Student Number (print neatly) | | | | | | | | |

Place a ✓ next to your teacher's name and your class

12MA22 *Ms Lakshmipathy*		12MA23 *Mr Sivasothy*	
12MA24 *Mr Carrozza / Mr Doolan*			

Homebush Boys High School

2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, all relevant mathematical reasoning and/or calculations need to be shown to be awarded with full marks

Total marks: 100

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

2021 Trial Higher School Certificate Examination
Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Section I
10 marks

Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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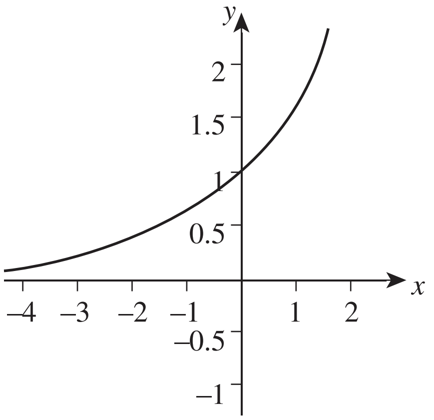
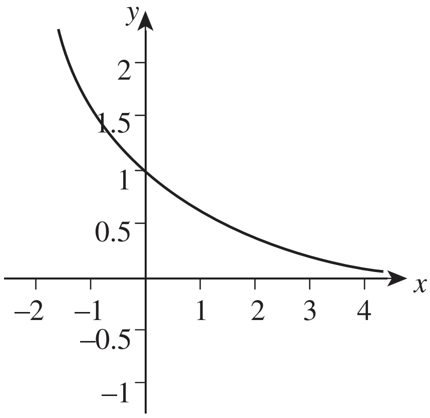
1. Which of the following graphs represents $y = -e^{ax}$, a < 0?

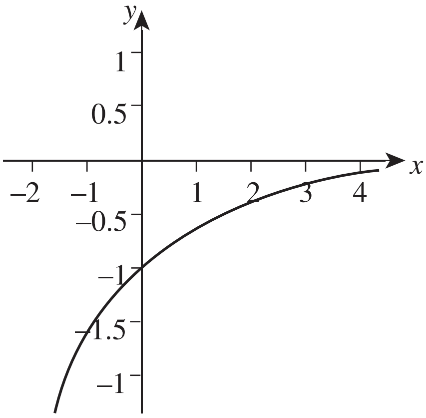
A.



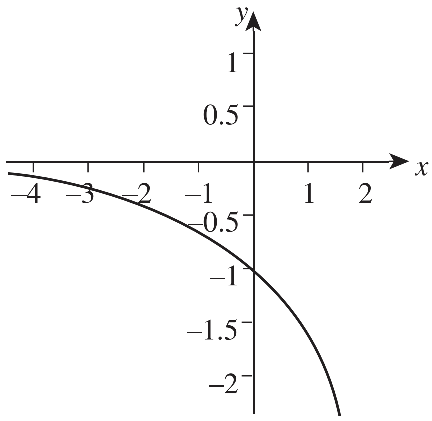
B.



C.



D.



2. What is the equation of the normal to the curve $y = \sin x$ at the origin?

A. $y = -x$

B. $y = \cos x$

C. $y = \sin x$

D. $y = x$

3. If the temperature of toffee (T) varies inversely with the square of the room temperature (C). Which equation correctly expresses T in terms of C and k, where k is the constant of variation.

A. $T = \dfrac{k}{C^2}$

B. $C = \dfrac{k}{T^2}$

C. $T = k\,C^2$

D. $T = k\,C^2 + 5$

4. Let $a = e^x$ Which expression is equal to $log_e(a^2)$?

A. e^{2x}

B. e^{x^2}

C. $2x$

D. x^2

5. The quadratic function $3x^2 - 5x + 2$ has roots α and β.

Which of the following statements is true?

(A) $2\alpha\beta = -\frac{4}{3}$

(B) $\alpha^2 + \beta^2 = \frac{13}{9}$

(C) $2\alpha + 2\beta = \frac{25}{3}$

(D) $\alpha^2\beta^2 = \frac{2}{9}$

6. The point $P\,(8,-3)$ lies on the graph of $y = f(x)$.

Find the coordinates of the image of P if the function is transformed to:

$y = -2f(x+7)+5$.

