NSW Education Standards Authority

2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–40)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which inequality gives the domain of $y = \sqrt{2x - 3}$?

A. $x < \frac{3}{2}$

B. $x > \frac{3}{2}$

C. $x \leq \frac{3}{2}$

D. $x \geq \frac{3}{2}$

2 The function $f(x) = x^3$ is transformed to $g(x) = (x - 2)^3 + 5$ by a horizontal translation of 2 units followed by a vertical translation of 5 units.

Which row of the table shows the directions of the translations?

	Horizontal translation of 2 units	*Vertical translation of 5 units*
A.	Left	Up
B.	Right	Up
C.	Left	Down
D.	Right	Down

3 John recently did a class test in each of three subjects. The class scores on each test were normally distributed.

The table shows the subjects and John's scores as well as the mean and standard deviation of the class scores on each test.

Subject	*John's score*	*Mean*	*Standard deviation*
French	82	70	8
Commerce	80	65	5
Music	74	50	12

Relative to the rest of the class, which row of the table below shows John's strongest subject and his weakest subject?

	Strongest subject	*Weakest subject*
A.	Commerce	French
B.	French	Music
C.	Music	French
D.	Commerce	Music

4 What is $\int e + e^{3x}\, dx$?

A. $ex + 3e^{3x} + c$

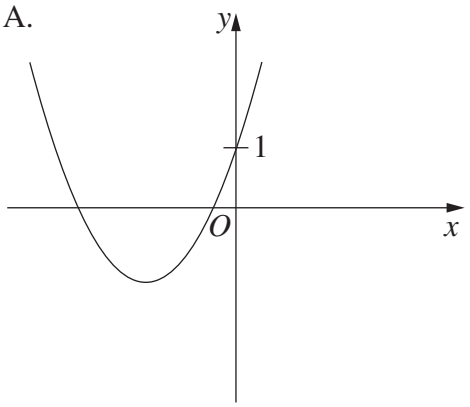
B. $ex + \frac{1}{3}e^{3x} + c$

C. $e + 3e^{3x} + c$

D. $e + \frac{1}{3}e^{3x} + c$

5 Which of the following could represent the graph of $y = -x^2 + bx + 1$, where $b > 0$?

A.



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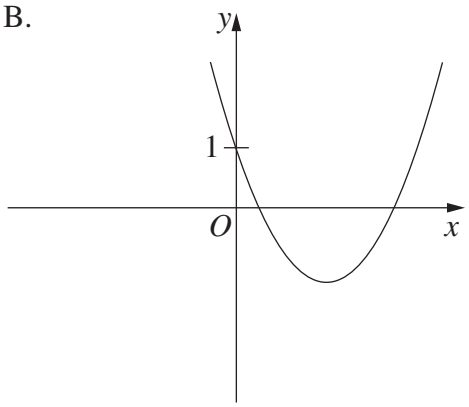

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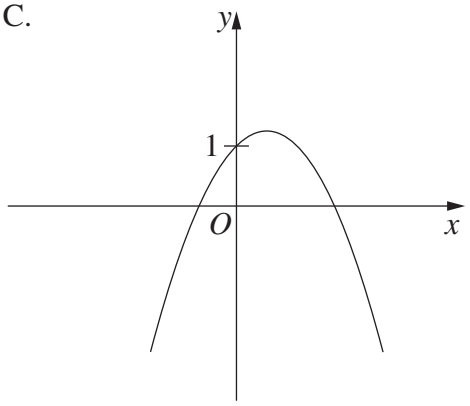
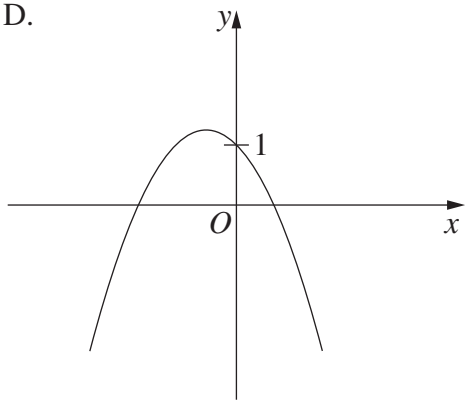


D.



