 \therefore\ C = 0$ $\therefore x = 2t - t^2$ $0 = 2t - t^2$ $0 = t(2 - t)$ $\therefore t = 2 \quad (t \neq 0)$ ✓	Aw 1 for demonstrating an understanding that $x = \int v\,dt$ Aw 2 for correct answer. Student must evaluate the constant of integration.
13(a)(iv)	When $t = 0,\ x = 0$. This is given in the initial conditions. When $t = 1,\ x = 2(1) - 1^2$. $= 1$ ✓ Examine this as this is when the particle comes to rest. When $t = 4,\ x = 2(4) - 4^2$ $= -8$ $\therefore$ The particle start at the origin, moves right to $x = 1$ and then travels left to $x = -8$.  ✓ $\therefore$ The particle travels a total of 10m in the first 4 seconds.	Aw 1 for, finding x when $t = 1$. **or** finding x when $t = 4$. **or** stating the particle has travelled 8m. **or** using integration to arrive at an answer of 8m. Aw 2 for, correct solution. Note that student may also arrive at the answer through integration. This is perfectly valid and Aw 2.

13(b)(i)	$\frac{32}{243} + k + k + \frac{40}{243} + \frac{10}{243} + \frac{1}{243} = 1$ ✓ $2k + \frac{83}{243} = 1$ $2k = \frac{160}{243}$ $k = \frac{80}{243}$ ✓	Aw 1 for summing the probabilities and equating to 1. Aw 2 for correct value of *k*.
13(b)(ii)	$E(X) = \left(0\left(\frac{32}{243}\right)\right) + \left(1\left(\frac{80}{243}\right)\right) + \left(2\left(\frac{80}{243}\right)\right) + \left(3\left(\frac{40}{243}\right)\right)$ .. $\ldots. + \left(4\left(\frac{10}{243}\right)\right) + \left(5\left(\frac{1}{243}\right)\right)$ $= \frac{405}{243}$ $= \frac{5}{3}$ ✓	Composite functions is new content. A question similar to this appeared in Q17 of the NESA specimen paper.
13(b)(iii)	$Var(X)$ $= E(X^2) - \mu^2$ $= \left(0\left(\frac{32}{243}\right)\right) + \left(1\left(\frac{80}{243}\right)\right) + \left(4\left(\frac{80}{243}\right)\right) + \left(9\left(\frac{40}{243}\right)\right) + \ldots$ $\ldots + \left(16\left(\frac{10}{243}\right)\right) + \left(25\left(\frac{1}{243}\right)\right) - \left(\frac{5}{3}\right)^2$ ✓ $= \frac{10}{9}$ $\therefore \sigma = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}$ ✓	Aw 1 for find $Var(X)$ Aw 2 for correct answer.

13(c)	In ΔBAD, $\tan 30° = \dfrac{BD}{18}$ $\dfrac{1}{\sqrt{3}} = \dfrac{BD}{18}$ $BD = \dfrac{18}{\sqrt{3}}$ $BD = 6\sqrt{3}$ ✓ $BC^2 = 18^2 + 12^2 - 2(18)(12)\cos 60^\circ$ ✓ $= 468 - 216$ $= 252$ $\therefore BC = \sqrt{252}$ $= 6\sqrt{7}$ ✓ $\tan\theta = \dfrac{BD}{BC}$ $= \dfrac{6\sqrt{3}}{6\sqrt{7}}$ $= \dfrac{\sqrt{3}}{\sqrt{7}}$ $\therefore \theta \approx 33°$ ✓	Aw 1 mark for each of the following. Finding the length of *BD*. Calculating $\angle BAC = 60^\circ$. Expression for BC^2 even if $\angle BAC$ is incorrect. Correct value of BC allowing for errors carried forward. Correct value of θ.
14(a)(i)	$f(x) = \dfrac{x^3}{6} - \dfrac{x^2}{4} - 3x + 1$ $\therefore f'(x) = \dfrac{x^2}{2} - \dfrac{x}{2} - 3$ Stationary points occur when $\therefore f'(x) = 0$ $\therefore 0 = \dfrac{x^2}{2} - \dfrac{x}{2} - 3$ $0 = x^2 - x - 6$ $0 = (x-3)(x+2)$ $\therefore x = -2$ and 3 $f(-2) = \dfrac{(-2)^3}{6} - \dfrac{(-2)^2}{4} - 3(-2) + 1$ $= -\dfrac{4}{3} - 1 + 6 + 1$ $= 4\dfrac{2}{3}$ continued over…..	Aw 1 for correct derivative. Aw 2 for correct derivative and finding both stationary points, including the *y* values. Aw 3 for complete solution. Students must give values for $f''(-2)$ and $f''(3)$ to determine nature. If using a table of values to determine the nature, numerical values must be given for $f'(x)$ and not just "+" and "–".