A. (1, 11)

B. (1, –1)

C. (15, 11)

D. (15, –1)

7. The equation of a circle is $(x-1)^2 + (y+1)^2 = 9$.
What is the centre and the range for this relation?

A. (1, –1) and $[-4,2]$

B. (–1, 1) and $[-4,2]$

C. (1, –1) and $[-2,4]$

D. (–1, 1) and $[-2,4]$

8. Sonia is installing a security keypad on her front door. It has the digits 0 – 9 and codes can be created which have either 4, 5 or 6 digits.

How many more codes are available if she uses a 6 digit code rather than a 4 digit code?

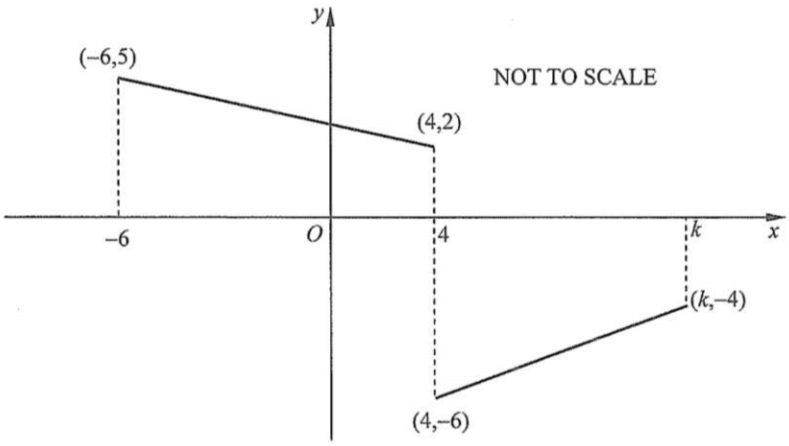
A. 9900

B. 521 441

C. 524 880

D. 990 000

9.



Use the graph above to find the value of k which satisfies

$$\int_{-6}^{k} f(x)dx = 0$$

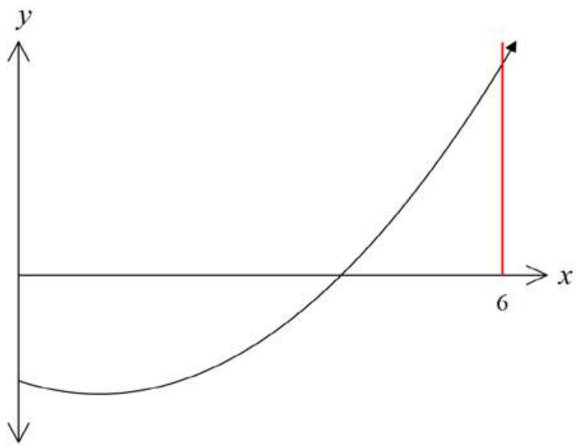
A. 6

B. 10

C. 11

D. 12

10. The diagram below shows the graph of $y = x^2 - 2x - 8$.



What is the correct expression for the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

A. $A = \int_0^5 (x^2 - 2x - 8)dx + \left|\int_5^6 (x^2 - 2x - 8)dx\right|$

B $A = \left|\int_0^4 (x^2 - 2x - 8)dx\right| + \int_4^6 (x^2 - 2x - 8)dx$

C $A = \int_0^4 (x^2 - 2x - 8)dx + \left|\int_4^6 (x^2 - 2x - 8)dx\right|$

D $A = \left|\int_0^5 (x^2 - 2x - 8)dx\right| + \int_5^6 (x^2 - 2x - 8)dx$

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2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced Section II Answer Booklet 1

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–20 (32 marks)

Booklet 2 — Attempt Questions 21–31 (58 marks)

Instructions

- Full marks may not be awarded if your responses do not show all relevant mathematical reasoning and/or calculations.