6 Which interval gives the range of the function $y = 5 + 2\cos 3x$?

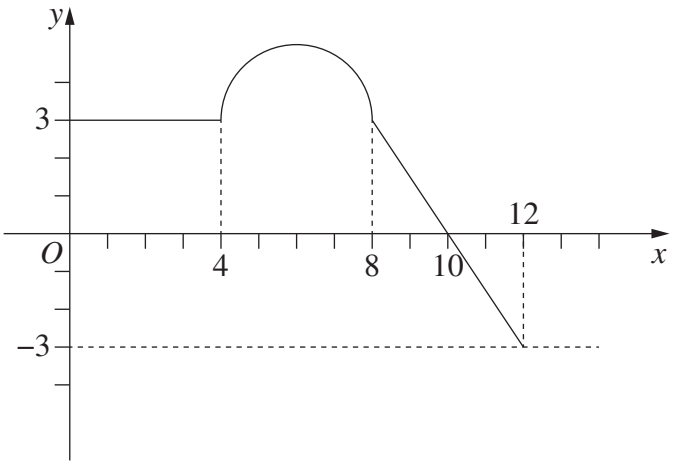
A. $[2, 8]$

B. $[3, 7]$

C. $[4, 6]$

D. $[5, 9]$

7 The diagram shows the graph $y = f(x)$, which is made up of line segments and a semicircle.



What is the value of $\int_0^{12} f(x)\,dx$?

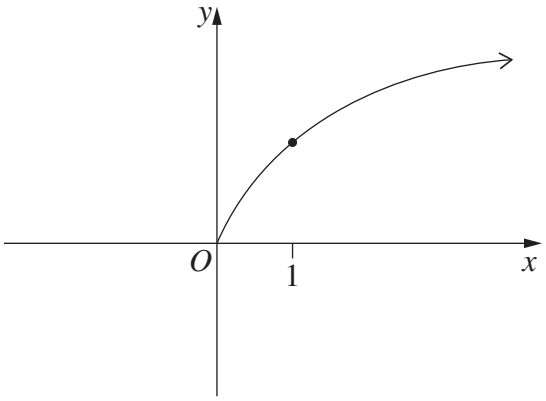
A. $24 + 2\pi$

B. $24 + 4\pi$

C. $30 + 2\pi$

D. $30 + 4\pi$

8 The graph of $y = f(x)$ is shown.



Which of the following inequalities is correct?

A. $f''(1) < 0 < f'(1) < f(1)$

B. $f''(1) < 0 < f(1) < f'(1)$

C. $0 < f''(1) < f'(1) < f(1)$

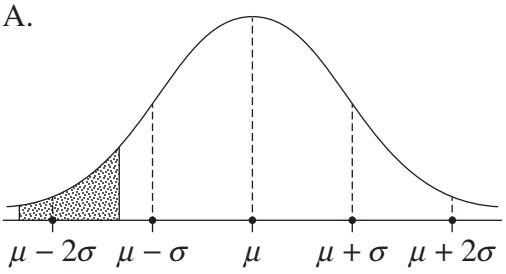
D. $0 < f''(1) < f(1) < f'(1)$

9 Suppose the weight of melons is normally distributed with a mean of μ and a standard deviation of σ.

A melon has a weight below the lower quartile of the distribution but NOT in the bottom 10% of the distribution.

Which of the following most accurately represents the region in which the weight of this melon lies?

A.



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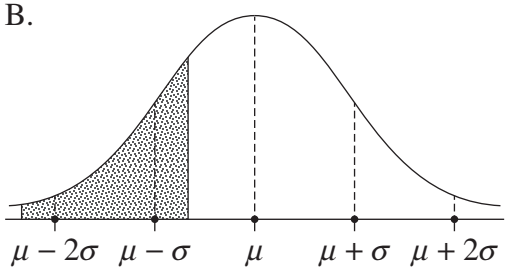

