KILLARA HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2021 **YEAR 12**

Mathematics Advanced

Trial Examination

Date: Exam block (18/8-26/8)

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks

- Allow about 15 minutes for this section

Section II - 90 marks

- Allow about 2 hours 45 minutes for this section

Q	Marks
MC	
11	
12	
13	
14	
15	
16	
17	
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19	
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22	
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27	
28	
29	
31	
32	
Total	

This question paper must not be removed from the examination room.

This assessment task constitutes 0% of the course.

Section I

10 marks
Allow about 15 minutes for this section.

Use the multiple-choice sheet for Question 1–10

1 Which of the following is equivalent to $\frac{1}{\sqrt{5}-2}$?

(A) $\dfrac{\sqrt{5}-2}{5}$

(B) $\sqrt{5}+2$

(C) $\dfrac{\sqrt{5}-2}{3}$

(D) $\sqrt{5}-2$

2 The derivative of e^{5x^2} is:

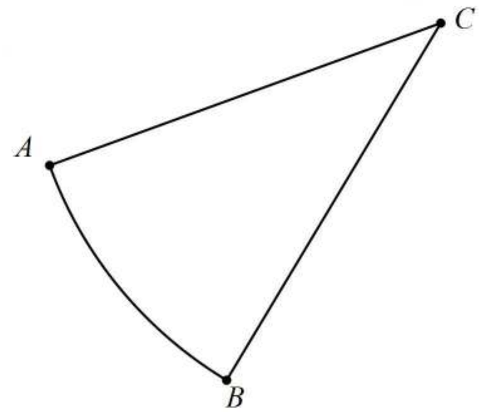
(A) $2e^{5x^2}$

(B) $10e^{5x^2}$

(C) $2xe^{5x^2}$

(D) $10xe^{5x^2}$

3 The sector ABC is shown in the diagram below. The length of arc AB is $\frac{6\pi}{5}$ units and the length of BC is 4 units.



The area of the sector is closest to which of the following?

(A) 0.942 units^2

(B) 6.472 units^2

(C) 7.540 units^2

(D) 15.080 units^2

4 Which of the following expressions is equivalent to $4 + \log_2 x^2$?

(A) $8 + 2\log_2(x)$

(B) $\log_2(16 + x^2)$

(C) $2\log_2(4x)$

(D) $\log_2(2x^2)$

5 At a point $P(k-2, y)$ on $f(x) = kx^2 - kx - k^2$ the gradient of the tangent is $3k - 8$. What is the value of k?

(A) 2

(B) 1

(C) 0

(D) -1

6 Consider the bivariate data given in the table below.

x	1.5	1.6	1.5	1.7	1.8
y	2.4	2.6	2.5	2.8	M

Given that the correlation coefficient is $r \approx 0.95$ to 2 decimal places, which of the following could be the value of M?

(A) $M = 3.1$

(B) $M = 3.2$

(C) $M = 3.3$

(D) $M = 3.4$

7 Evaluate:

$$\int_0^3 \frac{2x}{x^2+9}\,dx$$

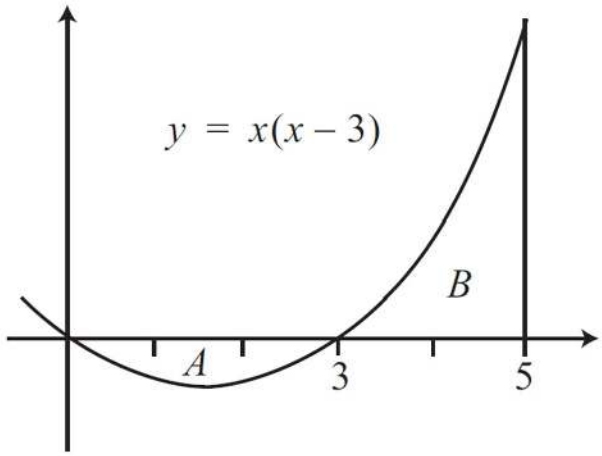
(A) $\frac{1}{2}\ln 2$

(B) $\ln 2$

(C) $2\ln 2$

(D) $\ln 18$

8 Find the total area bounded by $y = x(x-3)$, the x-axis and the ordinates $x = 0$ and $x = 5$.



(A) $4\frac{1}{6}$

(B) $4\frac{1}{2}$

(C) $8\frac{2}{3}$

(D) $13\frac{1}{6}$

9 Let $f(x) = x^3 - (m^2 - 4)x + 1$.
For which of the values of m below is $f(x)$ many-to-one?

(A) $m = -4$

(B) $m = -2$

(C) $m = 1$

(D) $m = 2$

10 Carol, Zachary, and Matilda are in a class together and they all completed the same test. They received their result as shown in the table below.

Name	Mark	z-score (2 d.p.)
Carol	32	0.46
Zachary	31	0.35
Matilda	30	0.12

After comparing results, they realised that one of the scores must be incorrect. Which of the following could be a correction to the results?

(A) Carol's mark should be corrected to 33

(B) Matilda's mark should be corrected to 29

(C) Matilda's z-score should be corrected to 0.22

(D) Zachary's z-score should be corrected to 0.38

End of Section I

Section II

90 marks
Allow about 2 hours and 45 minutes for this section.

In Questions 11 – 32, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

If $f(x) = \frac{3x}{x^2-2}$, find $f'(x)$. **2**

Question 12 (3 marks)

Find the primitives of the following functions:

(a) $f(x) = x^3 + \dfrac{2}{x^2}$ **2**

(b) $g(x) = \cos(2x + 1)$ **1**

Do NOT write in this area.

Question 13 (5 marks)

The random variable X has the probability distribution given in the following table.

X	1	2	3	4	5	6
$P(X = x)$	0.15	0.2	0.35	0.05	0.2	0.05

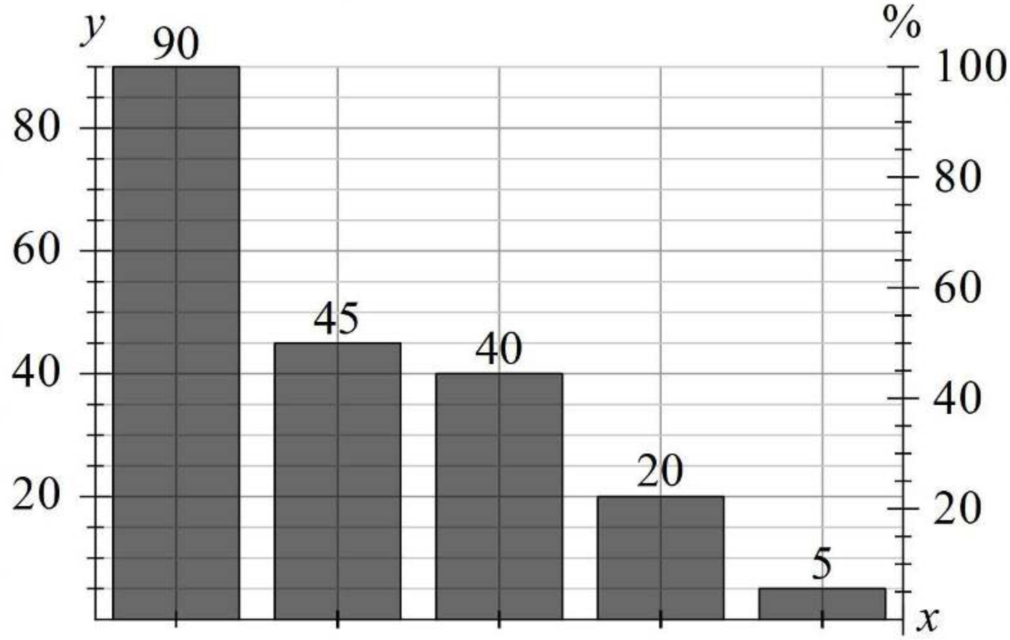
(a) Find $E(X)$. **1**

(b) Calculate the variance of X. **2**

(c) Find $P(X > 2|X \leq 5)$. **2**

Question 14 (2 marks)

A pareto chart below has been partially completed. The bar chart is complete, however, the line graph has not been included. **2**



Complete the pareto chart. You may use the space below for working.

Do NOT write in this area.

Question 15 (4 marks)

(a) Use the trapezoidal rule with four function values to approximate the region bounded by the curve $y = \frac{1}{x}$ and the x-axis between $x = 1$ and $x = 4$. Give your answer to 4 decimal places. **2**

(b) Explain whether this approximation is an underestimate or an overestimate of the true area. Include a sketch to support your explanation. **2**

Question 16 (6 marks)

In a music class of 28, 17 students play the violin (V) and 13 students play the clarinet (C). Five students play neither of these musical instruments.

(a) Draw a Venn diagram to represent these events using the labels V and C. **1**

(b) A student is chosen at random from the class.
Find the probability that the student plays:

(i) Both the violin and the clarinet **1**

....

....

....

