



NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Standard 2

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 15 marks (pages 2–8)

- Attempt Questions 1–15
- Allow about 25 minutes for this section

Section II – 85 marks (pages 9–44)

- Attempt Questions 16–37
- Allow about 2 hours and 5 minutes for this section

Section I

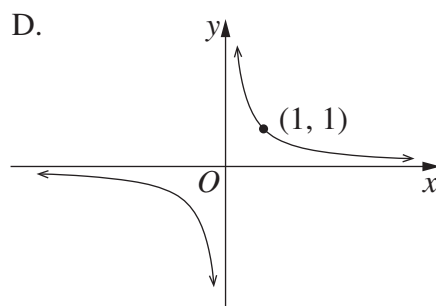
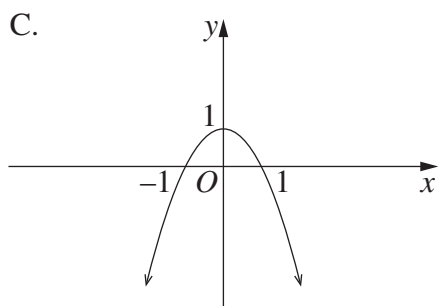
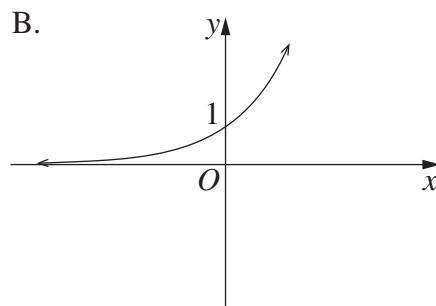
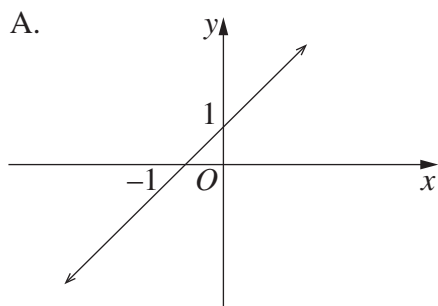
15 marks

Attempt Questions 1–15

Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1–15.

1 Which of the following could represent the graph of $y = -x^2 + 1$?



2 What is 0.002073 expressed in standard form with two significant figures?

- A. 2.07×10^{-2}
- B. 2.1×10^{-2}
- C. 2.07×10^{-3}
- D. 2.1×10^{-3}

- 3 The distance between Bricktown and Koala Creek is 75 km. A person travels from Bricktown to Koala Creek at an average speed of 50 km/h.

How long does it take the person to complete the journey?

- A. 40 minutes
- B. 1 hour 25 minutes
- C. 1 hour 30 minutes
- D. 1 hour 50 minutes

- 4 Joan invests \$200. She earns interest at 3% per annum, compounded monthly.

What is the future value of Joan's investment after 1.5 years?

- A. \$209.07
- B. \$209.19
- C. \$279.51
- D. \$311.93

- 5 A plant stem is measured to be 16.0 cm, correct to one decimal place.

What is the percentage error in this measurement?

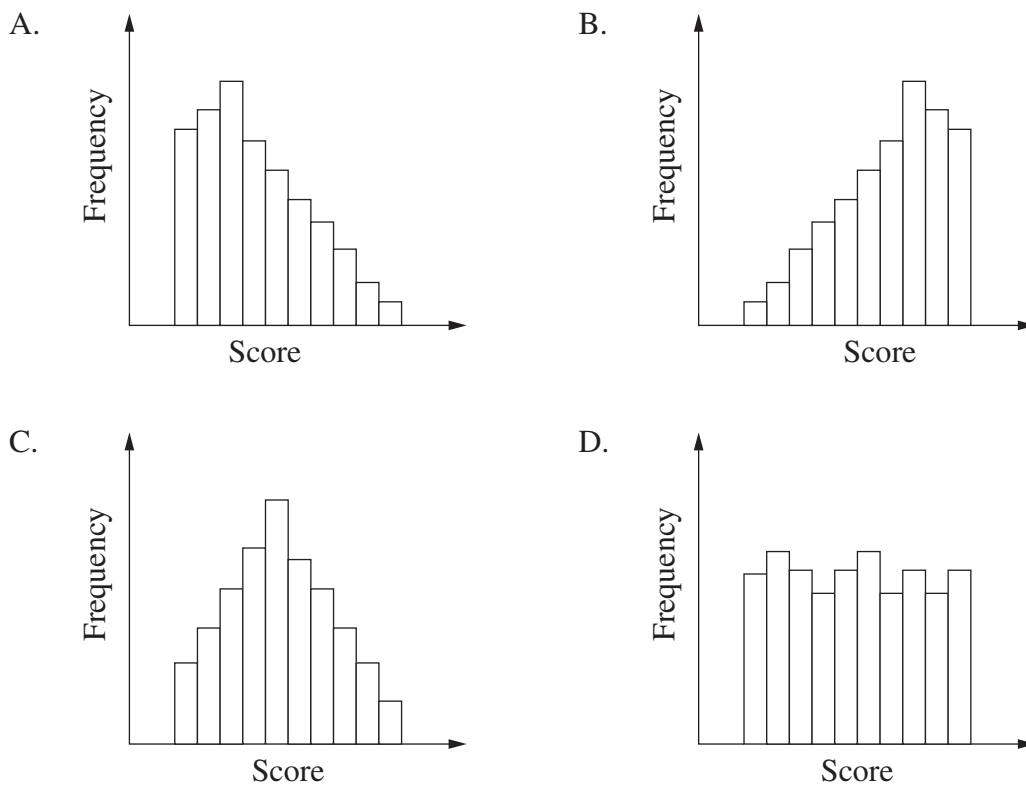
- A. 0.3125%
- B. 0.625%
- C. 3.125%
- D. 6.25%

- 6 Suppose $y = -1 - 2x$.