	$f(3)=\frac{(3)^3}{6}-\frac{(3)^2}{4}-3(3)+1$ $=\frac{27}{6}-\frac{9}{4}-9+1$ $=-5\frac{3}{4}$ Use $f''(x)$ to determine nature. $f''(x)=x-\frac{1}{2}$ $\therefore\ f''(-2)=-2-\frac{1}{2}$ $=-2\frac{1}{2}$ <0 $\therefore$ a local maximum occurs at $\left(-2,4\frac{2}{3}\right)$ $f''(x)=x-\frac{1}{2}$ $\therefore\ f''(3)=3-\frac{1}{2}$ $=2\frac{1}{2}$ >0 $\therefore$ a local minimum occurs at $\left(3,-5\frac{3}{4}\right)$	
14(a)(ii)	$f(-4)=\frac{(-4)^3}{6}-\frac{(-4)^2}{4}-3(-4)+1$ $=-10\frac{2}{3}-4+12+1$ $=-1\frac{2}{3}$ $f(6)=\frac{(6)^3}{6}-\frac{(6)^2}{4}-3(6)+1$ $=36-9-18+1$ $=10$ $\therefore$ the absolute maximum of $f(x)$ is 10.	Aw 1 for calculation of either $f(-4)$ or $f(6)$. Aw 2 for correct answer.

14(a)(iii)		Aw 1 for correct stationary points. Aw 1 for correct endpoints Aw 2 for correct stationary points and correct endpoints.
14(b)	Domain of $g(x)=\sqrt{x}$ is $x\geq 0$. $\therefore$ this is the domain of $f(g(x))$. $f(g(x))=(\sqrt{x})^4-2(\sqrt{x})^2$ ✓ $=x^2-2x, \quad (x\geq 0)$ ✓  ✓	Aw 1 for substitution of $g(x)=\sqrt{x}$ into $f(x)$. Aw 2 for $f(g(x))$ with correct restricted domain. Aw 3 for correct graph of $f(g(x))$.

14(c)(i)	Any integer in the range $56 < x < 64$.	Aw 1 for answer.
14(c)(ii)	$z = \dfrac{x - \mu}{\sigma}$ $= \dfrac{72 - 60}{8}$ ✓ $= 1.5$, as required.	Aw 1 for $\dfrac{72 - 60}{8}$
14(c)(iii)	$x = \mu + z\sigma$ $= 60 + 1(8)$ ✓ $= 68$	Aw 1 for answer.
14(c)(iv)	With a z-score of 1, Zoe's result will be better than 84% of scores according to the empirical results. Since Tyler's result is better than 93.3% of scores, the percentage between Zoe and Tyler is 9.3%. ✓ Students represented by the shaded region $= 9.3\% \times 85000$ $= 7905$ ✓	Aw 1 for an indication that Zoe's mark is better than 84% of scores. Aw 2 for 7905.
15(a)(i)	$\int_2^4 k(x-1)^2\,dx = 1$ ✓ $\left[\dfrac{k(x-1)^3}{3}\right]_2^4 = 1$ $\dfrac{k(4-1)^3}{3} - \dfrac{k(2-1)^3}{3} = 1$ $\dfrac{k}{3}\left(3^3 - 1^3\right) = 1$ $\dfrac{26k}{3} = 1$ $\therefore k = \dfrac{3}{26}$, as required. ✓	Aw 1 for setting the integral of the PDF equal to 1 as per tick. Award 2 for correct answer with adequate working given it is a show that question.
15(a)(ii)	$\text{CDF} = \int_2^x \dfrac{3}{26}(x-1)^2\,dx$ ✓ $= \left[\dfrac{(x-1)^3}{26}\right]_2^x$ $= \left(\dfrac{(x-1)^3}{26}\right) - \dfrac{1}{26}$ $= \dfrac{(x-1)^3 - 1}{26}$ ✓	Aw 1 mark for each line with a tick.