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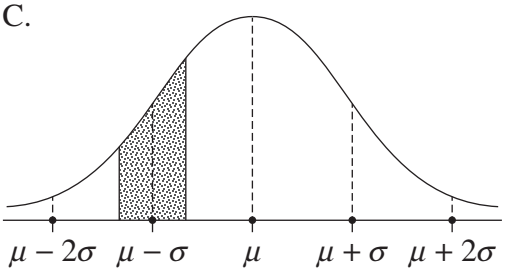
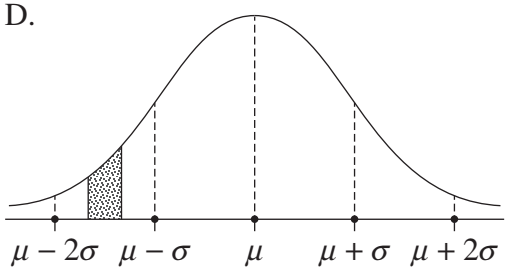
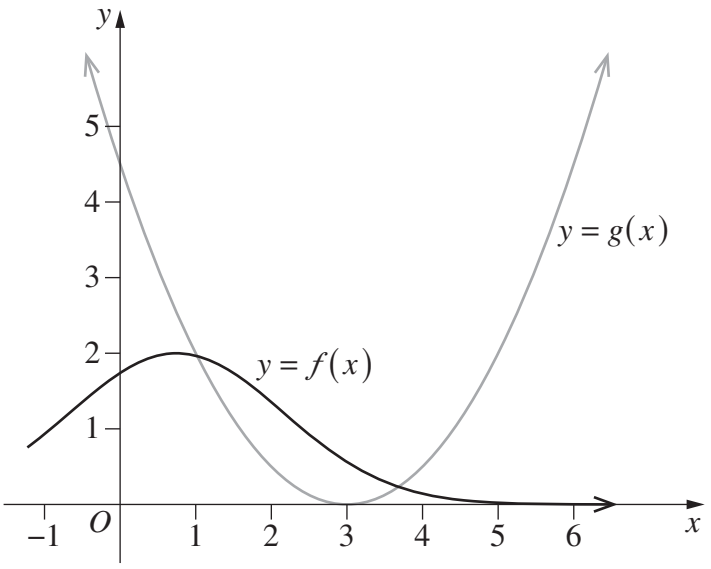


D.



10 The graph shows two functions $y = f(x)$ and $y = g(x)$.



Define $h(x) = f(g(x))$.

How many stationary points does $y = h(x)$ have for $1 \leq x \leq 5$?

A. 0

B. 1

C. 2

D. 3

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2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–20 (32 marks)
Booklet 2 — Attempt Questions 21–31 (58 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 19–20 of Booklet 1. If you use this space, clearly indicate which question you are answering.