(ii) Either the violin or the clarinet **1**

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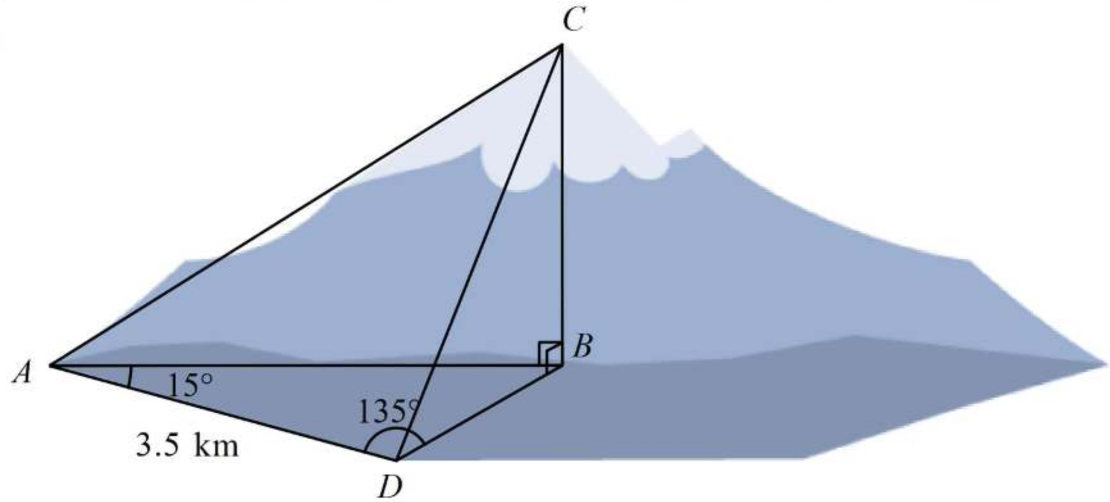
....

(c) A duet is a performance of two students, one playing the violin and the other playing the clarinet. **3**

If two students are chosen at random from the class, what is the probability that they can play a duet together?

Question 17 (3 marks)

A novice hiker is trying to determine whether she should begin her hike up a mountain from point A or point D, 3.5 km away. She measures the angle of inclination from point A to the peak of the mountain, C, to be 36°.



Assume points A, B and D are on a level plane, where B is the point directly below the peak of the mountain. $\angle DAB$ is 15° and $\angle BDA$ is 135°.

(a) Show that $AB = \frac{3.5 \sin 135°}{\sin 30°}$ **1**

(b) Hence, or otherwise, find the angle of inclination from point D to the peak of the mountain. Give your answer to the nearest minute. **2**

Do NOT write in this area.

Question 18 (3 marks)

Charlie is applying for membership at a new golf club that has a strong reputation. The scores of his current club have a lower quartile of 110 and an upper quartile of 118 and Charlie's score is not an outlier. **3**

Given that the club he plans to join has scores with a lower quartile of 90 and an upper quartile of 106, can you assure Charlie that his score will not be an outlier in his new club? Justify your answer with appropriate calculations.

Question 19 (6 marks)

Mani is a fruit grower. After his oranges are picked, they are sorted by a machine, according to size. The distribution of the diameter, in centimetres, of oranges is modelled by a continuous random variable, X, with probability density function given below.

$$f(x) = \begin{cases} -\dfrac{3x^3}{4} + 15x^2 - 99x + 216 & 6 \leq x \leq 8 \\ 0 & \text{Otherwise} \end{cases}$$

(a) Find the cumulative distribution function $F(x)$ for the domain $6 \leq x \leq 8$. **2**

(b) Hence or otherwise, find the probability that a randomly selected orange has a diameter greater than 7cm. **2**

Do NOT write in this area.

(c) Find the mode of the distribution. **2**

Question 20 (5 marks)

The time, X minutes, taken by Fred to install a satellite dish may be assumed to be normally distributed, with a mean of 134 and a standard deviation of 16.

The table below provides some values of the probabilities for the standard normal distribution.

i.e. $\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \phi(t)dt$

z	*first decimal place*									
	.0	**.1**	**.2**	**.3**	**.4**	**.5**	**.6**	**.7**	**.8**	**.9**
0.	0.500	0.540	0.579	0.618	0.655	0.692	0.726	0.758	0.788	0.816
1.	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971
2.	0.977	0.982	0.986	0.989	0.992	0.994	0.995	0.997	0.997	0.998
3.	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(a) Calculate $P(X < 150)$. **2**

(b) Fred's company policy states that safe installations require a minimum of a minutes to complete. Fred has been informed that 5.5% of his installation times do not meet this minimum standard. **3**

Find the value of a.

Do NOT write in this area.

Question 21 (2 marks)

Given the functions $f(x) = \frac{1}{x-1}$ and $g(x) = e^x$, state the domain of $f\big(g(x)\big)$. Give your answer in **interval notation**. **2**

Question 22 (8 marks)

Consider the function $y = 2xe^{\frac{x}{2}}$.

(a) Find the coordinates of any stationary points and determine their nature. **3**

(b) Find coordinates of any points of inflection. **2**

(c) For what values of x is the curve concave down? **1**

....

....

....

....

(d) Draw a sketch of the curve, showing all critical points. **2**

Question 23 (3 marks)

The effectiveness of a drug is measured by its ability to reduce the number of bacteria in a culture over time. The culture is reduced according to the model $N = N_0 e^{-kt}$ where N is the number of bacteria and t is the time since the drug was administered in minutes. **3**

A drug is tested in a culture initially with 1200 bacteria, which reduces to 200 in 6 minutes. How many bacteria are left in the culture after 3 minutes?

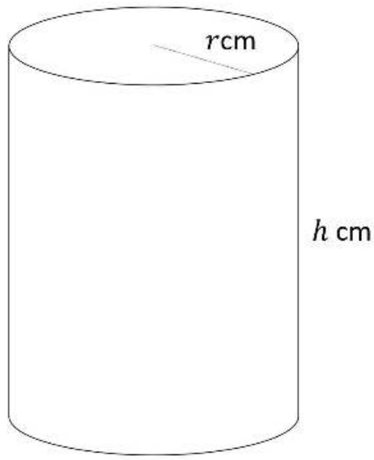
Do NOT write in this area.

Question 24 (2 marks)

Given that $\sin\theta = \frac{6+\sqrt{2}}{2\sqrt{19}}$ and θ is obtuse, find the exact value of $\tan\theta$ in simplest form. **2**

Question 25 (5 marks)

A tin can with a closed lid is made in the shape of a cylinder as shown below. The lid has a radius of r cm and a height of h cm.



The volume of the can is 192π cm^3.

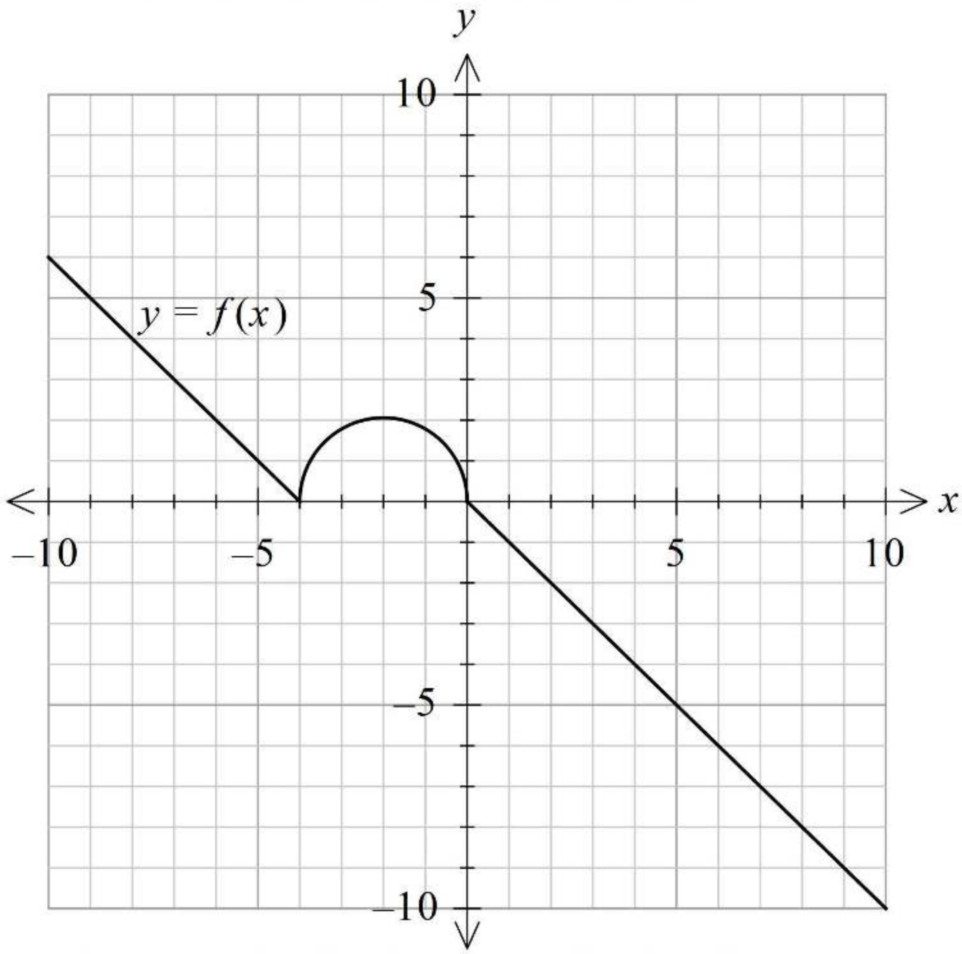
(a) Show that the total surface area of the can, A cm^2, is given by: **2**

$$A = 2\pi\left(r^2 + \frac{192}{r}\right)$$

(b) Find the value of r for which the tin has a minimum surface area. **3**

Question 26 (3 marks)

(a) The graph of $y = f(x)$ is given below. **2**
On the same set of axes, sketch $y = f(2(x+2))$.