15(a)(iii)	Upper quartile is found by solving CDF $= 0.75$ $0.75 = \dfrac{(x-1)^3 - 1}{26}$ ✓ $19.5 = (x-1)^3 - 1$ $20.5 = (x-1)^3$ $\therefore\ x - 1 = \sqrt[3]{20.5}$ $\therefore\ x = \sqrt[3]{20.5} + 1$ ≈ 3.736851837 (by calculator) ✓ ≈ 3.74 , as required.	Aw 1 for setting the CDF equal to 0.75 as per tick. Award 2 for correct answer with adequate working given it is a show that question.
15(b)(i)	$T = 22 + 162(1.2)^{-0.5t}$ when $t = 6$, $T = 22 + 162(1.2)^{-0.5(6)}$ ✓ $\approx 116°C$	Aw 1 for substitution of 6 for *t*.
15(b)(ii)	$T = 22 + 162(1.2)^{-0.5t}$ $\therefore\ \dfrac{dT}{dt} = 162(-0.5)(1.2)^{-0.5t}(\ln 1.2)$ ✓ $= -81(1.2)^{-0.5t}(\ln 1.2)$ when $t = 6$, $= -81(1.2)^{-0.5(6)}(\ln 1.2)$ $\approx -8.5°C$ per minute. ✓	Aw 1 for correct derivative Aw 2 for correct answer. Aw 1 if student omits the "ln(1.2)" from their derivative but proceeds to complete the question correctly. (They will get $-46.875°C$ per minute.)
15(c)(i)	The areas of the annuli moving outwards form the arithmetic sequence, $3\pi,\ 5\pi,\ 7\pi,\ \ldots$ $T_n = a + (n-1)d$ $\therefore\ T_{10} = 3\pi + (10-1)2\pi$ $= 21\pi$ ✓	
15(c)(ii)	$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $= \dfrac{n}{2}\left[2(3\pi) + (n-1)2\pi\right]$ ✓ $= n\left[3\pi + (n-1)\pi\right]$ $= n\left[3\pi + n\pi - \pi\right]$ $= n\left[n\pi + 2\pi\right]$ $\therefore\ S_n = \pi n^2 + 2n\pi$, as required. ✓	

15(c)(iii)	$\pi n^2 + 2n\pi \geq 300$ $\pi n^2 + 2n\pi - 300 \geq 0$ Solving, $\pi n^2 + 2n\pi - 300 = 0$, $n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{-2\pi \pm \sqrt{4\pi^2 + 1200\pi}}{2\pi}$ But $n > 0$, $\therefore\ n = \dfrac{-2\pi + \sqrt{4\pi^2 + 1200\pi}}{2\pi}$ only. So $n \approx 8.8$ So, considering the shape of the quadratic function, $f(x) = \pi n^2 + 2n\pi - 300$, The minimum value of n would be 9 meaning 9 annuli are required to give a total area of 300 u^2.	
16(a)(i)	$P = (x-1)(x-4)(x-9)$ $= (x-1)(x^2 - 13x + 36)$ $= x^3 - 13x^2 + 36x - x^2 + 13x - 36$ $= x^3 - 14x^2 + 49x - 36$, as required.	Aw 1 for establishment of the result through expansion.
16(a)(ii)	P is minimised when $P' = 0$. $\therefore 0 = 3x^2 - 28x + 49$ $= (3x-7)(x-7)$ $\therefore x = \dfrac{7}{3}$ or 7 But x is an integer. $\therefore x = 7$ only.	Aw 1 for $x = 7$. Aw 2 for $x = 7$ and establishing that this value minimises P. Aw 3 for complete solution.

	$P'' = 6x - 8$ When $x = 7$ $= 14$ > 0 $\therefore x = 7$ minimise P. When $x = 7$ $P = (7-1)(7-4)(7-9)$ $= -36$	
16(b)	$A_n = 700000(1.003)^n - 4000[1 + 1.003 + (1.003)^2 + \ldots + (1.003)^{n-1}]$ $400000 = 700000(1.003)^n - 4000\left(\frac{1.003^n - 1}{0.003}\right)$ $1200 = 2100(1.003)^n - 4000(1.003^n - 1)$ $1200 = 4000 - 1900(1.003^n)$ $1.003^n = \frac{28}{19}$ $n = \frac{\ln\left(\frac{28}{19}\right)}{\ln 1.003}$ ≈ 129.448963, by calculator ≈ 129 months.	Aw 1 for successful use of the sum to n terms of a GP formula. Aw 2 for $1.003^n = \frac{28}{19}$. Accept decimal equivalent on RHS. Aw 3 for answer **with** calculator display.
16(c)(i)	Area $\approx \frac{3}{2}\left[\ln 1 + 2(\ln 4) + \ln 7\right]$ ≈ 7.078	Aw 1 for $h = 3$ Aw 2 for correct answer.
16(c)(ii)	$\frac{d}{dx}(x \ln x) = x\left(\frac{1}{x}\right) + \ln x(1)$ $= 1 + \ln x$	Aw 1 for use of product rule. Award 2 for, $1 + \ln x$.
16(c)(iii)	From (b), $\int_1^7 (1 + \ln x)\,dx = [x \ln x]_1^7$ $\int_1^7 1\,dx + \int_1^7 \ln x\,dx = [x \ln x]_1^7$ $\int_1^7 (\ln x)\,dx = [x \ln x]_1^7 - \int_1^7 1\,dx$ $\int_1^7 (\ln x)\,dx = [x \ln x]_1^7 - [x]_1^7$ $= (7 \ln 7 - 0) - (7 - 1)$ $= 7 \ln 7 - 6$	Aw 1 for each line that has a tick.

16(c)(iv)	From (i) and (iii), $7\ln 7 - 6 \approx 7.078$ $7\ln 7 \approx 13.078$ $\therefore \ln 7 \approx 1.87$	Aw 1 for correct answer