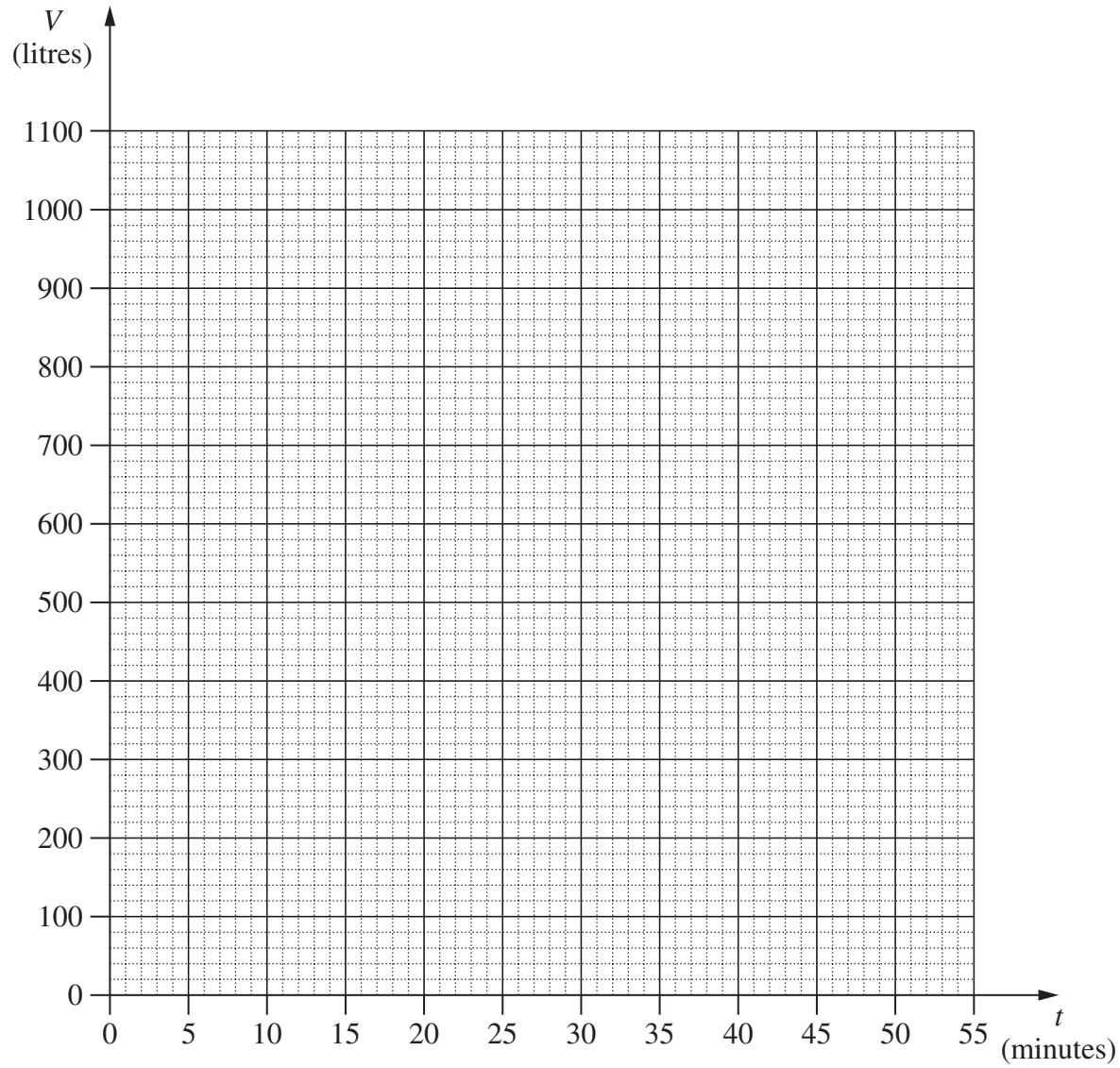
Please turn over

Question 11 (4 marks)

There are two tanks on a property, Tank *A* and Tank *B*. Initially, Tank *A* holds 1000 litres of water and Tank *B* is empty.

(a) Tank *A* begins to lose water at a constant rate of 20 litres per minute. The volume of water in Tank *A* is modelled by $V = 1000 - 20t$ where V is the volume in litres and t is the time in minutes from when the tank begins to lose water. **1**

On the grid below, draw the graph of this model and label it as Tank *A*.



Question 11 continues on page 11

(b) Tank *B* remains empty until $t = 15$ when water is added to it at a constant rate of 30 litres per minute. **2**

By drawing a line on the grid on the previous page, or otherwise, find the value of t when the two tanks contain the same volume of water.

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(c) Using the graphs drawn, or otherwise, find the value of t (where $t > 0$) when the total volume of water in the two tanks is 1000 litres. **1**

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End of Question 11

Question 12 (3 marks)

Calculate the sum of the arithmetic series $4 + 10 + 16 + \cdots + 1354$. **3**

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Question 13 (2 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$. **2**

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Question 14 (5 marks)

History and Geography are two of the subjects students may decide to study. For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

(a) A student is chosen at random. By using a Venn diagram, or otherwise, find the probability that the student studies both History and Geography. **2**

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(b) A student is chosen at random. Given that the student studies Geography, what is the probability that the student does NOT study History? **1**

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(c) Two different students are chosen at random, one after the other. What is the probability that the first student studies History and the second student does NOT study History? **2**

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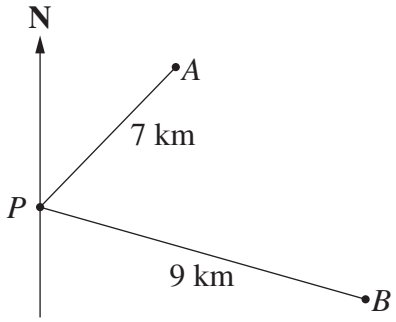
Question 15 (5 marks)

Mr Ali, Ms Brown and a group of students were camping at the site located at *P*. Mr Ali walked with some of the students on a bearing of 035° for 7 km to location *A*. Ms Brown, with the rest of the students, walked on a bearing of 100° for 9 km to location *B*.