(b) Hence, determine the number of solutions of the equation $f(2(x+2)) = |x+3|$. **1**

....

....

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Do NOT write in this area.

Question 27 (4 marks)

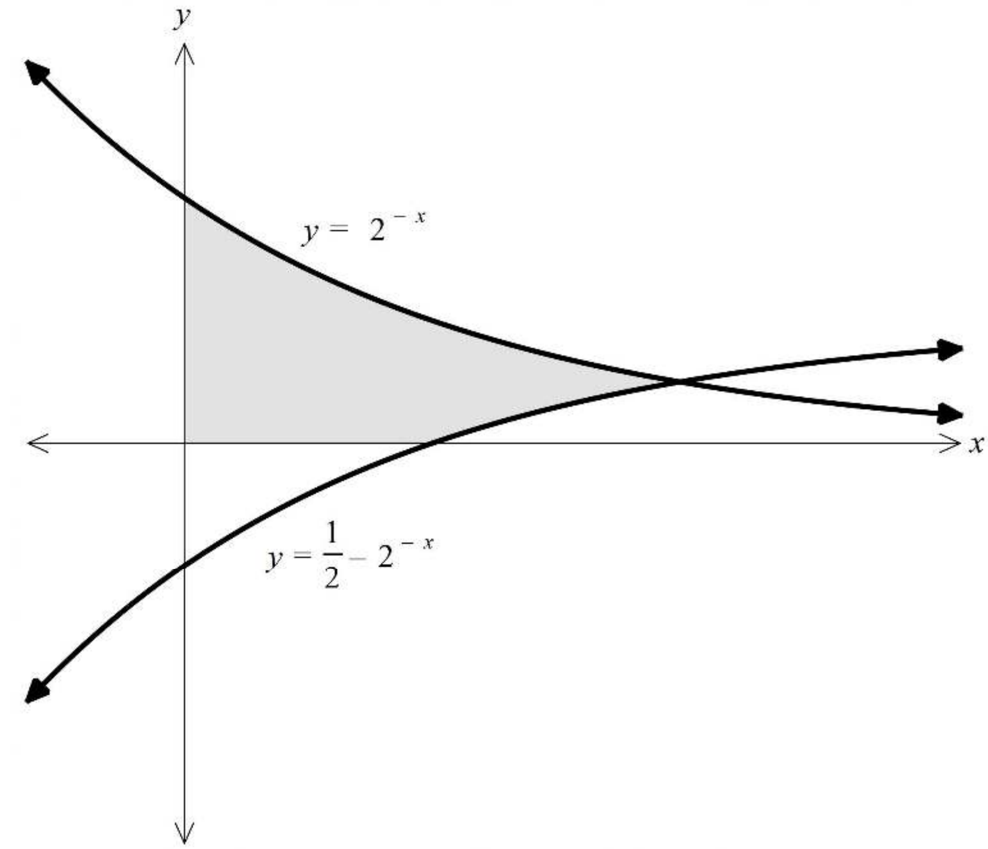
(a) Differentiate $\ln(\cos(2x))$. **2**

(b) Hence, or otherwise, evaluate: **2**

$$\int_0^{\frac{\pi}{6}} \tan(2x)\, dx$$

Question 28 (5 marks)

The diagram below shows the graphs of $y = 2^{-x}$ and $y = \frac{1}{2} - 2^{-x}$ and the point of intersection.



(a) By solving simultaneously, show that the point of intersection occurs when $x = 2$. **1**

Do NOT write in this area.

Do NOT write in this area.

(b) Hence, or otherwise, find the shaded area of the region bounded by the x-axis, the y-axis and the two curves, correct to 4 decimal places. **4**

Question 29 (4 marks)

(a) Show that **1**

$$\log_a b = \frac{1}{\log_b a}$$

(b) Hence, solve the equation **3**

$$6\log_x 2 + \log_2 x - 5 = 0$$

Do NOT write in this area.

Question 30 (5 marks)

A weight is attached to a spring that is connected to the ceiling. The weight is pulled away from the ceiling and then released so that it bounces up and down. **5**

Let the distance of the weight from the ceiling be x metres. The spring stretches and contracts such that x varies sinusoidally with time, between 1.2 metres and 1.8 metres. It takes 1 second for the weight to go from its highest point to its lowest point.

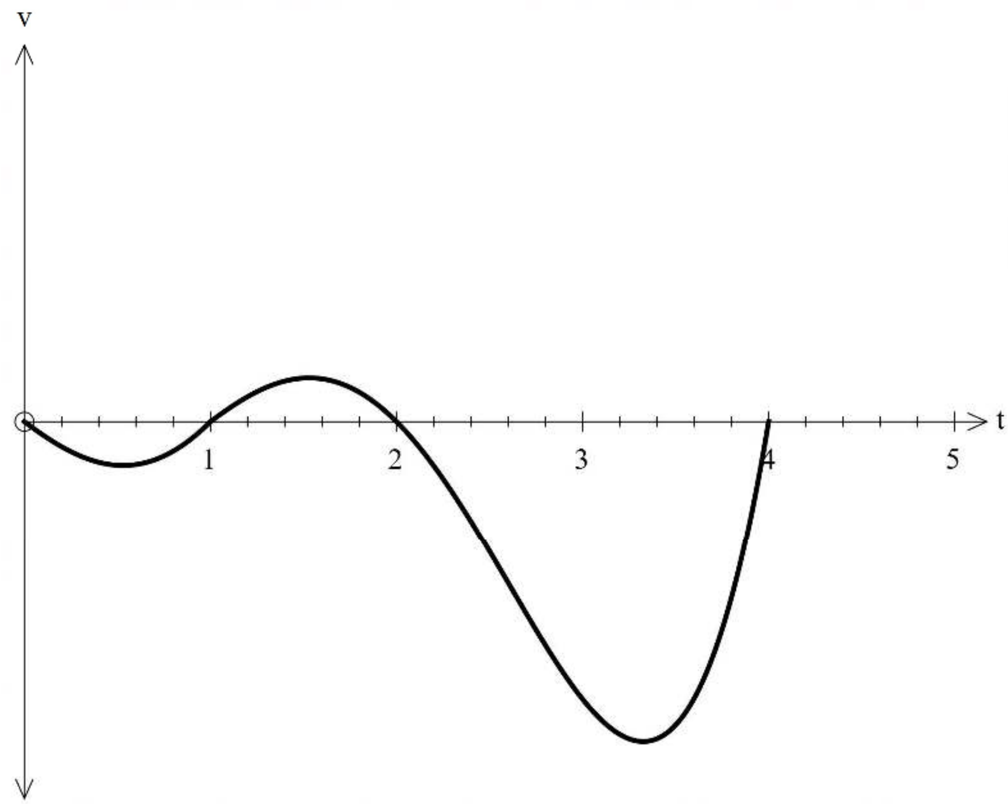
The motion of the weight can be modelled using a sine wave of the form:

$$x = a\sin(bt + c) + d$$

How far from the ceiling is the weight 2.7 seconds after it is at a high point?

Question 31 (6 marks)

The velocity of a particle moving in a straight line is shown below. The particle starts at the origin.



(a) When does the particle return to the origin? **1**

....

....

(b) When is the particle furthest from the origin? **1**

....

....