Question 11 (5 marks)

(a) Sketch the curve $y = 1 - sin2x \quad for \quad 0 \leq x \leq \pi.$ **3**

(b) On the sketch above draw $y = 2$ and write down the co-ordinates of any point(s) of intersection between $y = 1 - sin2x$ and $y = 2$ **2**

Question 12 (3 marks)

In any set of tennis between two players A and B, the probability that player A wins the set is $\frac{2}{3}$ and the probability that player B wins the set is $\frac{1}{3}$. If they continue to play sets of tennis until one of them is the first to win two sets, find the probability that it is player A who is the first to win two sets. **3**

Question 13 (4 marks)

Differentiate the following with respect to *x*:

(a) $\dfrac{e^{x^2}}{2x+3}$ **2**

(b) $ln\left(\frac{1+x}{1-x}\right)$ **2**

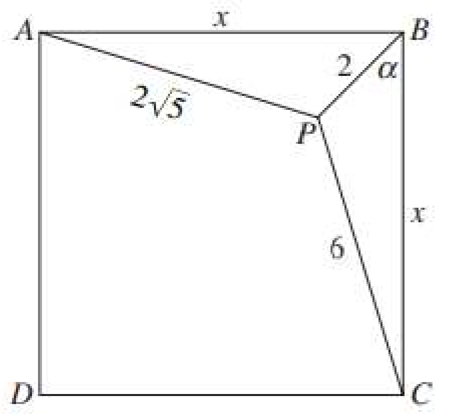
Question 14 (3 marks)

(a) Find $\int sec^2 5x\, dx$ **1**

(b)

Find $\int_{-2}^{1} \frac{2}{x+3} \mathrm{d}x$ **2**

Question 15 (5 marks)



NOT TO SCALE

The diagram shows a square *ABCD* of side *x* cm, with a point *P* within the square, such that *PC* = 6 cm, *PB* = 2 cm, and *AP* =$2\sqrt{5}$ .cm.

Let $\angle PBC = \alpha$.

(a) Using the cosine rule in triangle *PBC*, show that $cos\alpha = \frac{x^2-32}{4x}$ **2**

(b) By considering triangle *PBA,* show that $sin\alpha = \frac{x^2-16}{4x}$ **3**

Question 16 (2 marks)

Solve the pair of simultaneous equations $\log_2 x + \log_2 y = 3$ and $\log_x y = 2$ **2**

Question 17 (4 marks)

In the first week of the snow season 5 cm of snow falls. In each of the following weeks the snowfalls increase by 2 cm, so that in the second week there is a 7 cm snow fall, in the third week there is 9 cm. This continues to the middle week and, from then on, weekly snowfall decreases by 2 cm per week, the season lasting 21 weeks.

(a) How much snow falls in the 11^{th} week? **1**

(b) What is the total snowfall for the whole season? **3**

Question 18 (2 marks)

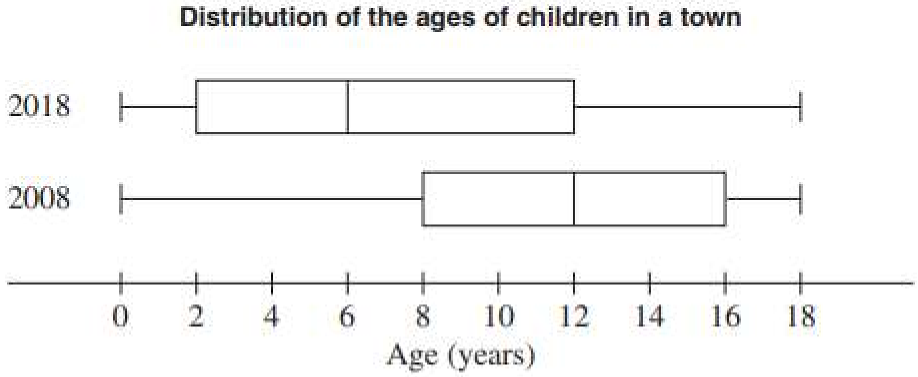
Solve $2sin^2x - 3sinx - 2 = 0 \quad for \quad 0 \le x \le 2\pi.$ **2**

Question 19 (2 marks)

By first differentiating $\ln(sinx)$ find $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cotx\,\mathrm{d}x.$ **2**

Question 20 (2 marks)

The diagram shows the distribution of the ages of children in a town in 2008 and 2018. **2**



In 2008 there were 1750 children aged 0–18 years.