When the value of x increases by 5, the value of y decreases by

- A. 1.
- B. 2.
- C. 5.
- D. 10.

7 Which histogram best represents a dataset that is positively skewed?



8 John recently did a class test in each of three subjects. The class scores on each test were normally distributed.

The table shows the subjects and John's scores as well as the mean and standard deviation of the class scores on each test.

<i>Subject</i>	<i>John's score</i>	<i>Mean</i>	<i>Standard deviation</i>
French	82	70	8
Commerce	80	65	5
Music	74	50	12

Relative to the rest of the class, which row of the table below shows John's strongest subject and his weakest subject?

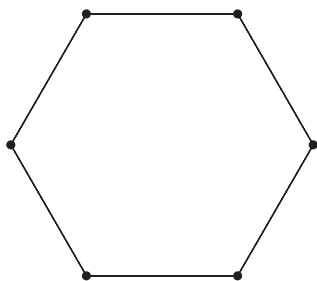
	<i>Strongest subject</i>	<i>Weakest subject</i>
A.	Commerce	French
B.	French	Music
C.	Music	French
D.	Commerce	Music

- 9 Team *A* and Team *B* have entered a chess competition.

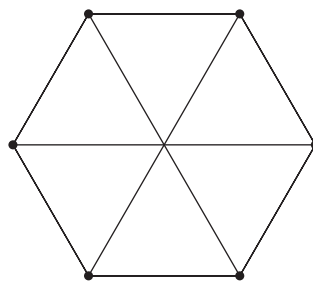
Team *A* and Team *B* have three members each. Each member of Team *A* must play each member of Team *B* once.

Which of the following network diagrams could represent the chess games to be played?

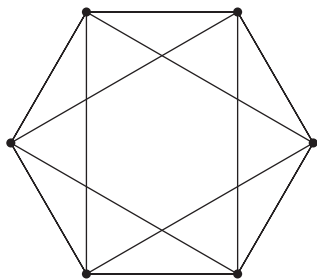
A.



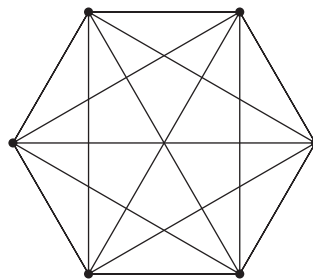
B.



C.



D.



- 10 A plumber charges a call-out fee of \$90 as well as \$2 per minute while working.

Suppose the plumber works for t hours.

Which equation expresses the amount the plumber charges (\$ C) as a function of time (t hours)?

- A. $C = 2 + 90t$
- B. $C = 90 + 2t$
- C. $C = 120 + 90t$
- D. $C = 90 + 120t$

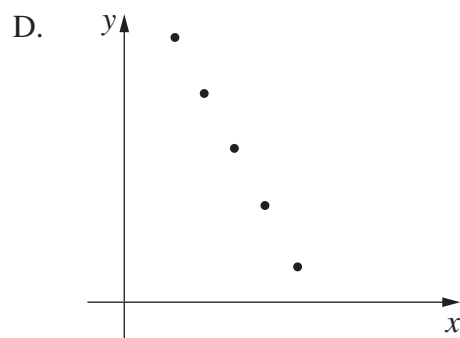
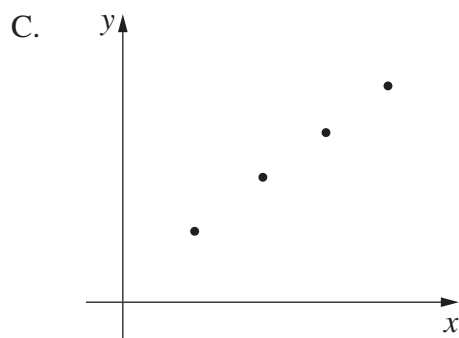
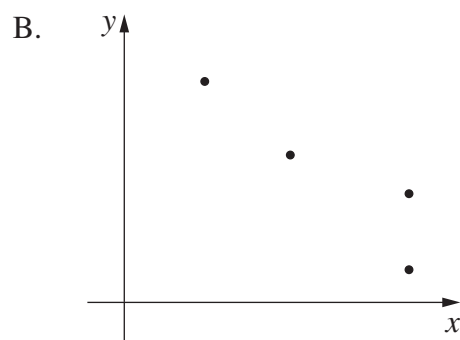
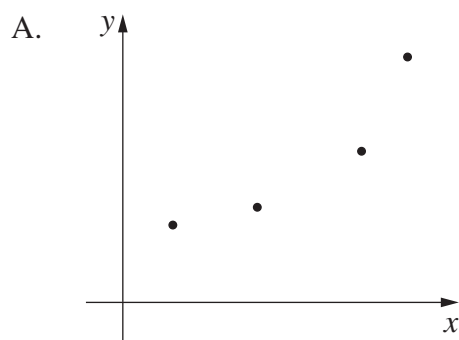
- 11** An asset is depreciated using the declining-balance method with a rate of depreciation of 8% per half year. The asset was bought for \$10 000.

What is the salvage value of the asset after 5 years?

- A. \$1749.01
- B. \$4182.12
- C. \$4343.88
- D. \$6590.82

- 12** For a set of bivariate data, Pearson's correlation coefficient is -1 .

Which graph could best represent this set of bivariate data?



- 13** When Jake stops drinking alcohol at 10:30 pm, he has a blood alcohol content (BAC) of 0.08375.