(c) When is the particle experiencing the greatest acceleration? **1**

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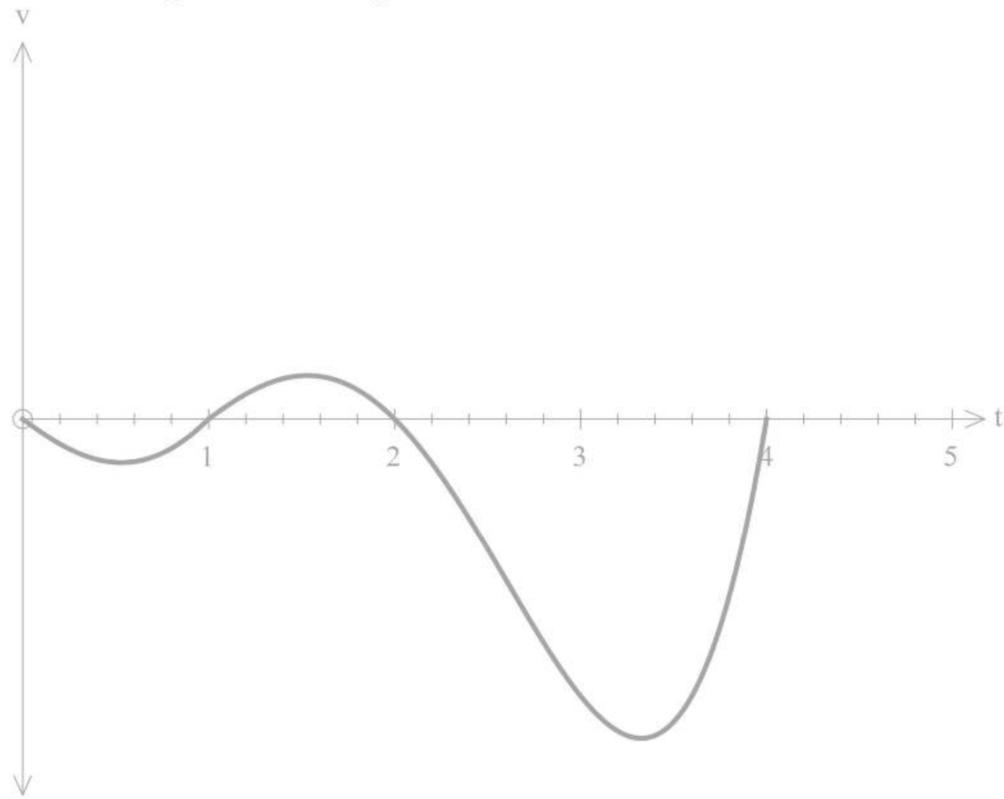
....

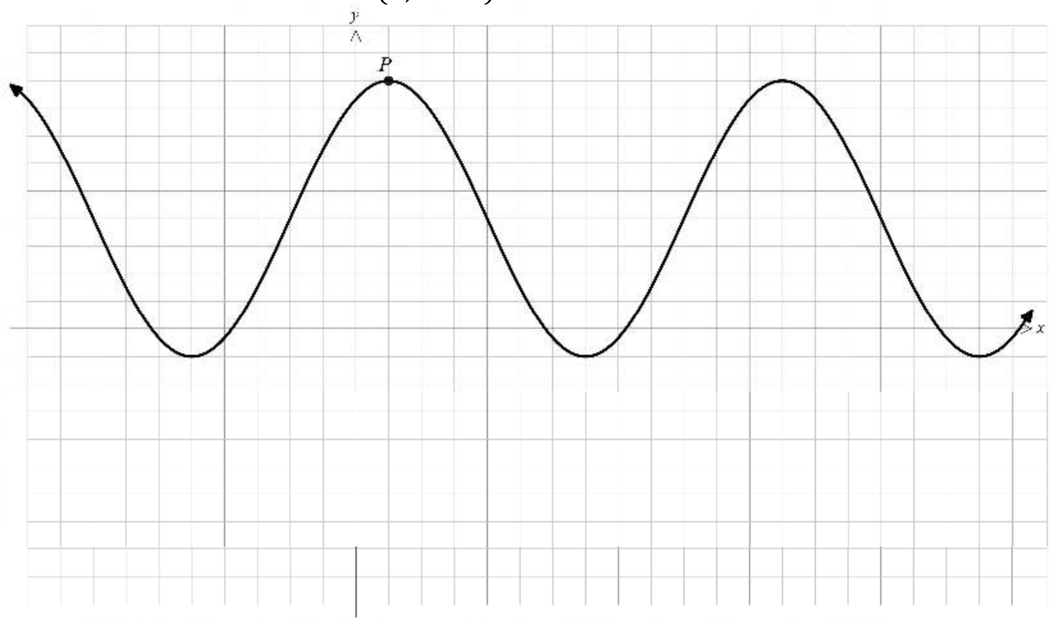
Do NOT write in this area.

(d) Sketch the displacement of the particle as a function of time. **3**



Question 32 (4 marks)

The graph of the function $y = \cos\big(2a(x-b)\big) + c$ is shown below, where a, b and c are constants. Point P has coordinates $(b, c+1)$. **4**



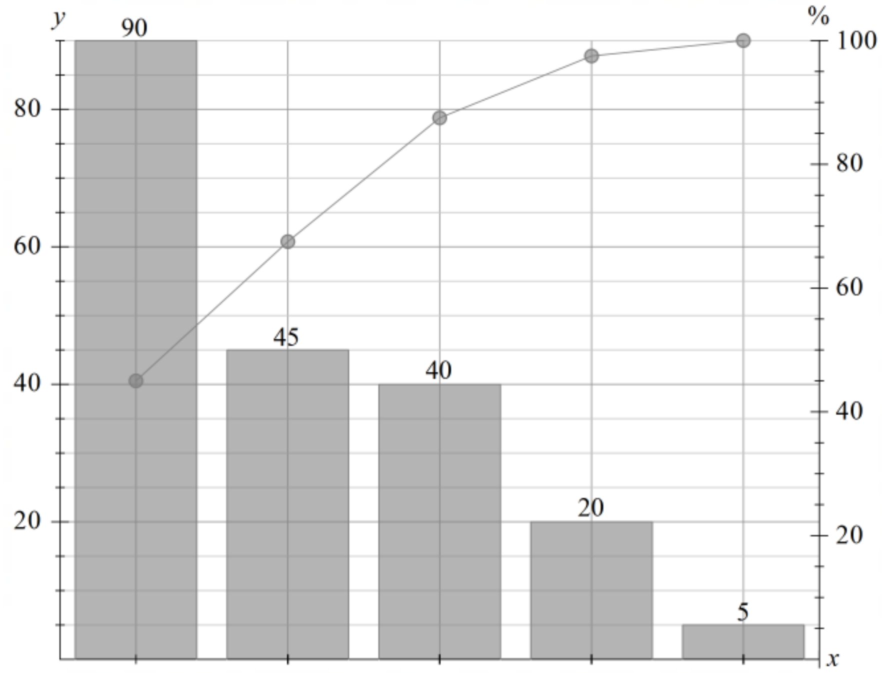
On the same set of axes, sketch:

$$y = -c\sin(3ax - 3ab)$$

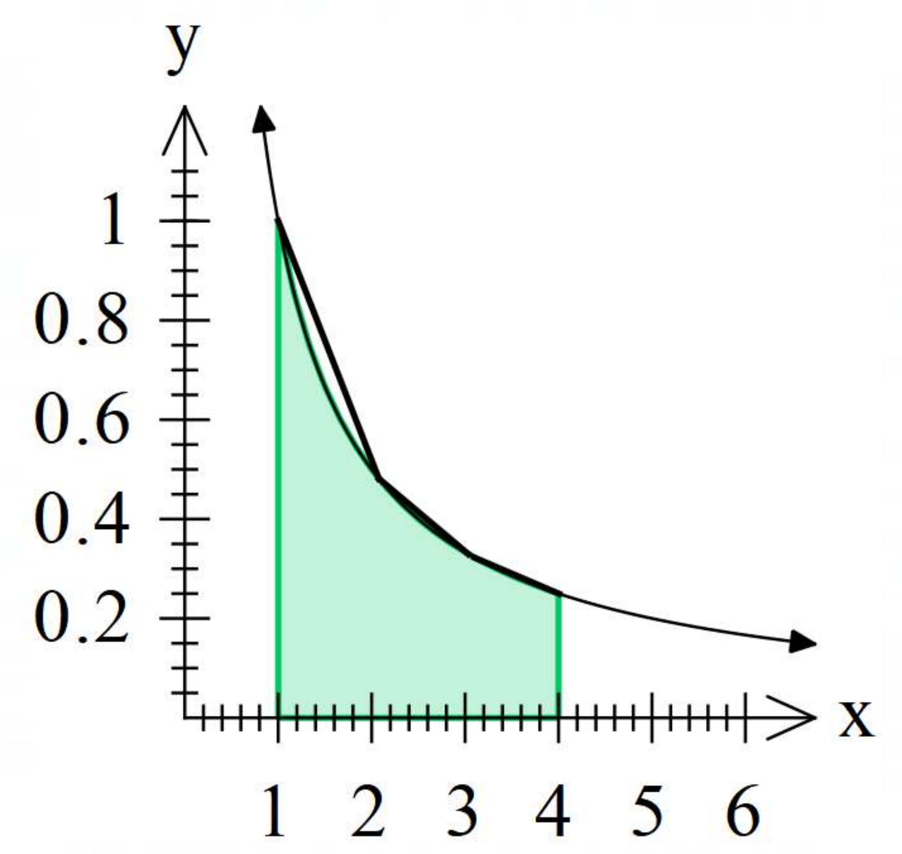
You may use the space below for working.