The number of children aged 12-18 years was the same in both 2008 and 2018.

How many children aged 0-18 were there in 2018?

End of Questions in Booklet 1

2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced Section II Answer Booklet 2

Booklet 2 — Attempt Questions 21–34 (58 marks)

Instructions

- Full marks may not be awarded if your responses do not show all relevant mathematical reasoning and/or calculations.

Question 21 (3 marks)

(a) Complete the table of values for $y = \sqrt{4 - x^2}$ **1**

Answer to 3 significant figures where required.

x	0	0.125	0.25	0.375	0.5
y			1.98		1.94

(b) By using the Trapezoidal rule with 4 sub intervals, estimate the integral **2**

$$\int_0^{0.5} \sqrt{4 - x^2}\,\mathrm{d}x \quad .$$

Question 22 (4 marks)

(a) Find the exact values of u for which $2u^2 + \sqrt{3}\,u - 3 = 0$ **2**

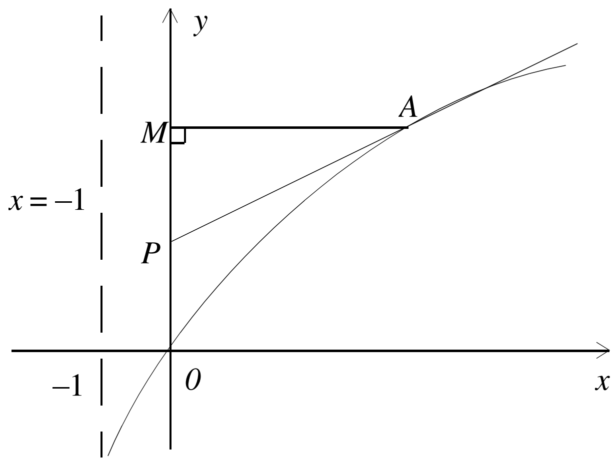
(b) Hence or otherwise solve $2\cos^2 x + \sqrt{3}\cos x - 3 = 0$ for $0 \leq x \leq 2\pi$. **2**

Question 23 (5 marks)

In the diagram, the point A whose y-coordinate is 1, lies on the curve $y = ln\,(x+1)$

The tangent to the curve at A cuts the y-axis at P.

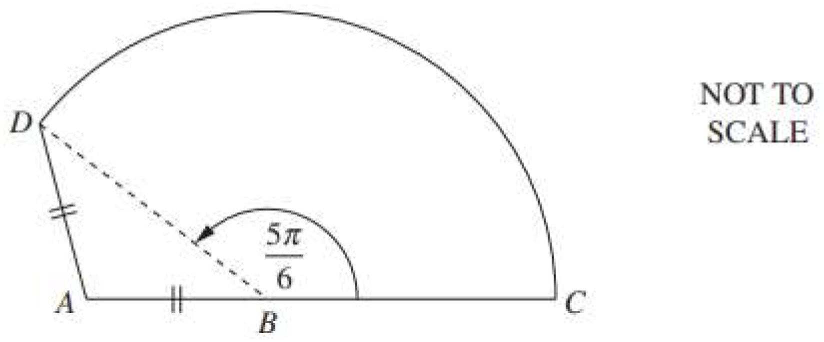
A straight line through A perpendicular to the y-axis meets the y-axis at M.



(a) Show that the x-coordinate of A is $(e-1)$. **1**

(b) Show that the equation of the tangent AP is $x - ey + 1 = 0$ **4**

Question 24 (4 marks)



In the diagram, $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \frac{5\pi}{6}$.

The points A, B and C lie on a straight line and $AB = AD = 3$ metres.