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(a) Show that the angle *APB* is 65°. **1**

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(b) Find the distance *AB*. **2**

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(c) Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree. **2**

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Question 16 (4 marks)

Sketch the graph of the curve $y = -x^3 + 3x^2 - 1$, labelling the stationary points and point of inflection. Do NOT determine the x intercepts of the curve. **4**

Question 17 (2 marks)

Find $\int \frac{x}{4+x^2}\,dx$. **2**

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Question 18 (3 marks)

(a) Differentiate $e^{2x}(2x+1)$. **2**

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(b) Hence, find $\int (x+1)e^{2x}\,dx$. **1**

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Question 19 (2 marks)

Prove that $\sec\theta - \cos\theta = \sin\theta\tan\theta$. **2**

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Question 20 (2 marks)

Kenzo is driving his car along a road while his friend records the velocity of the car, $v(t)$, in km/h every minute over a 5-minute period. The table gives the velocity $v(t)$ at time t hours. **2**

t	0	$\frac{1}{60}$	$\frac{2}{60}$	$\frac{3}{60}$	$\frac{4}{60}$	$\frac{5}{60}$
$v(t)$	60	55	65	68	70	67

The distance covered by the car over the 5-minute period is given by

$$\int_0^{\frac{5}{60}} v(t)\,dt.$$

Use the trapezoidal rule and the velocity at each of the six time values to find the approximate distance in kilometres the car has travelled in the 5-minute period. Give your answer correct to one decimal place.

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2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 21–31 (58 marks)

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 38–40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 21 (6 marks)

Hot tea is poured into a cup. The temperature of tea can be modelled by $T = 25 + 70(1.5)^{-0.4t}$, where T is the temperature of the tea, in degrees Celsius, t minutes after it is poured.

(a) What is the temperature of the tea 4 minutes after it has been poured? **1**

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(b) At what rate is the tea cooling 4 minutes after it has been poured? **2**

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(c) How long after the tea is poured will it take for its temperature to reach 55°C? **3**

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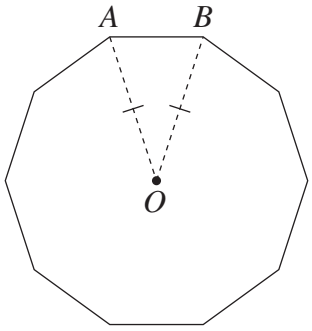
Question 22 (4 marks)

The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal). The decagon has centre O. **4**



The perimeter of the shape is 80 cm.

By considering triangle OAB, calculate the area of the ten-sided shape. Give your answer in square centimetres correct to one decimal place.

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Question 23 (4 marks)

A continuous random variable, X, has the following probability density function.

$$f(x) = \begin{cases} \sin x & \text{for } 0 \le x \le k \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Find the value of k. **2**

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(b) Find $P(X \le 1)$. Give your answer correct to four decimal places. **2**

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Questions 11–23 are worth 46 marks in total

Question 24 (3 marks)

The circle $x^2 - 6x + y^2 + 4y - 3 = 0$ is reflected in the x-axis. **3**

Sketch the reflected circle, showing the coordinates of the centre and the radius.

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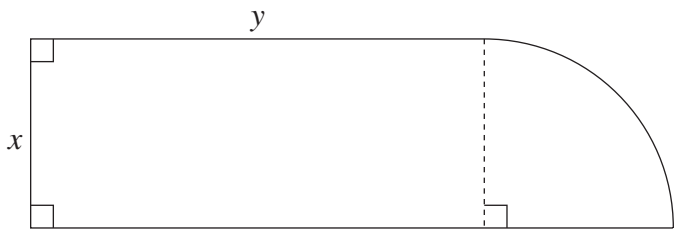
Question 25 (7 marks)

A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle. Let x and y be the dimensions of the rectangle in metres, as shown in the diagram.



The garden bed is required to have an area of 36 m^2 and to have a perimeter which is as small as possible. Let P metres be the perimeter of the garden bed.