Q	Solution	Marking Guidelines
1	B $$\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$ $$=\frac{\sqrt{5}+2}{5-4}$$ $$=\frac{\sqrt{5}+2}{1}$$ $$=\sqrt{5}+2$$	
2	D $$\frac{d}{dx}\left(e^{5x^2}\right)$$ $$=e^{5x^2}\times\frac{d}{dx}(5x^2)$$ $$=e^{5x^2}\times 10x$$ $$=10xe^{5x^2}$$	
3	$$l=r\theta=\frac{6\pi}{5}$$ $$r=4$$ $$A=\frac{1}{2}r^2\theta=\frac{1}{2}(r\theta)\times r$$ $$=\frac{1}{2}\times\frac{6\pi}{5}\times 4$$ $$=\frac{12\pi}{5}$$ $$=7.540\ (3\ d.p.)$$ C	
4	C $$4+\log_2 x^2$$ $$=4\log_2 2+\log_2 x^2$$ $$=\log_2 2^4+\log_2 x^2$$ $$=\log_2 16+\log_2 x^2$$ $$=\log_2 16x^2$$ $$=\log_2(4x)^2$$ $$=2\log_2 4x$$	
5	A $$f'(x)=2kx-k$$ $$f'(k-2)=2k(k-2)-k$$ $$=2k^2-4k-k$$ $$=2k^2-5k$$ $$2k^2-5k=3k-8$$ $$2k^2-8k+8=0$$ $$k^2-4k+4=0$$ $$(k-2)^2=0$$ $$k-2=0$$ $$k=2$$	
6	D	
7	B $$I=[\ln(x^2+9)]_0^3$$ $$=\ln 18-\ln 9$$ $$=\ln\frac{18}{9}$$ $$=\ln 2$$	
8	D Approximating using triangles.	

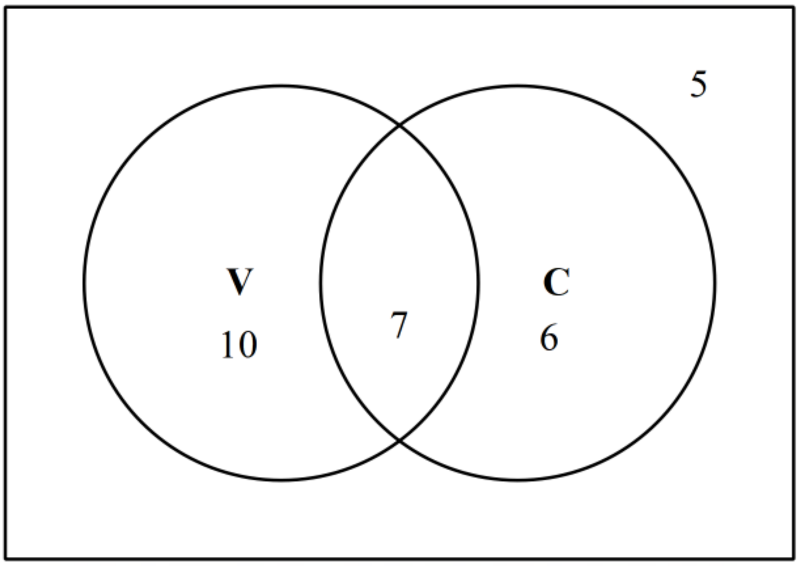
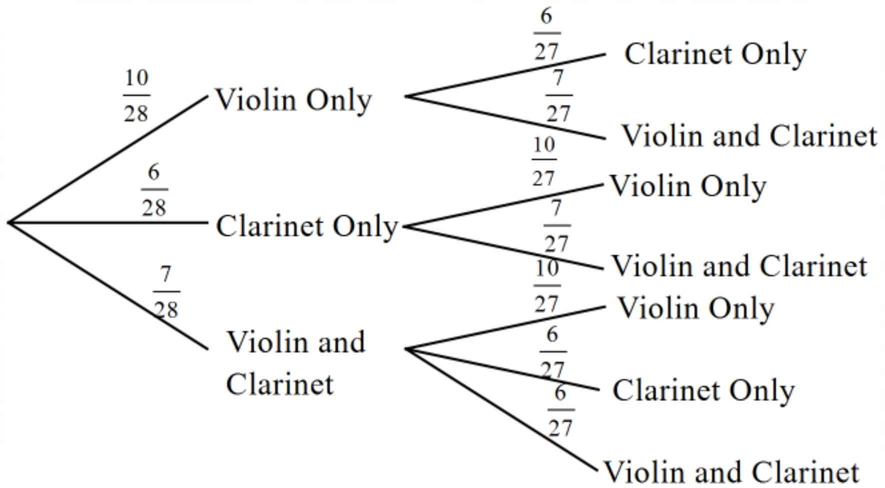
	When $x=\frac{3}{2}, y=-\frac{9}{4}$ Area A $\approx \frac{1}{2}\times 3\times\frac{9}{4}$ $$\approx\frac{27}{8}=3\frac{3}{8}$$ When $x=5, y=10$ Area B$\approx \frac{1}{2}\times 2\times 10$ $$\approx 10$$ Total area$=A+B$ $$\approx 3\frac{3}{8}+10$$ $$\approx 13\frac{3}{8}$$	
9	A Many-to-one occurs when the cubic has multiple stationary points. Therefore, discriminant of the first derivative is greater than zero. $$f'(x)=3x^2-(m^2-4)$$ $$\Delta=b^2-4ac$$ $$=0+12(m^2-4)$$ $\Delta>0$, when $12(m^2-4)>0$ $$m^2-4>0$$ $$(m+2)(m-2)>0$$ $$m>2 \text{ or } m<-2$$ Therefore, $m=-4$ is the correct value.	
10	B	
11	Quotient rule $$u=3x, u'=3$$ $$v=x^2-2, v'=2x$$ $$y'=\frac{3(x^2-2)-3x\times 2x}{(x^2-2)^2}$$ $$=\frac{3x^2-6-6x^2}{(x^2-2)^2}$$ $$=\frac{-3x^2-6}{(x^2-2)^2}$$	1 mark for applying quotient rule 1 mark for correct substitutions
12a	$$f(x)=x^3+2x^{-2}$$ $$F(x)=\frac{x^4}{4}-2x^{-1}+C$$	1 mark for integral 1 mark for +C
b	$$G(x)=\frac{\sin(2x+1)}{2}+C$$	No mark lost for +C
13a	$$E(X)=1\times 0.15+2\times 0.2+3\times 0.35+4\times 0.05+5\times 0.2+6\times 0.05$$ $$=3.1$$	
b	$$E(X^2)=1^2\times 0.15+2^2\times 0.2+3^2\times 0.35+4^2\times 0.05+5^2\times 0.2$$ $$+6^2\times 0.05$$ $$=11.7$$ $$E(X)^2=3.1^2$$ $$=9.61$$ $$Var(X)=11.7-9.61$$ $$=2.09$$	1 mark for $E(X^2)=11.7$ or $E(X)^2=9.61$ 1 mark for $Var(X)=2.09$ ECF allowed from part (a)
c	$$P(X>2\mid X\le 5)=\frac{0.35+0.05+0.2}{0.15+0.2+0.35+0.05+0.2}$$ $$=\frac{0.6}{0.95}$$ $$=\frac{12}{19}$$	1 mark for correct numerator or denominator 1 mark for simplified answer

<table>
<tr><td>14</td><td>$Total = 90 + 45 + 40 + 20 + 5 = 200$
</td><td>1 mark for noting Total=200

1 mark for line graph</td></tr>
<tr><td>15a</td><td>$h = 1$
$A \approx \frac{1}{2}\left(1 + \frac{1}{4} + 2\left(\frac{1}{2} + \frac{1}{3}\right)\right)$
$\approx \frac{35}{24}$
$\approx 1.4583\ (4\ d.p.)$</td><td>1 mark for $\frac{h}{2} = \frac{1}{2}$ or $\frac{b-a}{2n} = \frac{4-1}{2\times 3} = \frac{1}{2}$

1 mark for $\frac{35}{24}$ or decimal equivalent</td></tr>
<tr><td>b</td><td>
Trapezoidal rule gives an overestimate as the function is concave up for the given domain.</td><td>1 mark for diagram