The number of hours required for a person to reach zero BAC after they stop consuming alcohol is given by the formula

$$\text{Time} = \frac{\text{BAC}}{0.015}.$$

At what time on the next day should Jake expect his BAC to be 0.05?

- A. 12:45 am
 - B. 1:50 am
 - C. 2:15 am
 - D. 4:05 am
- 14** An annuity consists of ten payments, each equal to \$1000. Each payment is made on 30 June each year from 2021 through to 2030 inclusive.

The rate of compound interest is 5% per annum.

The present value of the annuity is calculated at 30 June 2020.

The future value of the annuity is calculated at 30 June 2030.

Without performing any calculations, which of the following statements is true?

- A. Present value of the annuity < \$10 000 < future value of the annuity
- B. \$10 000 < present value of the annuity < future value of the annuity
- C. Future value of the annuity < \$10 000 < present value of the annuity
- D. \$10 000 < future value of the annuity < present value of the annuity

- 15** The top of a rectangular table is divided into 8 equal sections as shown.

1	2	3	4
5	6	7	8

A standard die with faces labelled 1 to 6 is rolled onto the table. The die is equally likely to land in any of the 8 sections of the table. If the die does not land entirely in one section of the table, it is rolled again.

A score is calculated by multiplying the value shown on the top face of the die by the number shown in the section of the table where the die lands.

What is the probability of getting a score of 6?

- A. $\frac{1}{48}$
- B. $\frac{1}{12}$
- C. $\frac{1}{8}$
- D. $\frac{1}{6}$

--	--	--	--	--

Centre Number

Mathematics Standard 2

Section II Answer Booklet 1

--	--	--	--	--	--	--	--	--

Student Number

Section II

85 marks**Attempt Questions 16–37****Allow about 2 hours and 5 minutes for this section****Booklet 1 — Attempt Questions 16–28 (48 marks)****Booklet 2 — Attempt Questions 29–37 (37 marks)**

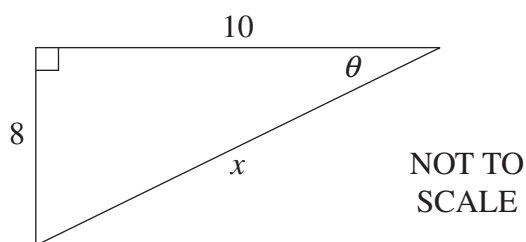
Instructions

- Write your Centre Number and Student Number at the top of this page.
 - Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided on pages 26–28 of Booklet 1. If you use this space, clearly indicate which question you are answering.
-

Please turn over

Question 16 (4 marks)

Consider the triangle shown.



- (a) Find the value of θ , correct to the nearest degree.

2

.....

.....

.....

.....

- (b) Find the value of x , correct to one decimal place.

2

.....

.....

.....

.....

Do NOT write in this area.

Question 17 (2 marks)

Ayla wishes to estimate the number of trees on a square block of land measuring 1000 m by 1000 m. She counts the number of trees on a 5 m by 5 m section of the block and finds there are 8 trees.

2

Based on this, estimate the number of trees on the entire square block of land.

.....

.....

.....

.....

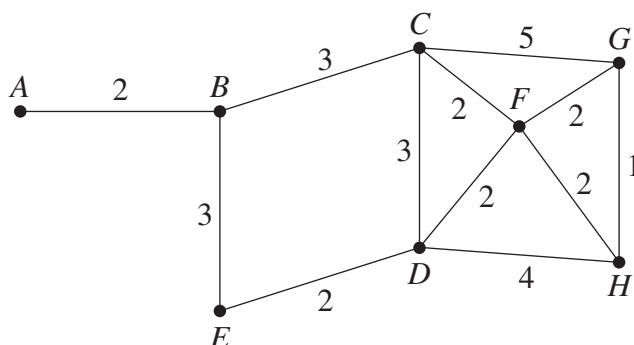
.....

.....

Please turn over

Question 18 (4 marks)

The diagram represents a network with weighted edges.



- (a) Draw a minimum spanning tree for this network in the space below and determine its length.

3

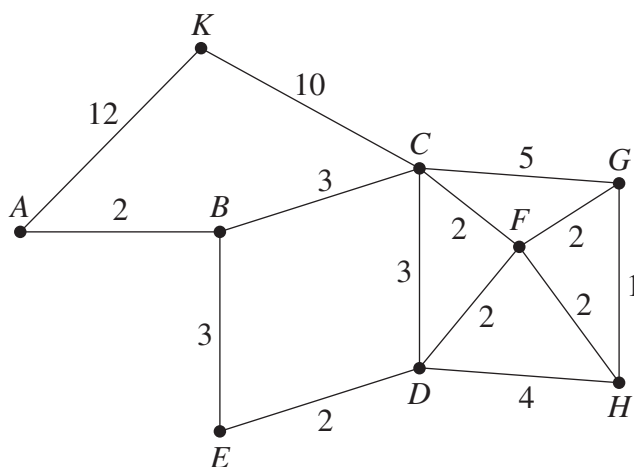
Minimum length of spanning tree =

Question 18 continues on page 13

Question 18 (continued)

- (b) The network is revised by adding another vertex, K . Edges AK and CK have weights of 12 and 10 respectively, as shown.

1



What is the length of the minimum spanning tree for this revised network?

.....

.....

.....