(a) Show that $P = 2x + \frac{72}{x}$. **3**

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Question 25 continues on page 27

Question 25 (continued)

(b) Find the smallest possible perimeter of the garden bed, showing why this is the minimum perimeter. **4**

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End of Question 25

Question 26 (7 marks)

Tina inherits \$60 000 and invests it in an account earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Tina withdraws \$800.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - 800,$$

where $n = 1, 2, 3, \ldots$ and $A_0 = 60\,000$.

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. **2**

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Question 26 continues on page 29

Question 26 (continued)

(b) Calculate the amount of interest earned in the first three months. **2**

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(c) Calculate the amount of money in the account immediately after the 94th withdrawal. **3**

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End of Question 26

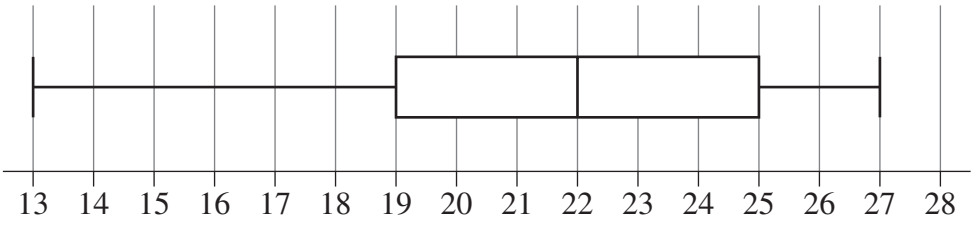
Question 27 (5 marks)

A cricket is an insect. The male cricket produces a chirping sound. **5**

A scientist wants to explore the relationship between the temperature in degrees Celsius and the number of cricket chirps heard in a 15-second time interval.

Once a day for 20 days, the scientist collects data. Based on the 20 data points, the scientist provides the information below.

- A box-plot of the temperature data is shown.



- The mean temperature in the dataset is 0.525°C below the median temperature in the dataset.

- A total of 684 chirps was counted when collecting the 20 data points.

The scientist fits a least-squares regression line using the data (x, y), where x is the temperature in degrees Celsius and y is the number of chirps heard in a 15-second time interval. The equation of this line is

$$y = -10.6063 + bx,$$

where b is the slope of the regression line.

The least-squares regression line passes through the point $(\overline{x}, \overline{y})$ where $\overline{x}$ is the sample mean of the temperature data and $\overline{y}$ is the sample mean of the chirp data.

Question 27 continues on page 31

Question 27 (continued)

Calculate the number of chirps expected in a 15-second interval when the temperature is 19° Celsius. Give your answer correct to the nearest whole number.

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End of Question 27

Question 28 (5 marks)

In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of \$25 and a standard deviation of \$5.

(a) Two adults who both work are chosen at random. **3**

Find the probability that at least one of them earns between \$15 and \$30 per hour.

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(b) The number of adults who work is equal to three times the number of adults who do not work. **2**

One adult is chosen at random.

Find the probability that the chosen adult works and earns more than \$25 per hour.

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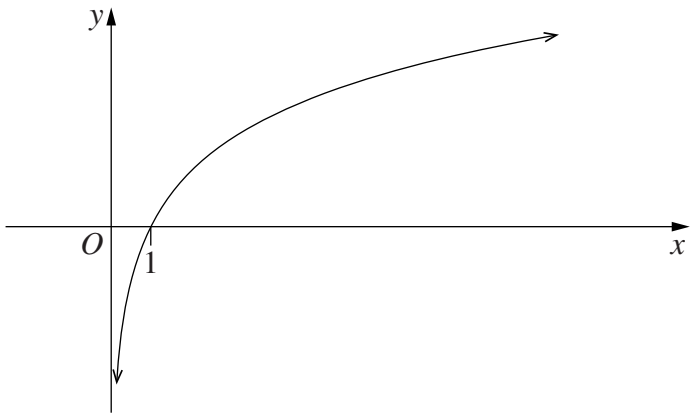
Question 29 (4 marks)

The diagram shows the graph of $y = c\ln x,\ c > 0$.