1 mark for overestimation stating “concave up”</td></tr>
</table>

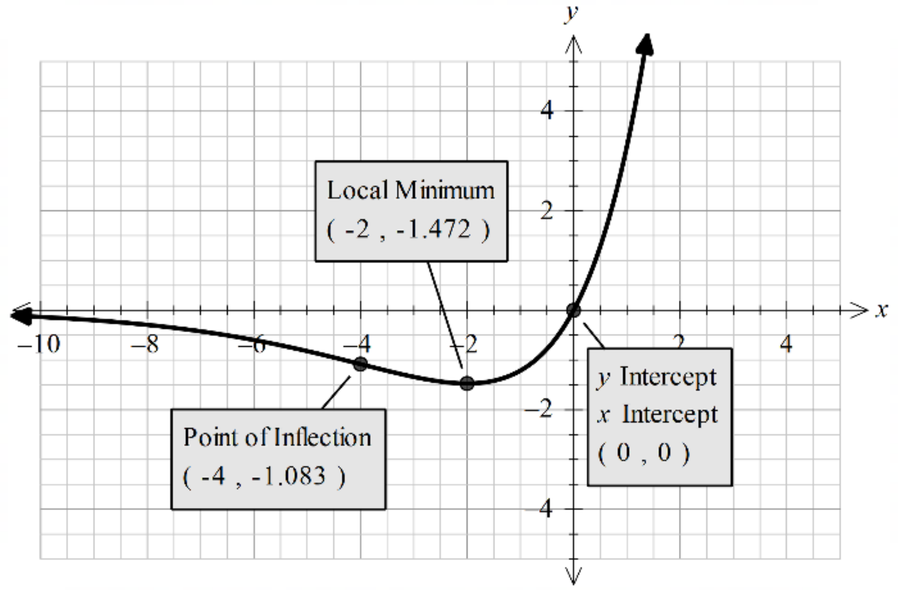
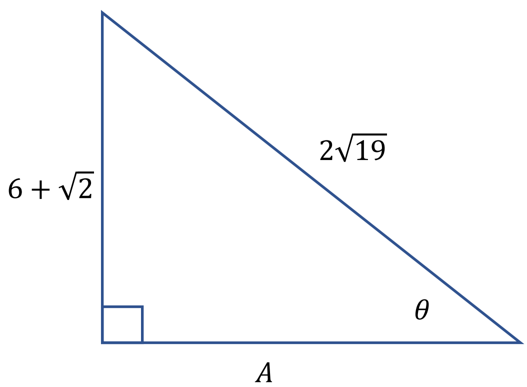
16a		Must give all correct values
bi	$\frac{7}{28} = \frac{1}{4}$	ECF allowed
ii	$\frac{23}{28}$	ECF allowed
c	 $P(duet) = P(V,C) + P(V,both) + P(C,V) + P(C,both) + P(both,V) + P(both,C) + P(both,both)$ $= (2 \times (P(V,C) + P(V,both) + P(C,both)) + P(both,both)$ $= 2 \times \left(\frac{10}{28} \times \frac{6}{27} + \frac{10}{28} \times \frac{7}{27} + \frac{6}{28} \times \frac{7}{27}\right) + (\frac{7}{28} \times \frac{6}{27})$ $= \frac{193}{378}$ $= 51.06\%$	3 marks for correct answer 2 marks for applying addition and multiplication rules with minor errors 1 mark for finding one possible duet or partially correct tree diagram
17a	$\angle ABD = 180 - 15 - 135$ $= 30°$ $\frac{AB}{\sin 135} = \frac{3.5}{\sin 30}$ $AB = \frac{3.5 \sin 135°}{\sin 30\,°}$ $\approx 4.950\ (3\ d.p.)$	Must show adequate working.
b	$\tan 36 = \frac{CB}{AB}$ $CB = \tan 36 \times 4.950$ ≈ 3.596 $\frac{BD}{\sin 15} = \frac{3.5}{\sin 30}$ $BD = \frac{3.5 \sin 15}{\sin 30}$ ≈ 1.812	1 mark for CB or stating $\theta = \tan^{-1}\left(\frac{CB}{BD}\right)$ 1 mark for final answer No penalty for rounding.

	$\tan\angle CDB = \frac{CB}{BD}$ $\angle CDB = \tan^{-1}\left(\frac{CB}{BD}\right)$ $= \tan^{-1}\frac{3.596}{1.812}$ $\approx 63°16'(Nearest\ minute)$	
18	$IQR = 118 - 110 = 8$ $1.5 \times IQR = 12$ $Upper\ bound = 118 + 12 = 130$ $Lower\ bound = 110 - 12 = 98$ $\therefore Charlie's\ score\ is\ between\ 98\ and\ 130$ For the new club $IQR = 106 - 90 = 16$ $1.5 \times IQR = 24$ $Upper\ bound = 106 + 24 = 130$ $Lower\ bound = 90 - 24 = 66$ ∴Charlie is not an outlier in the new club as he is not outside the bounds for the new club.	1 mark for finding IQR of either club 1 mark for finding range of charlie's score between 98 and 130 1 mark for comparing charlie's score to the bounds of the new club AND concluding statement
19a	$f(x) = -\frac{3x^3}{4} + 15x^2 - 99x + 216$ $F(x) = \int_6^x \frac{3t^3}{4} + 15t^2 - 99t + 216\,dt$ $= \left[\frac{3}{16}t^4 + 5t^3 - \frac{99}{2}t^2 + 216t\right]_6^x$ $= -\frac{3}{16}x^4 + 5x^3 - \frac{99}{2}x^2 + 216x - 351$	1 mark for stating integrals 1 mark for final cdf
b	$P(X > 7) = 1 - P(X < 7)$ $= 1 - F(7)$ $= 1 - \frac{5}{16}$ $= \frac{11}{16}$	2 marks for final answer 1 mark for $F(7) = \frac{5}{16}$
c	Mode occurs at global maximum (i.e. at turning points (stationary points) or at the ends of the domain) $f(6) = 0$ $f(8) = 0$ To find stationary points: $f'(x) = -\frac{3}{4}(3x^2 - 40x + 132)$ $f'(x) = 0$ when $3x^2 - 40x + 132 = 0$ $(3x - 22)(x - 6) = 0$ $x = 6\ or\ x = \frac{22}{3}$ $f(6) = 0$ $f\left(\frac{22}{3}\right) = \frac{8}{9}$ $\therefore Mode\ of\ the\ distribution\ is\ \frac{22}{3}$	1 mark for checking end points 1 mark for checking finding $x = \frac{22}{3}$ and checking $f\left(\frac{22}{3}\right)$
20a	$z = \frac{150 - 134}{16} = 1$ $P(X < 150) = P(Z < 1)$ $= 0.841\ (from\ table)$ $= 84.1\%$ OR	1 mark for z=1 1 mark for 84.1% from table of 84% from empirical rule

<table>
<tr><td></td><td>$$= 68\% + 13.5\% + 2.35\% + 0.15\%$$
$$= 84\% \ (empirical\ rule)$$</td><td></td></tr>
<tr><td>b</td><td>$$P(X < a) = 0.055$$
$$P(X > a) = 1 - 0.055$$
$$= 0.945$$
From the table $P(Z < 1.6) = 0.945$
And hence $P(Z > -1.6) = 0.945$
Therefore, a corresponds to a z-score of -1.6
$$a = 134 - 1.6 \times 16$$
$$= 108.4$$</td><td>1 mark for $1 - 0.055 = 0.945$
1 mark for $z = -1.6$
1 mark for $a = 108.4$</td></tr>
<tr><td>21</td><td>$$f(g(x)) = \frac{1}{e^x - 1}$$
Restrictions occur at:
$$e^x - 1 \neq 0$$
$$e^x \neq 1$$
$$x \neq 0$$
Hence, the domain of $f(g(x))$ is $(-\infty, 0) \cup (0, \infty)$.</td><td>1 mark for
$$x \neq 0$$

1 mark for interval notation</td></tr>
<tr><td>22a</td><td>$$f(x) = 2xe^{\frac{x}{2}}$$
$$u = 2x, u' = 2, v = e^{\frac{x}{2}}, v' = \frac{1}{2}e^{\frac{x}{2}}$$
$$f'(x) = 2e^{\frac{x}{2}} + xe^{\frac{x}{2}}$$
$$= (2 + x)e^{\frac{x}{2}}$$
$f'(x) = 0$, when
$$2 + x = 0$$
$$x = -2$$
$$f(-2) = \frac{-4}{e} \approx -1.47$$
<table><tr><td>x</td><td>-3</td><td>-2</td><td>0</td></tr><tr><td>$f'(x)$</td><td>-0.22</td><td>0</td><td>2</td></tr><tr><td></td><td>\</td><td>_</td><td>/</td></tr></table>
$\therefore$ Minimum turning point at $(-2, \frac{-4}{e})$</td><td>1 mark for derivative
1 mark for $x = -2$ and $y = \frac{-4}{e} \approx -1.47$
1 mark for checking nature using table or second derivative</td></tr>
<tr><td>b</td><td>$$f'(x) = (2 + x)e^{\frac{x}{2}}$$
$$u = (2 + x), u' = 1, v = e^{\frac{x}{2}}, v' = \frac{1}{2}e^{\frac{x}{2}}$$
$$f''(x) = e^{\frac{x}{2}} + \frac{2 + x}{2}e^{\frac{x}{2}}$$
$$= e^{\frac{x}{2}}\left(1 + \frac{2 + x}{2}\right) = e^{\frac{x}{2}}\left(\frac{4 + x}{2}\right)$$
$f''(x) = 0$ when
$$4 + x = 0$$
$$x = -4$$
<table><tr><td>x</td><td>-5</td><td>-4</td><td>-3</td></tr><tr><td>$f''(x)$</td><td>-0.041</td><td>0</td><td>0.11</td></tr><tr><td></td><td>∩</td><td>·</td><td>∪</td></tr></table>
$$f(-4) = -\frac{8}{e^2} \approx -1.083$$
$$\therefore \text{ Point of inflection at } (-4, -\frac{8}{e^2})$$</td><td>1 mark for $x = -4$