(a) Show that $\angle DAB = \frac{2\pi}{3}$. **1**

(b) Given that $BD = 3\sqrt{3}$, find the area of the garden $ABCD$. **3**

Question 25 (3 marks)

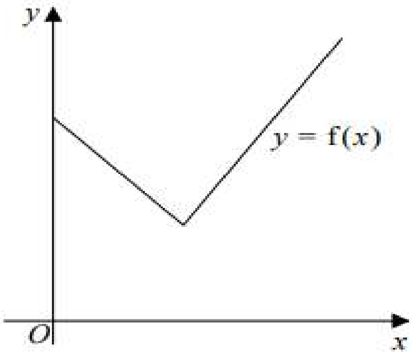
Consider the geometric series

$$1 + (\sqrt{5} - 2) + (\sqrt{5} - 2)^2 + \cdots$$

(a) Show with calculations why this geometric series has a limiting sum. **1**

(b) Find the exact value of the limiting sum with a rational denominator. **2**

Question 26 (4 marks)



(a) The figure above shows a sketch of part of the graph $y = f(x)$, where $f(x) = 2|3 - x| + 5, \quad x \geq 0$. **3**

Solve the equation $f(x) = \frac{1}{2}x + 30$.

(b) Given that the equation $f(x) = k$, where k is a constant, has two distinct roots, state the possible values of k. **1**

Question 27 (5 marks)

Stephen considers two options that will allow him to purchase a property worth $350 000 within the next fifteen years.

(a) One option is to invest an amount at the end of each year into an annuity which pays interest at 3% per annum. **1**

This table shows the ***future values of an annuity of $1*** for periods between 10 and 16 years, for different annual interest rates. The contributions are made at the end of each year.

Years	Interest Rate per annum					
	3%	4%	5%	6%	7%	8%
10	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866
11	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455
12	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771
13	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953
14	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149
15	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521
16	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243

Use the table to determine how much he would need to invest each year to achieve his target of $350 000 after 15 years.

(b) His second option is to borrow $350 000 now at the same interest rate and pay it back in monthly repayments over 15 years. **2**

This table shows the ***present values of an annuity of $1*** for periods between 165 and 172 months, for different monthly interest rates.

Months	Interest Rate per month					
	0.15%	0.20%	0.25%	0.30%	0.35%	0.40%
176	154.5830	148.2363	142.2439	136.5834	131.2338	126.1755
177	155.3500	148.9384	142.8867	137.1719	131.7726	126.6688
178	156.1158	149.6391	143.5279	137.7586	132.3095	127.1602
179	156.8805	150.3385	144.1675	138.3436	132.8446	127.6496
180	157.6440	151.0364	144.8055	138.9268	133.3777	128.1370
181	158.4064	151.7329	145.4419	139.5083	133.9090	128.6226
182	159.1677	152.4281	146.0767	140.0880	134.4385	129.1061
183	159.9278	153.1218	146.7099	140.6660	134.9661	129.5878

Use the table to find the monthly repayment necessary to pay off the loan of $350 000 in fifteen years.

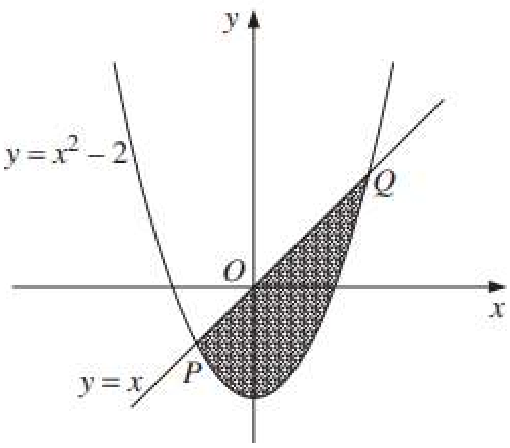
Continued on the next page

(c) Based on your answers to (a) and (b), determine which option would cost Stephen less each year in payments. **1**

(d) Outline one other factor he should consider when comparing these options. **1**

Question 28 (4 marks)

The diagram shows the graphs of $y = x^2 - 2$ and $y = x$.