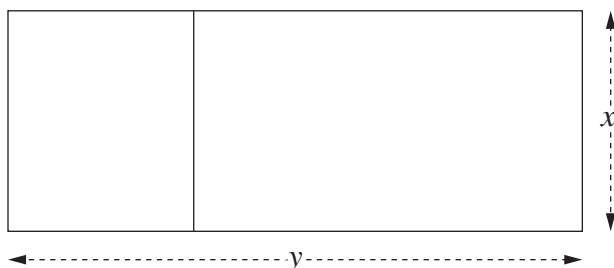
End of Question 18

Please turn over

Question 19 (4 marks)

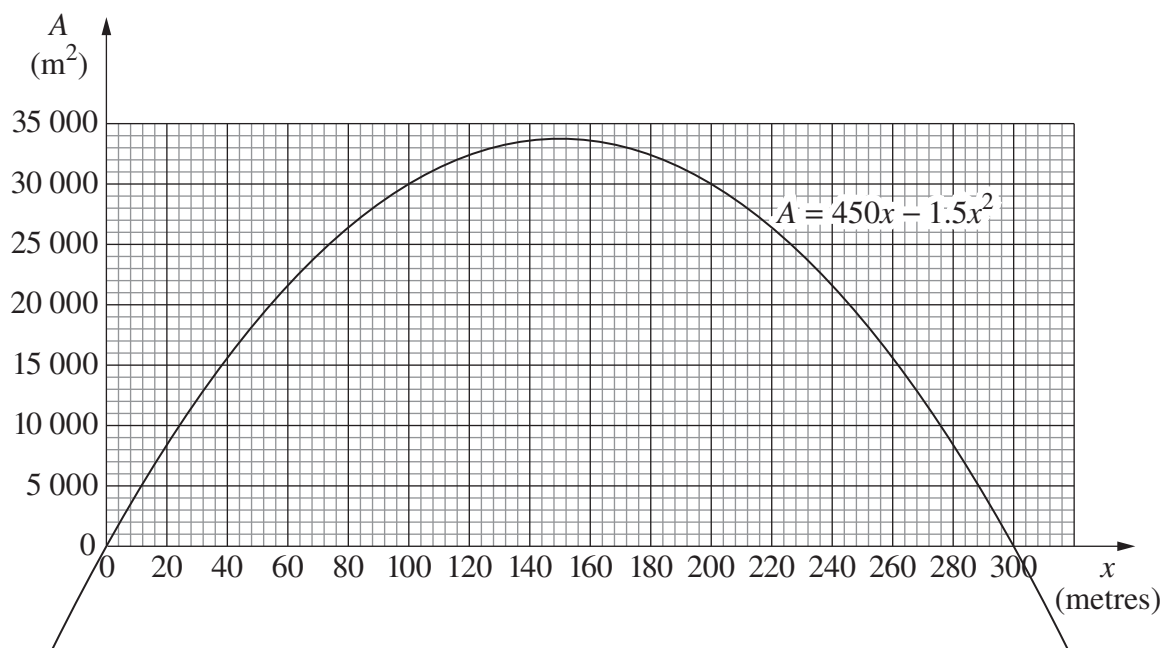
A fence is to be built around the outside of a rectangular paddock. An internal fence is also to be built.

The side lengths of the paddock are x metres and y metres, as shown in the diagram.



A total of 900 metres of fencing is to be used. Therefore $3x + 2y = 900$.

The area, A , in square metres, of the rectangular paddock is given by $A = 450x - 1.5x^2$.
The graph of this equation is shown.



- (a) If the area of the paddock is 30 000 m^2 , what is the largest possible value of x ? 1

.....

.....

Question 19 continues on page 15

Question 19 (continued)

- (b) Find the values of x and y so that the area of the paddock is as large as possible. **2**

.....

.....

.....

.....

.....

.....

.....

- (c) Using your values from part (b), find the largest possible area of the paddock. **1**

.....

.....

.....

.....

End of Question 19

Please turn over

Question 20 (3 marks)

The table shows the income tax rates for the 2019–2020 financial year.

3

<i>Taxable income</i>	<i>Tax on this income</i>
0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$90 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$90 001 – \$180 000	\$20 797 plus 37c for each \$1 over \$90 000
\$180 001 and over	\$54 097 plus 45c for each \$1 over \$180 000

For the 2019–2020 financial year, Wally had a taxable income of \$122 680. During the year, he paid \$3000 per month in Pay As You Go (PAYG) tax.

Calculate Wally’s tax refund, ignoring the Medicare levy.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Do NOT write in this area.

Question 21 (2 marks)

The inflation rate over the year from January 2019 to January 2020 was 2%.

2

The cost of a school jumper in January 2020 was \$122.

Calculate the cost of the jumper in January 2019 assuming that the only change in the cost of the jumper was due to inflation.

.....

.....

.....

.....

.....

Please turn over

Question 22 (3 marks)

Nisa has a credit card on which interest at 17% per annum, compounded daily, is charged on the amount owing.

3

At the beginning of the month, Nisa owes \$500 on her credit card. She makes no other purchases using the credit card, but fifteen days later, she repays \$250.

Assuming that interest is charged for the fifteen days, calculate the amount owing on the credit card immediately after the \$250 payment is made.

.....

.....

.....

.....

.....

.....

.....

Do NOT write in this area.

Question 23 (5 marks)

In a tropical drink, the ratio of pineapple juice to mango juice to orange juice is 15 : 9 : 4.

- (a) How much orange juice is needed if the tropical drink is to contain 3 litres of pineapple juice? 2

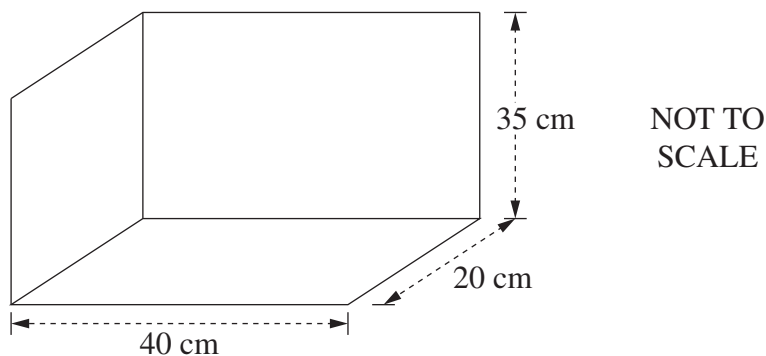
.....