1 mark for checking change in concavity using a table or equivalent</td></tr>
<tr><td>c</td><td>$$x < -4$$</td><td></td></tr>
</table>

d	y-intercepts at $x = 0$ $$f(0) = 0$$ x-intercepts at $y = 0$ $$2xe^{\frac{x}{2}} = 0$$ $$x = 0$$ 	1 mark for x/y-intercept and marking the critical points from part (a) and (b) 1 mark for correct shape (must tend towards left asymptote $y = 0$
23	When $t = 0, N = 1200$ $$\therefore 1200 = N_0 e^0$$ $$N_0 = 1200$$ $$N = 1200e^{-kt}$$ When $t = 6, N = 200$ $$200 = 1200e^{-6k}$$ $$e^{-6k} = \frac{200}{1200}$$ $$= \frac{1}{6}$$ $$-6k = \ln\frac{1}{6}$$ $$k = \frac{\ln\frac{1}{6}}{-6}$$ $$k \approx 0.299\ (3\ d.p.)$$ $$N = 1200e^{-0.299t}$$ When $t = 3$, $$N = 1200e^{-0.299\times 3}$$ $$= 489.898\ (3\ d.p.)$$	1 mark for $$N_0 = 1200$$ 1 mark for k 1 mark for answer
24	Constructing a right-angled triangle in quadrant 2. 	1 mark for $$A^2 = \left(2\sqrt{19}\right)^2 - \left(6 + \sqrt{2}\right)^2$$ 1 mark for $$\tan\theta = -\frac{6+\sqrt{2}}{6-\sqrt{2}}$$ $$OR$$ $$\tan\theta = \frac{6+\sqrt{2}}{\sqrt{2}-6}$$

<table>
<tr><td></td><td>

$$A^2 = \left(2\sqrt{19}\right)^2 - \left(6+\sqrt{2}\right)^2$$
$$= 76 - \left(36 + 12\sqrt{2} + 2\right)$$
$$= 76 - \left(38 + 12\sqrt{2}\right)$$
$$= 38 - 12\sqrt{2}$$
$$= 36 - 12\sqrt{2} + 2$$
$$= \left(6 - \sqrt{2}\right)^2$$
$$A = 6 - \sqrt{2}$$
$$\therefore \tan\theta = -\frac{6+\sqrt{2}}{6-\sqrt{2}}$$
$$OR$$
$$\tan\theta = \frac{6+\sqrt{2}}{\sqrt{2}-6}$$

</td><td></td></tr>
<tr><td>25a</td><td>

Given that $V = 192\pi = \pi r^2 h$

$$h = \frac{192\pi}{\pi r^2}$$
$$= \frac{192}{r^2} \quad (1)$$

For surface area of a cylinder $A = 2\pi r^2 + 2\pi rh$

$$= 2\pi(r^2 + rh) \ (2)$$

Substituting (1) in (2)

$$A = 2\pi(r^2 + r\left(\frac{192}{r^2}\right))$$
$$= 2\pi\left(r^2 + \frac{192}{r}\right)$$

</td><td>1 mark for substituting 192π into volume formula

1 mark for substituting into surface area and finding answer</td></tr>
<tr><td>b</td><td>

$$A = 2\pi\left(r^2 + \frac{192}{r}\right)$$
$$A = 2\pi(r^2 + 192r^{-1})$$
$$A' = 2\pi(2r - 192r^{-2})$$
$$A'' = 2\pi(2 + 384r^{-3})$$

For stat points

$$A' = 0$$
$$2r - \frac{192}{r^2} = 0$$
$$2r = \frac{192}{r^2}$$
$$2r^3 = 192$$
$$r^3 = 96$$
$$r = \sqrt[3]{96}$$

When $r = \sqrt[3]{96}$,

$$A'' = 2\pi\left(2 + \frac{384}{r^3}\right)$$
$$= 2\pi\left(2 + \frac{384}{96}\right)$$
$$A'' > 0$$
$$\therefore \mathit{Minimum\ turning\ point\ when}\ r = \sqrt[3]{96}$$

Hence, the minimum surface area occurs when

$$r = \sqrt[3]{96}$$

</td><td>1 mark for derivative
1 mark for
$r = \sqrt[3]{96}$

1 mark for proving nature using table of second derivative</td></tr>
<tr><td>26a</td><td>

Transformations:

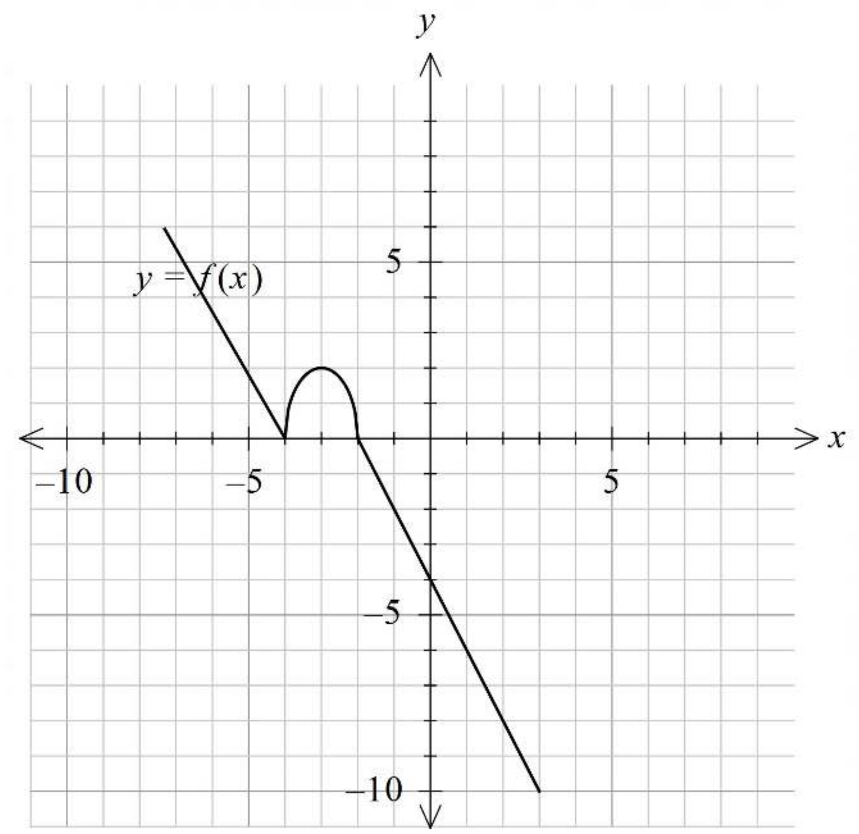
1. Horizontal dilation by ½
2. Horizontal translation left by 2

Transformation of Key Points:

<table><tr><td>Original Point</td><td>After Dilation</td><td>After Translation</td></tr><tr><td>$(-10,6)$</td><td>$(-5,6)$</td><td>$(-7,6)$</td></tr></table>

</td><td>2 marks for correct sketch

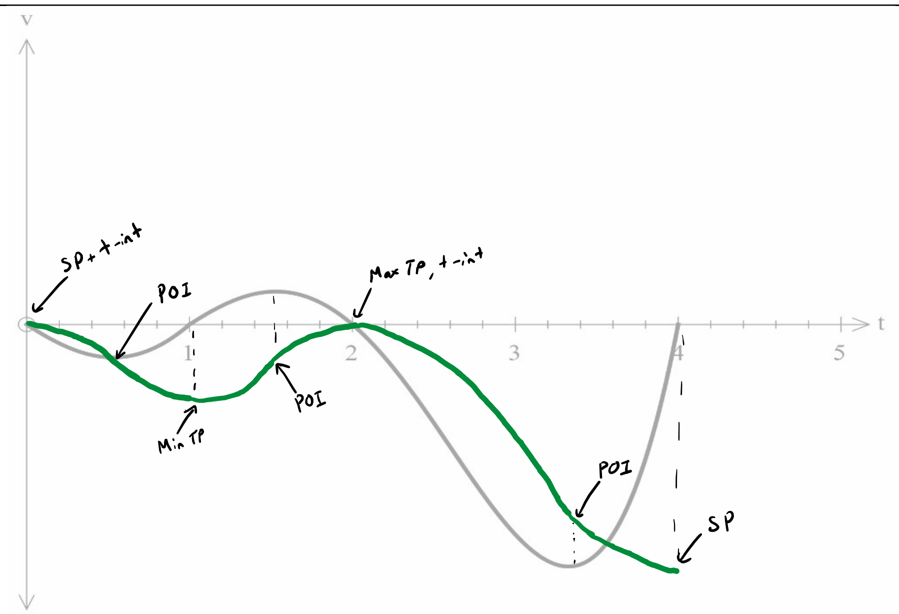
1 mark if any one of the following:
- Function has been dilated correctly
- Function has been translated correctly</td></tr>
</table>