(a) Show, algebraically, that the coordinates of P are (-1,-1) and find the coordinates of Q. **1**

(b) Find the area of the shaded region. **3**

Question 29 (3 marks)

Show that $$\frac{2\cos\theta\sin^2\theta + 2\cos^3\theta}{4\sin\theta} = \frac{1}{2}\cot\theta .$$ **3**

Question 30 (7 marks)

A particle is moving in a straight line, starting from the origin. At time t seconds the particle has a displacement of x metres from the origin and a velocity $v\ ms^{-1}$. The displacement is given by $x = 2t - 3\log_e(t+1)$

(a) Find an expression for the velocity v. **2**

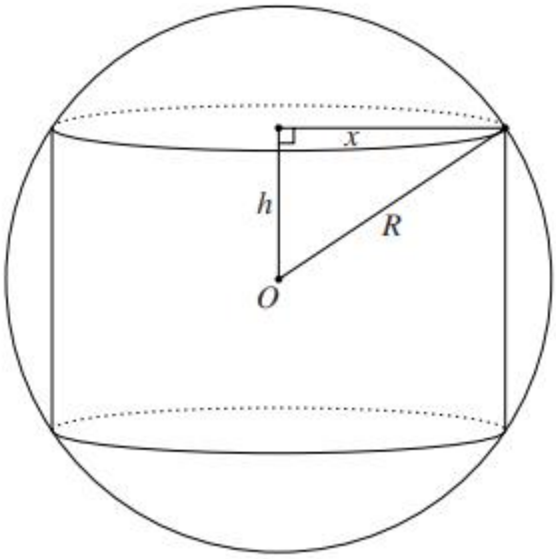
(b) Find the initial velocity. **1**

(c) Find when the particle comes to rest. **1**

(d) Find the distance travelled by the particle in the first three seconds. **3**

Question 31 (4 marks)

4



A cylinder of radius x and height $2h$ is to be inscribed in a sphere of radius R centred at O as shown.

If the volume of a cylinder of radius r and height h is $V = \pi r^2 h$, show that the volume of the cylinder in the diagram above is given by

$$V = 2\pi h(R^2 - h^2)$$

Hence show that the cylinder has a maximum volume when $h = \frac{R}{\sqrt{3}}$

Question 32 (3 marks)

On the same set of axes, draw a sketch of $y = \frac{1}{x}$ and use this to help you draw a sketch of **3**

$y = g(x) = \frac{1}{x+2} + 1$.

Question 33 (4 marks)

George invests \$100 000 which was an inheritance, in an account which earns 4% interest, compounded annually. He intends to withdraw $\$M$ at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for 25 years, after which the account will be empty.

What is the amount $\$M$, that George will receive each year? **4**

Question 34 (5 marks)

A curve is defined by $y = x^3 - 3x^2$ **5**

Sketch the curve , showing all key features including classifying the nature of stationary points and finding any inflections.
Space to draw the graph is provided on the next page. On the graph label the stationary points and point of inflection

End of Paper

2020 Trial HSC Examination

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

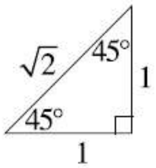
$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

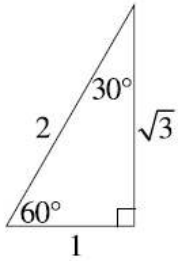


$A = \frac{1}{2}ab\sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \frac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

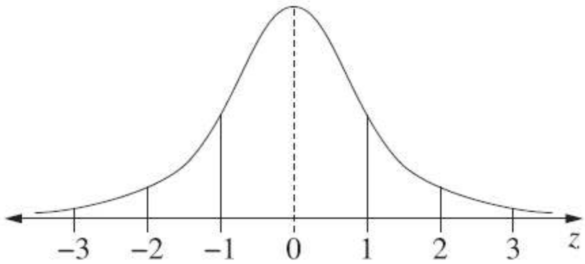
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \int_a^x f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$

$X \sim \text{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\Big\{f(a) + f(b) + 2\big[f(x_1) + \cdots + f(x_{n-1})\big]\Big\}$$

where $a = x_0$ and $b = x_n$