.....

.....

.....

- (b) The internal dimensions of a drink container, in the shape of a rectangular prism, are shown. 3



To completely fill the container with the tropical drink, how many litres of mango juice are required?

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

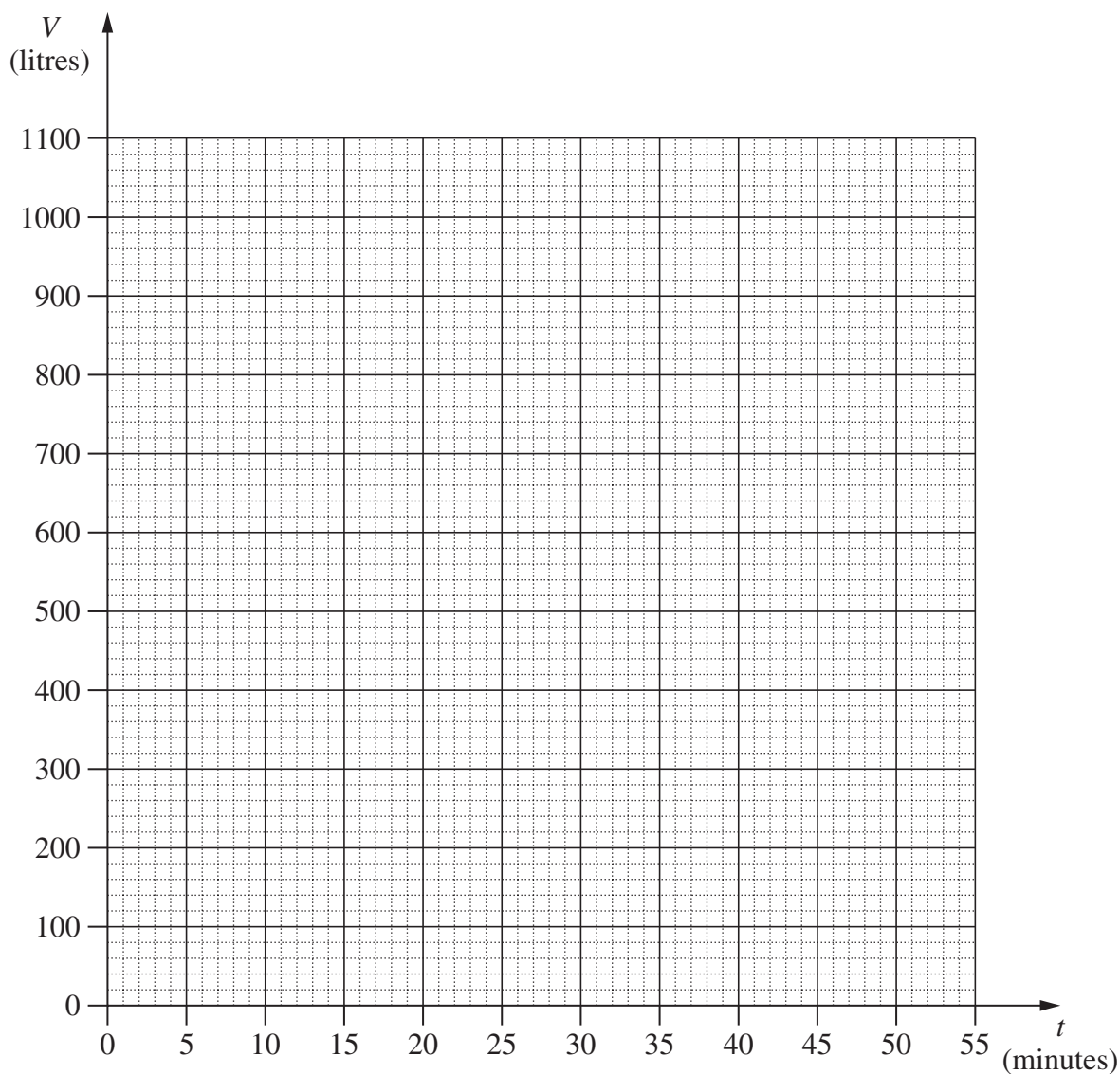
Question 24 (4 marks)

There are two tanks on a property, Tank *A* and Tank *B*. Initially, Tank *A* holds 1000 litres of water and Tank *B* is empty.

- (a) Tank *A* begins to lose water at a constant rate of 20 litres per minute. The volume of water in Tank *A* is modelled by $V = 1000 - 20t$ where V is the volume in litres and t is the time in minutes from when the tank begins to lose water.

1

On the grid below, draw the graph of this model and label it as Tank *A*.



Do NOT write in this area.

Question 24 continues on page 21

Question 24 (continued)

- (b) Tank *B* remains empty until $t = 15$ when water is added to it at a constant rate of 30 litres per minute. 2

By drawing a line on the grid on the previous page, or otherwise, find the value of t when the two tanks contain the same volume of water.

.....

.....

.....

.....

.....

.....

- (c) Using the graphs drawn, or otherwise, find the value of t (where $t > 0$) when the total volume of water in the two tanks is 1000 litres. 1

.....

.....

.....