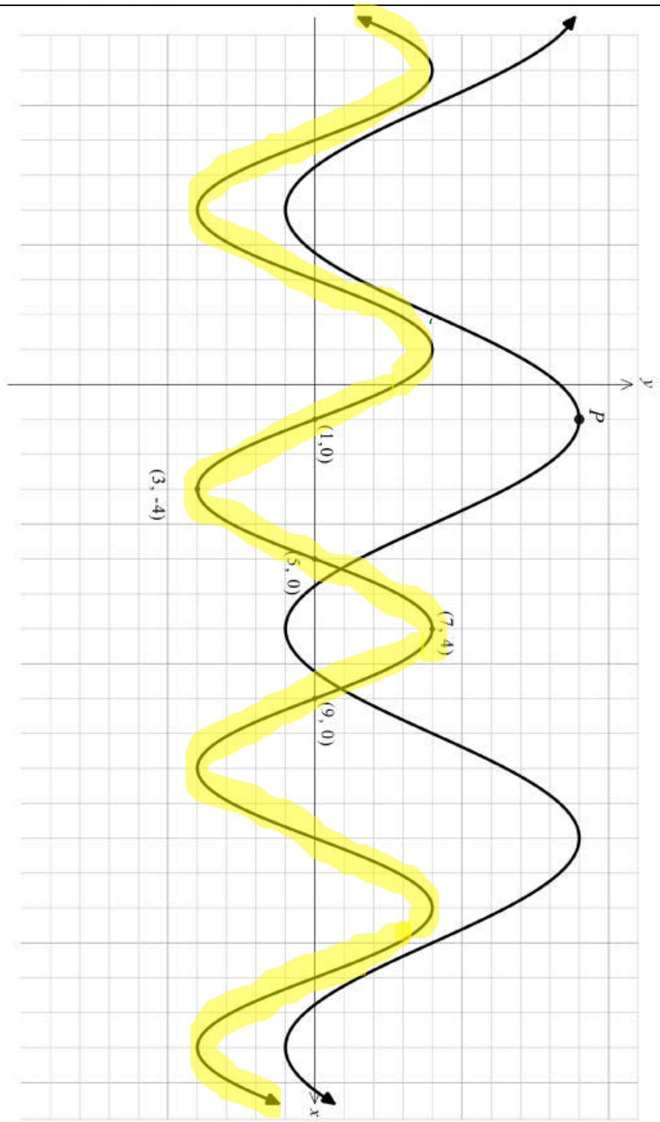
<table>
<tr>
<td></td>
<td>
<table>
<tr><td>$(-4,0)$</td><td>$(-2,0)$</td><td>$(-4,0)$</td></tr>
<tr><td>$(-2,2)$</td><td>$(-1,2)$</td><td>$(-3,2)$</td></tr>
<tr><td>$(0,0)$</td><td>$(0,0)$</td><td>$(-2,0)$</td></tr>
<tr><td>$(10,-10)$</td><td>$(5,-10)$</td><td>$(3,-10)$</td></tr>
</table>


</td>
<td>- A section of the function has been transformed correctly</td>
</tr>
<tr>
<td>b</td>
<td>$3\ solutions$</td>
<td>ECF allowed from part (a)</td>
</tr>
<tr>
<td>27a</td>
<td>Let $u = \cos 2x$
$$u' = -2\sin 2x$$
$$y = \ln u$$
$$y' = \frac{u'}{u}$$
$$= \frac{-2\sin 2x}{\cos 2x}$$
$$= -2\tan 2x$$</td>
<td>2 marks for final answer
1 mark for single mistake</td>
</tr>
<tr>
<td>b</td>
<td>$$I = \int_0^{\frac{\pi}{6}} \tan(2x)\,dx$$
$$= -\frac{1}{2}\int_0^{\frac{\pi}{6}} -2\tan(2x)\,dx$$
$$= -\frac{1}{2}[\ln(\cos(2x)]_0^{\frac{\pi}{6}}$$
$$= -\frac{1}{2}\left(\ln\left(\cos\frac{\pi}{3}\right) - \ln(\cos 0)\right)$$
$$= -\frac{1}{2}\left(\ln\left(\frac{1}{2}\right) - \ln(1)\right)$$
$$= -\frac{1}{2}\left(\ln\left(\frac{1}{2}\right)\right)$$
$$= \frac{1}{2}\ln 2$$</td>
<td>2 marks for final answer in exact form
$-\frac{1}{2}\left(\ln\left(\frac{1}{2}\right)\right)$ OR $\frac{1}{2}\ln 2$

1 mark for correct integration
ECF allowed from part (a)</td>
</tr>
<tr>
<td>28a</td>
<td>$$\frac{1}{2} - 2^{-x} = 2^{-x}$$
$$\frac{1}{2} = 2 \times 2^{-x}$$
$$\frac{1}{2} = 2^{1-x}$$</td>
<td>Must solve simultaneously</td>
</tr>
</table>

	$2^{-1}=2^{1-x}$ $-1=1-x$ $x=2$	
b	For the x-intercept of $y=\frac{1}{2}-2^{-x}$ $\frac{1}{2}-2^{-x}=0$ $\frac{1}{2}=2^{-x}$ $2^{-1}=2^{-x}$ $x=1$ Hence, $A=\int_0^2 2^{-x}\,dx-\int_1^2 \frac{1}{2}-2^{-x}\,dx$ $=-\left[\frac{2^{-x}}{\ln 2}\right]_0^2-\left[\frac{x}{2}+\frac{2^{-x}}{\ln 2}\right]_1^2$ $=-\left(\frac{1}{4\ln 2}-\frac{1}{\ln 2}\right)$ $-(\left(\frac{2}{2}+\frac{1}{4\ln 2}\right)-\left(\frac{1}{2}+\frac{1}{2\ln 2}\right))$ $=\frac{1}{\ln 2}-\frac{1}{2}$ $=0.9427\ units^2$	1 mark for finding x-intercept 1 mark for expressing area using the correct integrals 1 mark for correct integrals 1 mark for final answer (no penalty for rounding error)
29a	RTP: $\log_a b=\frac{1}{\log_b a}$ $LHS=\log_a b$ $=\frac{\log_b b}{\log_b a}$ $=\frac{1}{\log_b a}$ $=RHS$	Must use proper setting out for proof.
b	$6\log_x 2+\log_2 x-5=0$ $\frac{6}{\log_2 x}+\log_2 x-5=0$ $6+(\log_2 x)^2-5\log_2 x=0$ $Let\ u=\log_2 x$ $u^2-5u+6=0$ $(u-3)(u-2)=0$ $u=3\ or\ u=2$ For $\log_2 x=3$ $x=2^3$ $x=8$ For $\log_2 x=2$ $x=2^2$ $x=4$ $\therefore x=4\ or\ x=8$	1 mark for applying part (a) 1 mark for creating quadratic 1 mark for solutions in x
30	As x is **distance from ceiling**, <u>high point at $x=1.2$</u> and <u>low point at $x=1.8$</u> Hence, amplitude $a=\frac{1.8-1.2}{2}=0.3$ And centre of motion is at $d=\frac{1.8+1.2}{2}$ $d=1.5$ Time taken from high to low is 1s. Hence, $period=2\times 1=\frac{2\pi}{b}$ $b=\frac{2\pi}{2}$ $b=\pi$ Assuming the weight starts ($t=0$) at the centre of motion	1 mark for amplitude and centre of motion 1 mark for $b=\pi$ 1 mark for recognising that highest point corresponds to $x=1.2$ 1 mark for finding $t=1.5$ (or equivalent for the x used for the highest point)

	Distance from ceiling can be modelled as: $$x = 0.3\sin(\pi t) + 1.5$$ Using this model, weight is at high point when $x = 1.2$ at $t = 1.5$ $$2.7 + 1.5 = 4.2$$ When $t = 4.2$ $$x = 0.3\sin(\pi \times 4.2) + 1.5$$ $$= 1.676\, m\ (3\, d.p.)$$ (Note: as period is 2, you could also use $t = 1.5 + 0.7$ or $t = 0.2$)	1 mark for substituting $t = 4.2\ (or\ 2.2\ or\ 0.2)$ (or equivalent for the x used for the highest point) to find answer
31a	$t = 2$ as signed area of $\int_0^t f(x)\,dx = 0$ when $t = 2$	
b	$t = 4$ as $\lvert \int_0^t f(x)\,dx \rvert$ is largest when $t = 4$	
c	As $f(x)$ is velocity, $f'(x)$ is acceleration. Greatest acceleration is when gradient of the function is greatest. $t = 4$	
d		Mark lost for each of the following criteria missing - Function is never positive - t-intercepts at 0 and 2 - stationary points at $t = 0,1,2,4$ - greatest displacement at $t = 4$
32	**Original function:** $y = \cos\big(2a(x-b)\big) + c$ **New function:** $y = -c\sin(3a(x-b))$ - Centre of motion at c corresponds to the amplitude of the new function - Both functions have been translated horizontally by b - Ratio of periods of the original function to the new function: $2a : 3a = 2 : 3$. Therefore, the new function will sketch 3 periods within the same time that the original function sketched 2 periods. (i.e. 3 periods in 24 boxes or 1 period in 8 boxes) - The new function will be a negative sine curve.	1 mark for each of the following: - Recognises the functions have the same horizontal translation and/or starts the centre of motion of the new function below point P at $x = b$ - Sketches a negative sine curve from $x = b$ or $x = 0$ - Matches amplitude of the new function to the centre of motion of the original function - New function has period of 8 boxes.



<u>Note: The coordinates marked are not the real coordinates. They are using the grid as a reference for marking/to check your answer.</u>