(a) Show that the equation of the tangent to $y = c\ln x$ at $x = p$, where $p > 0$, is **2**

$$y = \frac{c}{p}x - c + c\ln p.$$

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(b) Find the value of c such that the tangent from part (a) has a gradient of 1 and passes through the origin. **2**

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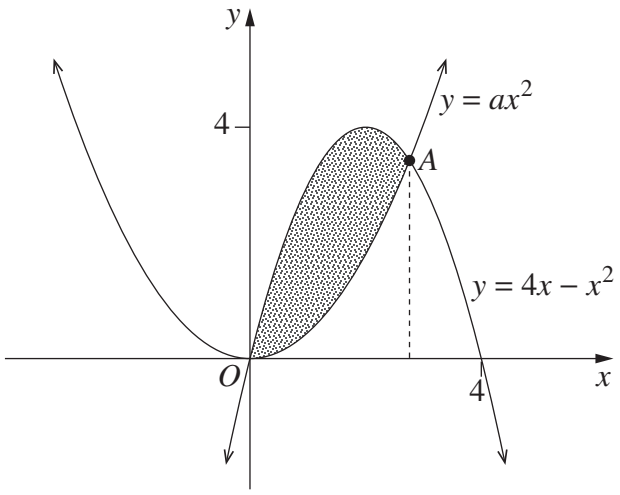
Question 30 (6 marks)

The diagram shows two parabolas $y = 4x - x^2$ and $y = ax^2$, where $a > 0$. The two parabolas intersect at the origin, O, and at A.



(a) Show that the x-coordinate of A is $\frac{4}{a+1}$. **2**

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Question 30 continues on page 35

Question 30 (continued)

(b) Find the value of a such that the shaded area is $\frac{16}{3}$. **4**

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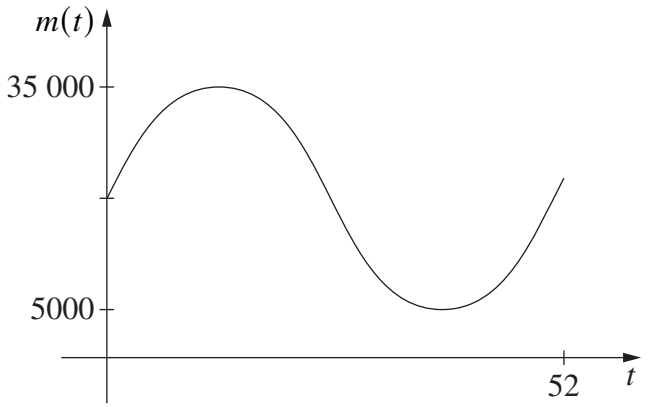
End of Question 30

Question 31 (7 marks)

The population of mice on an isolated island can be modelled by the function

$$m(t) = a\sin\left(\frac{\pi}{26}t\right) + b,$$

where t is the time in weeks and $0 \le t \le 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$. The graph of $m(t)$ is shown.



(a) What are the values of a and b? **2**

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Question 31 continues on page 37

Question 31 (continued)

(b) On the same island, the population of cats can be modelled by the function **3**

$$c(t) = -80\cos\left(\frac{\pi}{26}(t-10)\right) + 120.$$

Consider the graph of $m(t)$ and the graph of $c(t)$.

Find the values of t, $0 \le t \le 52$, for which both populations are increasing.

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(c) Find the rate of change of the mice population when the cat population reaches a maximum. **2**

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NSW Education Standards Authority

2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

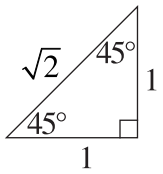
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

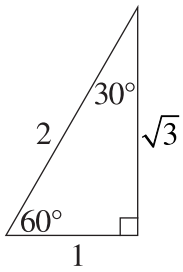
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

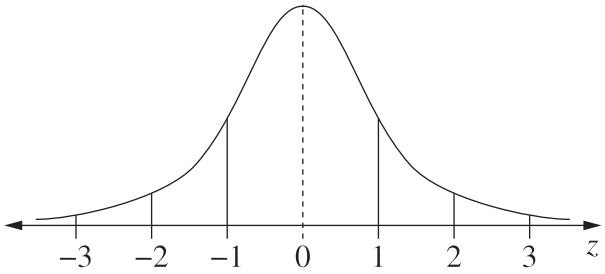
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow\ P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$