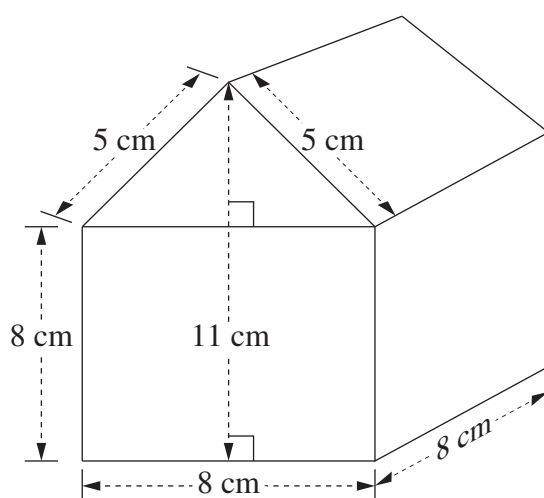
.....

End of Question 24

Question 25 (3 marks)

A composite solid consists of a triangular prism which fits exactly on top of a cube, as shown.

3



NOT TO
SCALE

Find the surface area of the composite solid.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

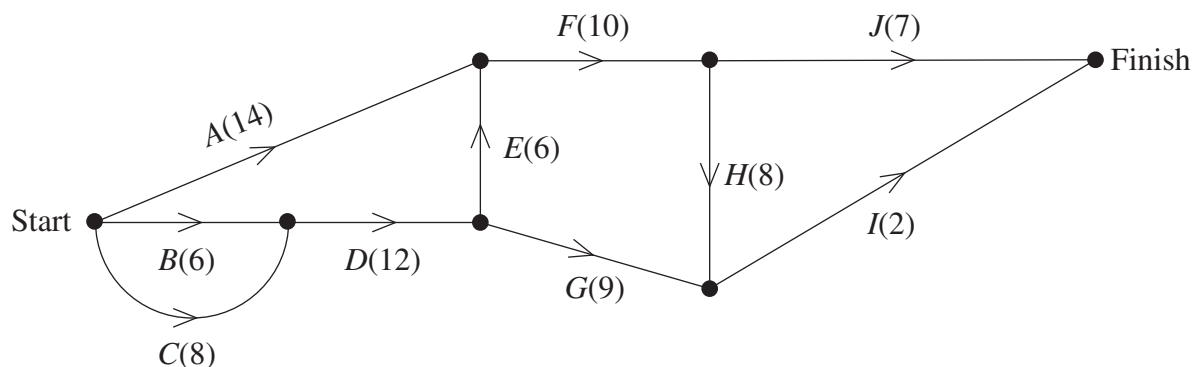
.....

.....

Do NOT write in this area.

Question 26 (5 marks)

The preparation of a meal requires the completion of all ten activities A to J. The network diagram shows the activities and their completion times in minutes.



- (a) What is the minimum time needed to prepare the meal? 1

.....

.....

.....

- (b) List the activities which make up the critical path for this network. 2

.....

.....

.....

.....

- (c) Complete the table below, showing the earliest start time and float time for activities A and G. 2

Activity	Earliest start time (minutes)	Float time (minutes)
A		
G		

Question 27 (5 marks)

The shaded region on the diagram represents a garden. The scale is 1 cm = 5 m.



- (a) Use two applications of the trapezoidal rule to calculate the approximate area of the garden.

3

.....

.....

.....

.....

.....

.....

.....

- (b) Should the answer to part (a) be more than, equal to or less than the actual area of the garden? Referring to the diagram above, briefly explain your answer.

2

.....

.....

.....

.....

.....

Questions 16–27 are worth 44 marks in total

Question 28 (4 marks)

Consider the following dataset.

4

1 5 9 10 15

Suppose a new value, x , is added to this dataset, giving the following.

1 5 9 10 15 x

It is known that x is greater than 15. It is also known that the difference between the means of the two datasets is equal to ten times the difference between the medians of the two datasets.

Calculate the value of x .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Do NOT write in this area.

Do NOT write in this area.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Do NOT write in this area.

--	--	--	--	--

Centre Number

Mathematics Standard 2

Section II Answer Booklet 2

--	--	--	--	--	--	--	--	--

Student Number

Booklet 2 — Attempt Questions 29–37 (37 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 43–44 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 29 (3 marks)

Jana owns a share portfolio. Details of her share portfolio at 30 June 2020 are given in the table.

3

<i>Company name</i>	<i>Number of shares in Jana's portfolio</i>	<i>Dividend yield (per annum)</i>	<i>Market price per share</i>
<i>ABC</i>	200	6.0%	\$5.50
<i>XYZ</i>	?	4.0%	\$6.00

Jana received a total annual dividend of \$149.52 from her share portfolio.

Calculate the number of shares Jana has in company XYZ on 30 June 2020.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

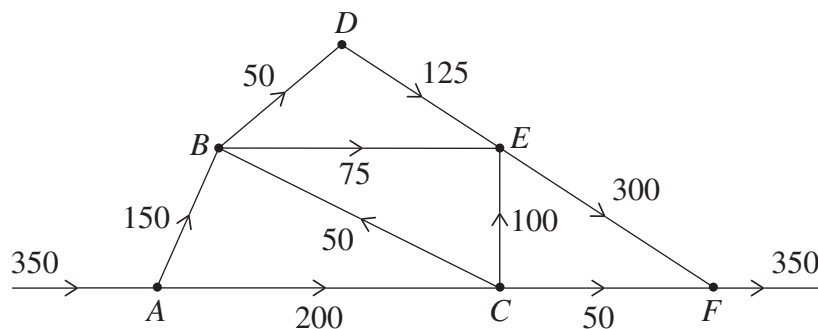
.....

.....

Do NOT write in this area.

Question 30 (3 marks)

The network diagram shows a series of water channels and ponds in a garden. The vertices A, B, C, D, E and F represent six ponds. The edges represent the water channels which connect the ponds. The numbers on the edges indicate the maximum capacity of the channels.



- (a) Determine the maximum flow of the network.

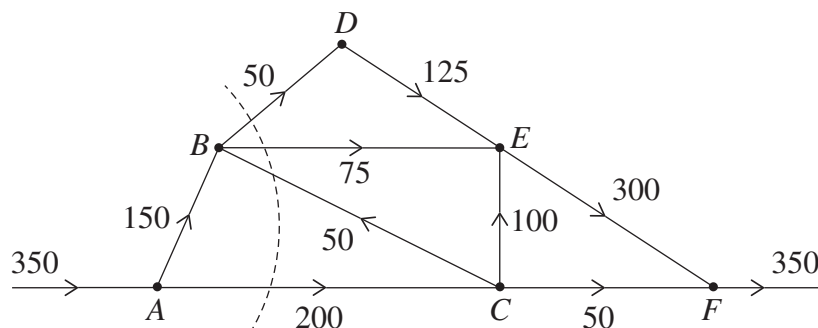
2

.....

.....

- (b) A cut is added to the network, as shown.

1



Is the cut shown a minimum cut? Give a reason for your answer.

.....

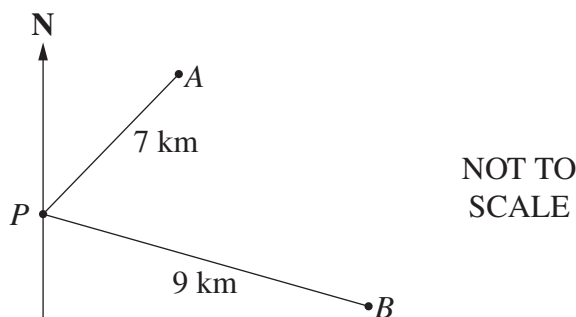
.....

.....

.....

Question 31 (5 marks)

Mr Ali, Ms Brown and a group of students were camping at the site located at P . Mr Ali walked with some of the students on a bearing of 035° for 7 km to location A . Ms Brown, with the rest of the students, walked on a bearing of 100° for 9 km to location B .



- (a) Show that the angle APB is 65° .

1

.....

.....

- (b) Find the distance AB .

2

.....

.....

.....

.....

.....

- (c) Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree.

2

.....

.....

.....

.....

.....

.....

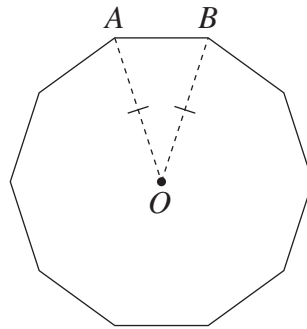
.....

.....

Question 32 (4 marks)

The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal). The decagon has centre O .

4



The perimeter of the shape is 80 cm.

By considering triangle OAB , calculate the area of the ten-sided shape. Give your answer in square centimetres correct to one decimal place.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

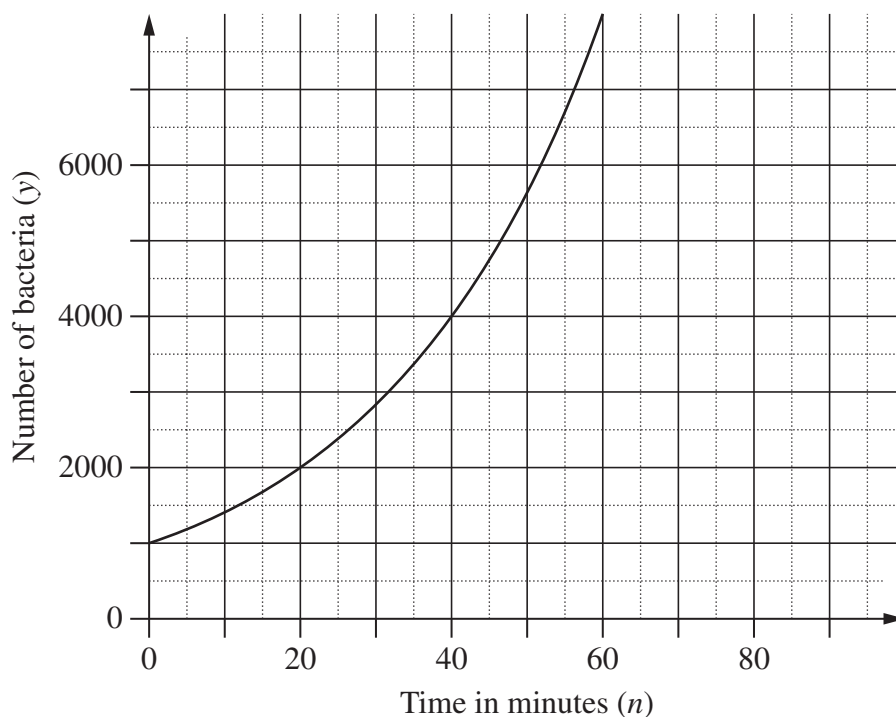
.....

.....

.....

Question 33 (3 marks)

The graph shows the number of bacteria, y , at time n minutes. Initially (when $n = 0$) the number of bacteria is 1000.



- (a) Find the number of bacteria at 40 minutes.

1

.....

.....

Question 33 continues on page 35

Question 33 (continued)

- (b) The number of bacteria can be modelled by the equation $y = A \times b^n$, where A and b are constants. 2

Use the guess and check method to find, to two decimal places, an upper and lower estimate for the value of b . The upper and lower estimates must differ by 0.01.

.....

.....

.....

.....

.....

.....

End of Question 33

Please turn over

Question 34 (4 marks)

Tina inherits \$60 000 and invests it in an account earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Tina withdraws \$800.

The amount in the account immediately after the n th withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - 800,$$

where $n = 1, 2, 3, \dots$ and $A_0 = 60\,000$.

- (a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 34 continues on page 37

Question 34 (continued)

- (b) Calculate the amount of interest earned in the first three months.

2

.....

.....

.....

.....

.....

.....

.....

.....

End of Question 34

Please turn over

Question 35 (7 marks)

The Intelligence Quotient (IQ) scores for adults in City *A* are normally distributed with a mean of 108 and a standard deviation of 10.

The IQ scores for adults in City *B* are normally distributed with a mean of 112 and a standard deviation of 16.

- (a) Yin is an adult who lives in City *A* and has an IQ score of 128.

2

What percentage of the adults in this city have an IQ score higher than Yin's?

.....

.....

.....

.....

.....

.....

- (b) There are 1 000 000 adults living in City *B*.

2

Calculate the number of adults in City *B* that would be expected to have an IQ score lower than Yin's.

.....

.....

.....

.....

.....

.....

Question 35 continues on page 39

Question 35 (continued)

- (c) Simon, an adult who lives in City *A*, moves to City *B*. The z -score corresponding to his IQ score in City *A* is the same as the z -score corresponding to his IQ score in City *B*.

3

By first forming an equation, calculate Simon's IQ score. Give your answer correct to one decimal place.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of Question 35

Question 36 (5 marks)

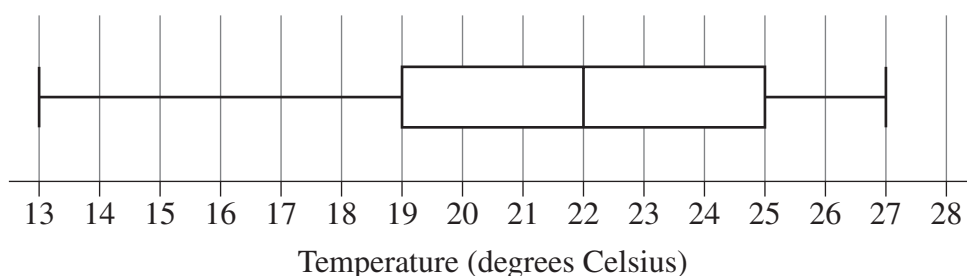
A cricket is an insect. The male cricket produces a chirping sound.

5

A scientist wants to explore the relationship between the temperature in degrees Celsius and the number of cricket chirps heard in a 15-second time interval.

Once a day for 20 days, the scientist collects data. Based on the 20 data points, the scientist provides the information below.

- A box-plot of the temperature data is shown.



- The mean temperature in the dataset is 0.525°C below the median temperature in the dataset.
- A total of 684 chirps was counted when collecting the 20 data points.

The scientist fits a least-squares regression line using the data (x, y) , where x is the temperature in degrees Celsius and y is the number of chirps heard in a 15-second time interval. The equation of this line is

$$y = -10.6063 + bx,$$

where b is the slope of the regression line.

The least-squares regression line passes through the point (\bar{x}, \bar{y}) where \bar{x} is the sample mean of the temperature data and \bar{y} is the sample mean of the chirp data.

Question 36 continues on page 41

Question 36 (continued)

Calculate the number of chirps expected in a 15-second interval when the temperature is 19° Celsius. Give your answer correct to the nearest whole number.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of Question 36

Please turn over

Question 37 (3 marks)

Wilma deposited a lump sum into a new bank account which earns 2% per annum compound interest.

3

Present value interest factors for an annuity of \$1 for various interest rates (r) and numbers of periods (N) are given in the table.

Table of present value interest factors

$N \backslash r$	<i>Interest rate per period as a decimal</i>			
	0.01	0.015	0.02	0.025
10	9.471	9.222	8.983	8.752
20	18.046	17.169	16.351	15.589
30	25.808	24.016	22.396	20.930

Wilma was able to make the following withdrawals from this account.

- \$1000 at the end of each year for twenty years (starting one year after the account is opened)
- \$3000 each year for ten years starting 21 years after the account is opened.

Calculate the minimum lump sum Wilma must have deposited when she opened the new account.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of paper

Do NOT write in this area.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Handwriting practice lines for Section II extra writing space.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Do NOT write in this area.

Mathematics Standard 1

Mathematics Standard 2

REFERENCE SHEET

Measurement

Limits of accuracy

$$\text{Absolute error} = \frac{1}{2} \times \text{precision}$$

$$\text{Upper bound} = \text{measurement} + \text{absolute error}$$

$$\text{Lower bound} = \text{measurement} - \text{absolute error}$$

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

Statistical Analysis

An outlier is a score

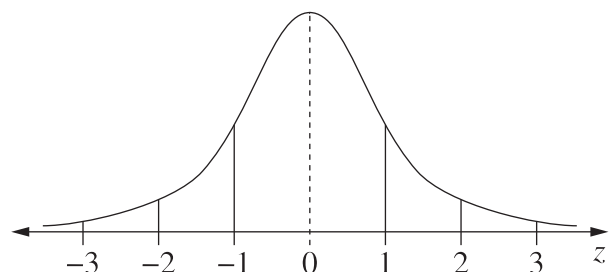
$$\text{less than } Q_1 - 1.5 \times IQR$$

or

$$\text{more than } Q_3 + 1.5 \times IQR$$

$$z = \frac{x - \mu}{\sigma}$$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

BLANK PAGE